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CONSTRUCTION OF APPROXIMATE SOLUTIONS OF SOME PLANE PROBLEMS OF THERMOELASTICITY FOR TRANSVERSAL ISOTROPIC BODIES

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Abstract. In this work the algorithm of the approximate solution of two-dimensional boundary value problems of thermoelasticity is offered for transversal isotropic body. The offered algorithm is based on use of representation of the general solution of system of the equations of balance by means of harmonic functions.

Keywords and phrases: Transversal isotropic body, two-dimensional boundary value problems of thermoelasticity, approximate solution.

AMS subject classification: 74K25, 74B20.

1. Main equations. Let $Oxyz$ be a rectangular Cartesian coordinate system. Let there be a case of the plane deformed state parallel to the plane Oxy for the uniform transversal isotropic thermoelastic body. If the plane of an isotropy is parallel to the plane Oxy, then the uniform system of the equations of static balance in displacements has the form [1]

$$\begin{aligned} &\mu\Delta u+\frac{1}{2}\frac{E_1E_2}{(1-\nu_1)E_2-2\nu_2^2E_1}\partial_x(\partial_x u+\partial_y v)-\beta\partial_x T=0,\\ &\mu\Delta v+\frac{1}{2}\frac{E_1E_2}{(1-\nu_1)E_2-2\nu_2^2E_1}\partial_y(\partial_x u+\partial_y v)-\beta\partial_y T=0, \end{aligned} \tag{1}$$

where $\partial_x\equiv\dfrac{\partial}{\partial x}$, $\partial_y\equiv\dfrac{\partial}{\partial y}$, $\Delta=\partial_{xx}+\partial_{yy}$; μ shear modulus $\mu=\dfrac{E_1}{2(1-\nu_1)}$, ν_1, E_1 and ν_2, E_2 Poisson's coefficients and Young's modules in the plane of an isotropy and in the direction of perpendicular to it, respectively; u and v are components of displacement vector; β coefficient which depends on thermal properties of a material $\beta=\dfrac{E_1E_2(\alpha_1+\nu_2\alpha_2)}{(1-\nu_1)E_2-2\nu_2^2E_1}$, α_1,α_2 coefficients of linear thermal expansion in the plane of an isotropy and in the direction of perpendicular to it, respectively; T temperature change which satisfies to the equation

$$\Delta T=0. \tag{2}$$

Duhamel-Neumann law, connecting stresses and displacements has the form [1]

$$\begin{aligned} &\sigma_{xx}=\frac{2\mu}{(1-\nu_1)E_2-2\nu_2^2E_1}[(E_2-\nu_2^2E_1)\partial_x u+(\nu_1E_2+\nu_2^2E_1)\partial_y v]-\beta T,\\ &\sigma_{yy}=\frac{2\mu}{(1-\nu_1)E_2-2\nu_2^2E_1}[(E_2-\nu_2^2E_1)\partial_y v+(\nu_1E_2+\nu_2^2E_1)\partial_x u]-\beta T,\\ &\sigma_{xy}=\sigma_{yx}=\mu(\partial_x v+\partial_y u),\\ &\sigma_{zz}=\frac{\nu_2E_1E_2}{(1-\nu_1)E_2-2\nu_2^2E_1}[\partial_x u+\partial_y v]-\beta T, \end{aligned} \tag{3}$$

where $\sigma_{xx}, \sigma_{yy}, \sigma_{xy}, \sigma_{zz}$ are components of a tensor of stresses. Other components of a tensor of stresses in case of plane deformation are equal to zero.

2. The general solution of system (1). It is possible to show that the general solution of system of balance (1) is represented by means of three harmonic functions by means of Kolosov-Muskhelishvili formulas [2],[3]. We will give these representations without conclusion details.

Representation of displacements

$$\begin{aligned} 2\mu u &= \frac{c+2\mu}{2c}\varphi^* + \frac{1}{2}(y\partial_y\varphi^* - x\partial_y\tilde{\varphi}) + \partial_y\psi + \frac{\mu\beta}{c}T^*, \\ 2\mu v &= \frac{c+2\mu}{2c}\tilde{\varphi} + \frac{1}{2}(x\partial_x\tilde{\varphi} - y\partial_x\varphi^*) - \partial_x\psi + \frac{\mu\beta}{c}\tilde{T}, \end{aligned} \tag{4}$$

where φ^* and $\tilde{\varphi}$ any adjoint harmonic functions ($\partial_x\varphi^* = \partial_y\tilde{\varphi} = \varphi, \partial_y\varphi^* = -\partial_x\tilde{\varphi}$), ψ is any harmonic function; T^* and $\tilde{T}$ are adjoint harmonic functions ($\partial_x T^* = \partial_y\tilde{T} = T, \partial_y T^* = -\partial_x\tilde{T}$); $c = \dfrac{1}{2}\dfrac{E_1E_2}{(1-\nu_1)E_2 - 2\nu_2^2E_1}$.

