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STRAIN CONTROL OF THE ELASTIC INFINITE BODIES WITH ELLIPTIC HOLE AND CRACKS BY MEANS OF BOUNDARY CONDITIONS VARIATION

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Abstract. A two-dimensional boundary value problem of elastic equilibrium of a plane-deformed infinite body with an elliptic opening is studied. A part of the opening is fixed and from some points of the unfixed part of the cylindrical boundary there come curvilinear finite cracks. The problem is to find conditions for the fixed parts of the opening so that the damage caused by the crack, i.e. stresses on its surface, should be minimal. We should note that the crack ends inside the body are curved. The curve radii vary similar to boundary conditions.

Keywords and phrases: Boundary element method, fictitious load, displacement discontinuity, crack, hole.

AMS subject classification: 65N38, 74B05, 74S15.

1. Introduction. To research of cracks in an elastic body many works are devoted. Among them works [1],[2],[3] which are to some extent close to our work, but don't coincide with it.

In the above-considered work the two-dimensional boundary value problem of elasticity about elastic equilibrium of an infinite body with an elliptic opening is investigated. A part of the opening is fixed and from some points of the unfixed part of the cylindrical boundary there come curvilinear finite cracks. The problem is to find conditions for the fixed parts of the opening so that the damage caused by the crack, i.e. stresses on its surface, should be minimal. We should note that the crack ends inside the body are curved. The curve radii vary similar to boundary conditions.

The solution of a problem can be used for constructions of various buildings, and in particular, underground buildings. The problem is solved by a boundary element method [4].

2. Equilibrium equations, physical equations and statement of boundary value problems in elliptic system of coordinates. The equilibrium equations of the state of plane deformation, when absence of mass forces, in elliptic system of coordinates $\xi, \eta (0 \leq \xi < \infty,\ \ 0 \leq \eta < 2\pi)$ have the following form [5]:

$$\begin{aligned} &\frac{\partial D}{\partial \xi} - \frac{\partial K}{\partial \eta} = 0, \qquad && \frac{\partial \bar{u}}{\partial \xi} + \frac{\partial \bar{v}}{\partial \eta} = \frac{\kappa - 2}{\kappa \mu} h_0^2 D, \\ &\frac{\partial D}{\partial \eta} + \frac{\partial K}{\partial \xi} = 0, && \frac{\partial \bar{v}}{\partial \xi} - \frac{\partial \bar{u}}{\partial \eta} = \frac{1}{\mu} h_0^2 K, \end{aligned} \tag{1}$$

where $\kappa = 4\,(1-\nu)\,,\ \ \mu = \frac{E}{2(1-\nu)},\ h_0 = \sqrt{\cosh{(2\xi)} - \cos{(2\eta)}},\ \bar{v} = \frac{2v}{c^2};\ u$ and $-v$ are components of the displacement vector $\vec{U},\ \frac{\kappa-2}{\kappa\mu} h_0^2 D$ - is divergence of the displacement vector, $\frac{1}{\mu}h_0^2K$ is rotor of the displacement vector, ν is Poisson's coefficient, E is the elasticity module.

At infinity the considered elastic body $\tilde{\Omega} = \{\xi_1 < \xi < \infty,\ \ 0 \le \eta < 2\pi\}$ stretches along coordinate y with stress $Y_y = P$ (see Fig.1a).

