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THE CONSISTENT CRITERIA FOR GAUSSIAN HOMOGENEOUS ISOTROPIC STATISTICAL STRUCTURES

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Abstract. In this paper we prove the necessary and sufficient conditions for the existence of the consistent criteria for Gaussian strucrures.

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Here we recall some definitions [1]. Let (E, S) be a measurable space with a given family of probability measures: $\{\mu_i, i \in I\}$.

Definition 1. A statistical structure $\{E, S, \mu_i, i \in I\}$ is called orthogonal (singular) if μ_i and μ_j are orthogonal for each $i \neq j$.

Definition 2. A statistical structure $\{E, S, \mu_i, i \in I\}$ is called strongly separable if there exists a disjoint family of S-measurable sets $\{X_i, i \in I\}$ such that the relations are fulfilled $\mu_i (X_i) = 1, \quad \forall i \in I$.

Let $|\mathrm{H}|$ be the set of hypotheses and let $\mathrm{B}(|\mathrm{H}|)$ be σ-algebra of subsets of $|\mathrm{H}|$ which contains all finite subsets of $|\mathrm{H}|$.

Definition 3. A statistical structure $\{E, S, \mu_H, H \in |\mathrm{H}|\}$ is said to admit a consistent criteria of hypotheses if there exists at least one measurable map $\delta : (E, S) \to (|\mathrm{H}|, \mathrm{B}(|\mathrm{H}|))$, such that $\mu_H (x : \delta(x) = H) = 1, \forall H \in |\mathrm{H}|$.

Let M^σ be a linear space of all finite measures with alternating signs on S [2].

Definition 4. A linear subset of measures $M_H \subset M^\sigma$ is said to be a Hilbert space of measures if:

1. One can introduce space on M_H a scalar product $\langle \mu, \nu \rangle$, $\mu, \nu \in M_H$ such that M_H is the Hilbert space and for every mutually singular measures μ and ν, $\mu, \nu \in M_H$ is the scalar product $\langle \mu, \nu \rangle = 0$;

2. If $\nu \in M_H$ and $|f| \leq 1$, then $\nu_f (A) = \int_A f(x) \nu(dx) \in M_H$ and $\langle \nu_f, \nu_f \rangle \leq \langle \nu, \nu \rangle$.

Let $\xi(t_1, t_2, ..., t_n), \quad (t_1, t_2, ..., t_n) \in D$, (let D be a closed bounded Doman in R^n) be a Gaussian homogeneous random field with the same correlation functions and different mean values $H(t_1, t_2, ..., t_n), \quad (t_1, t_2, ..., t_n) \in D$, let $\mu_H, \quad H \in |H|$ be corresponding probability measures given on S and let $f_H(\lambda_1, \lambda_2, ..., \lambda_n), \quad H \in |H|$ be spectral densities of these fields. Let $\text{supvrai}\, f_H(\lambda_1, \lambda_2, ..., \lambda_n) = c < +\infty, \quad \forall H \in |H|$.

Theorem 1. *The Gaussian homogenous statistical structure* $\{E, S, \mu_{H_i}, i \in N\}, N = \{1, 2, ..., n, ..\}$ *admits a consistent criterion* δ *of a Hypotheses if and only if this formal*

$$\int_{-\infty}^{+\infty} ... \int_{-\infty}^{+\infty} |H_K(t_1, t_2, ..., t_n)|^2 \, dt_1 ... dt_n = +\infty \tag{1}$$

or

$$\int_{-\infty}^{+\infty} ... \int_{-\infty}^{+\infty} \frac{\left|\tilde{H}_K(\lambda_1, \lambda_2, ..., \lambda_n)\right|^2}{f_{H_K}(\lambda_1, \lambda_2, ..., \lambda_n)} d\lambda_1 ... d\lambda_n = +\infty \tag{2}$$

for all $k \in N$, *where* $\tilde{H}_k(\lambda_1, ..., \lambda_n) = \frac{1}{(2\pi)^n} \int_{-\infty}^{+\infty} ... \int_{-\infty}^{+\infty} e^{i(\lambda_1 t_1 + ... + \lambda_n t_n)} H_k(t_1 ... t_n)\, dt_1 ... dt_n$.

Necessity. Since a statistical structure $\{E, S, \mu_{H_k}, k \in N\}$ admits a consistent criterion of hypotheses, then exists measurable map $\delta : (E, S) \to (|\mathrm{H}|, \mathrm{B}(|\mathrm{H}|))$, such that $\mu_{H_k}(x : \delta(x) = H_k) = 1$, $\forall k \in N$, then it is obvious, that $\mu_{H_k}(X_k) = 1$, $\forall k \in N$. Therefore, a statistical structure $\{E, S, \mu_{H_k}, k \in N\}$ is orthogonal and as formula (1) or formula (2) is fulfilled.