Substituting formulas (4) in (3), the following expressions for stresses turn out

$$\begin{aligned} \sigma_{xx} &= \varphi + \frac{1}{2}(y\partial_{xy}\varphi^* - x\partial_{xy}\tilde{\varphi}) + \partial_{xy}\psi, \\ \sigma_{yy} &= \varphi - \frac{1}{2}(y\partial_{xy}\varphi^* - x\partial_{xy}\tilde{\varphi}) - \partial_{xy}\psi, \\ \sigma_{xy} &= \frac{1}{2}(y\partial_{yy}\varphi^* + x\partial_{xx}\tilde{\varphi}) + \partial_{yy}\psi, \\ \sigma_{zz} &= 2\nu_2\varphi - (1-2\nu_2)\beta T. \end{aligned} \tag{5}$$

3. Construction of approximate solutions of boundary value problems. Formulas (4) and (5) can be used for creation of approximate solutions of classical and nonclassical problems of thermoelasticity. In case of a finite simply connected domain Ω of function T^*, $\tilde{T}$, φ^*, $\tilde{\varphi}$ and ψ are also representated in the form

$$\begin{aligned} (T^*, \varphi^*) &= \sum_{j=1}^{N}(T_j, A_j)[\ln\sqrt{(x-\xi_j)^2 + (y-\eta_j)^2} - f(x-\xi_j, y-\eta_j)], \\ (\tilde{T}, \tilde{\varphi}) &= \sum_{j=1}^{N}(T_j, A_j)[\ln\sqrt{(x-\xi_j)^2 + (y-\eta_j)^2} + f(x-\xi_j, y-\eta_j)], \\ \psi(x,y) &= \sum_{j=1}^{N} B_j \ln\sqrt{(x-\xi_j)^2 + (y-\eta_j)^2}, \end{aligned} \tag{6}$$

where

$$f(x-\xi_j, y-\eta_j) = \begin{cases} \arctan\dfrac{y-\eta_j}{x-\xi_j}, & x > \xi_j, \\ \arctan\dfrac{y-\eta_j}{x-\xi_j} + \pi, & x < \xi_j, \quad y \geq \eta_j, \\ \arctan\dfrac{y-\eta_j}{x-\xi_j} - \pi, & x < \xi_j, \quad y < \eta_j, \\ \dfrac{\pi}{2}, & x = \xi_j, \quad y \geq \eta_j, \\ -\dfrac{\pi}{2}, & x = \xi_j, \quad y < \eta_j. \end{cases}$$

Points (ξ_j, η_j) are located on a contour, covering domain Ω.

Further the method of fundamental solutions is applied [4]. If the classical boundary value problem is solved, then on the boundary of domain Ω with N points is marked (Fig. 1). On noted points using representations (4), (5) and transformation formulas, meet the boundary conditions. As a result the system of the linear algebraic equations of strictly required coefficients of T_j, A_j, B_j turns out. As a static case consider, the solution of the received system which is reduced to the solution of two independent systems.The first of them will represent system of the equations with unknowns $T_1, T_2, ..., T_N$, and the second of them will represent system of the equations with unknowns $A_1, A_2, ..., A_N, B_1, B_2, ..., B_N$. After the solution of the specified systems the received values of coefficients are brought in formulas (6). As a result of substitution of the constructed approximations of harmonic functions into formulas (4)and (5), we will have an approximate solution of the boundary value problem.

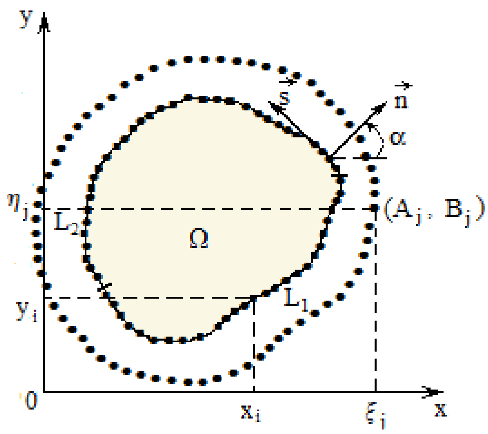

Fig. 1. Considered domain of Ω

Let the infinite multiply connected domains area which is limited to simple contours of F be considered now, last of which covers the others. Harmonic functions T^* and $\tilde{T}$ are represented in the form

$$T^* = \sum_{k=1}^{m} C_k[x \ln \sqrt{(x - x_{(k)})^2 + (y - y_{(k)})^2} - yF(x - x_{(k)}, y - y_{(k)})]$$

$$+ \sum_{k=1}^{m} [\alpha_k \ln \sqrt{(x - x_{(k)})^2 + (y - y_{(k)})^2} - \beta_k F(x - x_{(k)}, y - y_{(k)})]$$

$$+ \sum_{j=1}^{N} T_j \frac{x - x_j}{(x - \xi_j)^2 + (y - \eta_j)^2},$$

$$\tilde{T} = \sum_{k=1}^{m} C_k[x \ln \sqrt{(x - x_{(k)})^2 + (y - y_{(k)})^2} + yF(x - x_{(k)}, y - y_{(k)})]$$

$$+ \sum_{k=1}^{m} [\alpha_k \ln \sqrt{(x - x_{(k)})^2 + (y - y_{(k)})^2} - \beta_k F(x - x_{(k)}, y - y_{(k)})]$$

$$-\sum_{j=1}^{N} T_j \frac{x - x_j}{(x-\xi_j)^2 + (y-\eta_j)^2},$$

where points of $(x_{(k)}, y_{(k)})$ are in contours of $L_k, k = 1, 2, ..., m$;

$$F(x - x_{(k)}, y - y_{(k)}) = Arg((x - x_{(k)}) + i(y - y_{(k)})).$$

Functions φ^*, $\tilde{\varphi}$ and ψ are represented respectively also, and further construction of approximate solutions of problems is carried out similar to a case of a simply connected body, but the condition of unambiguity of displacement is considered.

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