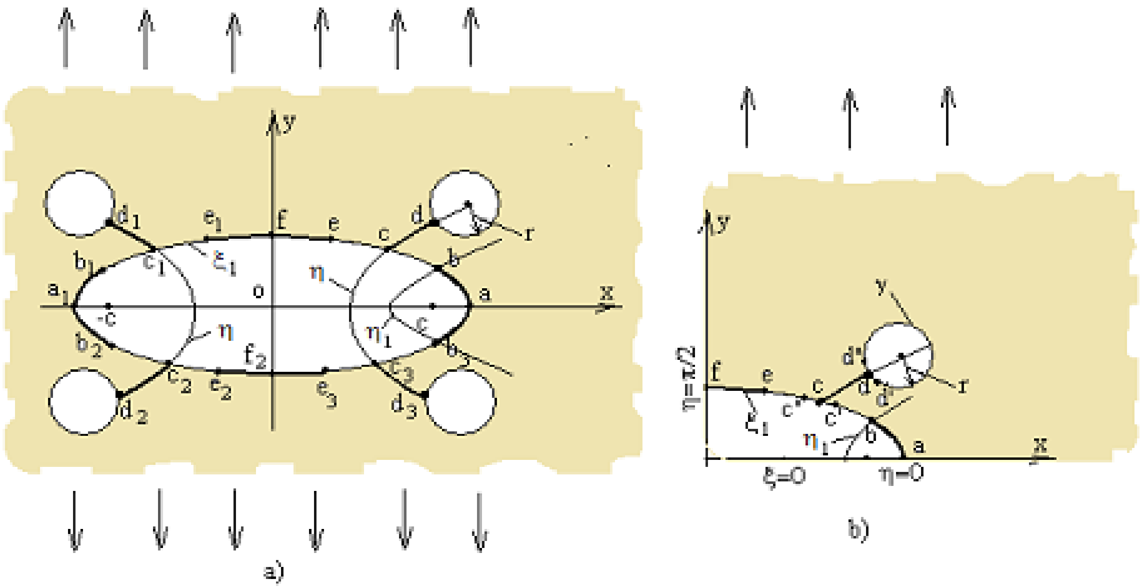

Fig. 1. Considered elastic area and its quarter

Stresses through displacements are expressed by the following equalities:

$$
\begin{aligned}
\frac{h_0^2}{2\mu}\sigma_{\xi\xi} &= \frac{h_0^2}{2\mu}D - \frac{\partial \bar{u}}{\partial \eta} - \frac{1}{h_0^2}\left[\sinh(2\xi)\,\bar{u} - \sin(2\eta)\,\bar{v}\right], \\
\frac{h_0^2}{2\mu}\sigma_{\eta\eta} &= \frac{h_0^2}{2\mu}D - \frac{\partial \bar{u}}{\partial \xi} + \frac{1}{h_0^2}\left[\sinh(2\xi)\,\bar{u} - \sin(2\eta)\,\bar{v}\right], \\
\frac{h_0^2}{2\mu}\tau_{\xi\eta} &= \frac{h_0^2}{2\mu}K + \frac{\partial \bar{u}}{\partial \eta} - \frac{1}{h_0^2}\left[\sin(2\eta)\,\bar{u} + \sinh(2\xi)\,\bar{v}\right].
\end{aligned}
\tag{2}
$$

Here $\sigma_{\xi\xi}$, $\sigma_{\eta\eta}$ are normal stresses, $\tau_{\xi\eta}$ is a tangential stress.

Based on the principle of symmetry the final statement of a problem will be realized for quarter areas $\tilde{\Omega}$, i.e. for area $\Omega = \left\{\xi_1 < \xi < \infty, 0 < \eta < \frac{\pi}{2}\right\}$

Before final set up a problem, we will write out boundary conditions on a contour of the quarter, represented in Fig.1b.

On ab and ef arches

$$
\begin{aligned}
&a)\ \ u = 0,\ \ v = 0\ \ \text{or}\ \ b)\ \ u = 0,\ \ \tau_{\xi\eta} = 0,\ \ \text{or}\ \ b')\ \ u = 0,\ \ K = 0,\ \ \text{or} \\
&c)\ \ \sigma_{\xi\xi} = 0,\ \ v = 0,\ \ \text{or}\ \ c')\ \ D = 0,\ \ v = 0,\ \ \text{or}\ \ d)\ \ \sigma_{\xi\xi} = 0,\ \ \tau_{\xi\eta} = 0.
\end{aligned}
\tag{3}
$$

On be arc

$$\sigma_{\xi\xi} = 0, \quad \tau_{\xi\eta} = 0. \tag{4}$$

On cd segment of crack

$$\sigma_{\eta\eta} = 0, \quad \tau_{\xi\eta} = 0. \tag{5}$$

On the circle γ passing through a point d:

$$\sigma_n = 0, \quad \tau_n = 0, \tag{6}$$

where σ_n and τ_n-normal and tangential stresses in a point of a circle γ with a normal n.

When $\eta = 0$ and $\eta = \dfrac{\pi}{2}$:

$$v = 0, \quad \tau_{\xi\eta} = 0 \quad \Leftrightarrow \quad v = 0, \quad \frac{\partial u}{\partial \eta} = 0. \tag{7}$$

The aim of this work is to study the stress-strain state quarter under various boundary conditions on the arcs ab and ef and for different values of the radius η of the circle γ (see Figure 1).