Sufficiency. A statistical structure $\{E, S, \mu_{H_k}, k \in N\}$ is orthogonal, so $cardN = \chi_0$, the statistical structure is strongly separable then (see theorem 3 [1]) admits a consistent criterion of Hypotheses. Theorem 1 is proved.

Theorem 2. *Let* $M_H = \bigoplus_{i \in N} M_H(\mu_{H_i})$ *be a Hilbert space of measures. The Gaussian Homogeneous isotropic orthogonal statistical structure* $\{E, S, \mu_{H_k}, k \in N\}$ *admits a consistent criteria of Hypotheses if and only if the correspondence* $f \to \psi_f$, *given by the equality* $\int f(x)\, \nu(dx) = \langle \psi_f, \nu \rangle$, $\forall \nu \in M_H$ *be one-to-one* ($f \in F$, *where* f *is the set of those* f *for which* $\int f(x)\, \nu(dx)$ *is defined* $\forall \nu \in M_H$.

Proof. Necessity. Since the statistical structure $\{E, S, \mu_{H_k}, k \in N\}$ admits a consistent criterion of Hypotheses and this family is strongly separable, there exist S-measurable sets $X_i, i \in N$, such that

$$\mu_{H_i}(X_j) = \begin{cases} 1, & \text{if } i = j, \\ 0, & \text{if } i \neq j. \end{cases}$$

Let the function $\mathrm{I}_{X_i}(x) \in F$ be correspond with $\mu_{H_i} \in M_H(\mu_{H_i})$. Then $\int \mathrm{I}_{X_i}(x)\, \mu_{H_i}(dx) = \int \mathrm{I}_{X_i}(x) \cdot \mathrm{I}_{X_i}(x)\, \mu_{H_i}(dx) = \langle \mu_{H_i}, \mu_{H_i} \rangle$. Let the function $f_\psi(x) = f_1(x)\, \mathrm{I}_{X_i}(x)$ be corresponded with $\psi_1 \in M_H(\mu_{H_i})$. Then any $\psi_2 \in M_H(\mu_{H_i})$ $\int f_\psi(x)\, \psi_2(x)\, \mu_{H_i}(dx) = \int f_\psi(x) \cdot f_2(x)\, \mathrm{I}_{X_i}(x)\, \mu_{H_i}(dx) = \int f_\psi(x)\, f_2(x)\, \mu_{H_i}(dx) = \langle \psi_1, \psi_2 \rangle$. Let the function $f(x) = \sum_{i \in N} g_i(x)\, \mathrm{I}_{X_i}(x) \in F$ be corresponded with the measure $\nu \in M_H$, $\nu = \sum_{i \in N} \int g_i(x)\, \mu_{H_i}(dx)$ then $\nu_1(B) = \sum_{i \in N} \int_B g_i^1(x)\, \mu_{H_i}(dx)$, have $\int f(x)\, \nu_1(dx) = \langle \nu_1, \nu \rangle$. so the necessity is proved.

Sufficiency. Let $f \in F$ be corresponded with $\nu_f \in M_H$ for which $\int f(x)\, \nu(dx) = \langle \nu_f, \nu \rangle$, then $\psi_1, \psi_2 \in M_H(\mu_{H_i})$ we have $\int f_{\psi_1}(x)\, \psi_2(dx) = \langle \psi_1, \psi_2 \rangle = \int f_1(x)\, f_2(x)\, \mu_{H_i}(dx)$. So $f_{\psi_1} = f_1$ for a. e. M_{H_i} measures and $f_{H_i}(x) > 0$, $\int f_{H_i}^2(x)\, \mu_{H_i}(dx) < +\infty$, $\mu_{H_i}^* = \int f_{H_i}(x)\, \mu_{H_i}(dx)$, then $\langle \mu_{H_i}, \mu_{H_j} \rangle = 0 \quad \forall i \neq j$, so the statistical structure $\{E, S, \mu_{H_i}, i \in N\}$ is weakly separable, so $cardN = \chi_0$, so the statistical structure $\{E, S, \mu_{H_i}, i \in N\}$ is strongly separable, then (see Theorem 3 [1]) admits a consistent criterion of Hypotheses. The Theorem 2 is proved.

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R E F E R E N C E S

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