3. The solution of boundary value problems and discussion of the received results. First of all, we note that incomplete boundary value problem (1), (2), (4), (5), (6), (7) in the future will be denoted by A, for example, the boundary value problem (1), (2) (3a), (4), (5), (6), (7) is denoted by A3a, the boundary value problem (1), (2), (3b), (4), (5), (6), (7) through A3b etc.

Below two graphs are shown in Fig. 2 and Fig. 3. Both graphs reproduced in the case where $\xi_1 = 100$cm, $\eta_1 = \frac{\pi}{8}$, cd $= 40$cm, $P = 10$ kg$/cm^2$, $E = 2 \cdot 10^6$kg$/$cm^2, $\nu = 0,3$.

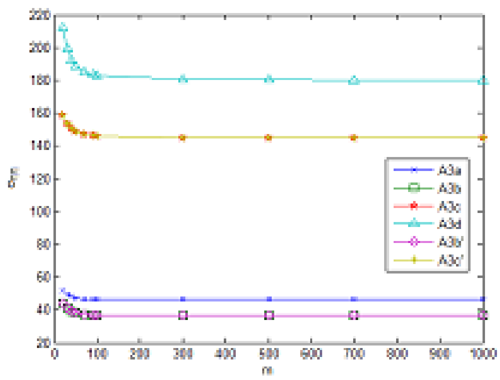

Fig. 2. Tensile stresses in the neighborhood of point c under various conditions on the arcs ab and ef, and various values of the radius $r = \frac{\xi_1}{m}$ ($m = 20 - 1000$, $\xi_1 = 100$cm, cd $= 40$cm)

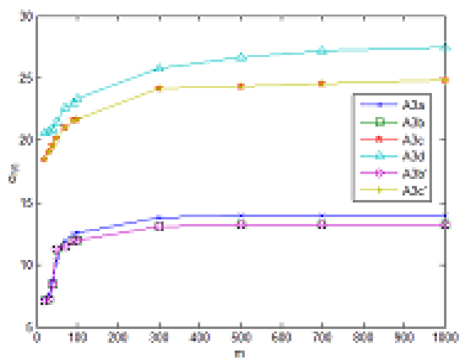

Fig. 3. Tensile stresses in the neighborhood of point d under various conditions on the arcs ab and ef, and various values of the radius $r = \frac{\xi_1}{m}$ ($m = 20 - 1000$, $\xi_1 = 100$cm, cd $= 40$cm)

Both graphs are built on material obtained after the decision of tasks boundary element method, and in particular, the fictitious load method and displacement discontinuity method[4].

4. Conclusion. Comparing the graphs of Fig. 2 and Fig. 3 we can say that in all cases the boundary conditions (3) and the radius r in Fig. 2 is observed a decrease values of $\sigma_{\eta\eta}$ with increasing values m with subsequent stabilization of this stress, and Fig. 3 picture is reversed, i.e. values of $\sigma_{\eta\eta}$ increase monotonically.

Based on the above-said, we will continue to be guided only by the graphs in Fig. 3. As for the main purpose of our work, i.e. finding such conditions on the arcs ab and ef to stress $\sigma_{\eta\eta}$ would be minimal in the neighborhood of point d, we can say the following.

Most expedient to put on the arcs ab and ef conditions ($3b$) (the first curve from below in Fig. 3) corresponding to the boundary value problem A3b, although approximately the same values of $\sigma_{\eta\eta}$ are observed under the conditions ($3a$), ($3b'$) and ($3c'$) (the second curve from below in Fig. 3), corresponding to boundary value problems A3a, A3b′, A3c′ respectively. With regard to the third curve from below in Fig. 3 (task A3d), the higher values corresponding to it would be expected, although not very clear values of $\sigma_{\eta\eta}$ corresponding to the first curve from above in Fig. 3 (task A3c). The behavior of stress $\sigma_{\eta\eta}$ in the neighborhood of point d depending on the radius r, as already mentioned, is as follows: for all boundary conditions (3) with increasing of m stress is increasing.

It is easy to see that conditions ($3b$) and ($3b'$), and also conditions ($3c$) and ($3c'$) are respectively similar, though such fixing of border of an elastic body when conditions ($3b$) or ($3c$) technically unrealizable whereas as it is specified in work [6], fixing of the borders of a body according to conditions ($3b'$) and ($3c'$) are realizable.

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