Reports of Enlarged Session of the
Seminar of I. Vekua Institute
of Applied Mathematics
Volume 28, 2014

ON ONE MATHEMATICAL MODEL OF OIL PENETRATION INTO NON-HOMOGENEOUS SOIL

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Abstract. In the present article a mathematical model of the accidental spilled liquid's penetration into the soil having non-homogeneous structure in the vertical direction is discussed. The mathematical model is based on the integration of the non-linear and non-stationary systems of the hydrodynamic equations. The numerical model is taking into consideration the spilled liquid's evaporation process, the main characteristic parameters of soil and some physical-chemical processes characterizing non-stationary processes in the soil. Some results of numerical calculations are presented.

Keywords and phrases: Mathematical modeling, liquid penetration, non-homogeneous soil.

AMS subject classification: 65M12, 35Q80, 65L07.

1. Introduction. Let us consider a case when there is an accidental oil spill on the horizontal located soil surface heaving non-homogeneous structure to the vertical direction i.e. we assume (suppose) that oil penetrates into soil which is constructed by horizontal homogeneous but to the vertical direction non-homogeneous different kinds of layers $\Omega_i, i = 1, n$ characterized by different physical parameters $\Omega_i(\rho_i, K_i, \sigma_i, h_i)$, $(i = 1, n)$, where ρ_i are soil density, K_i are filtration coefficients, σ_i are soil porosities and h_i are height of layers. If we direct axis Ox along the earth (soil) surface and axes Oz vertically down then, process of oil infiltration into different kinds of layers of the complex soil can be described by the following nonstationary nonlinear parabolic type equations [1, 2]

$$\frac{\partial S_i}{\partial t} + K_i \frac{\partial S_i^n}{\partial z} = D_i \frac{\partial^2 S^k}{\partial x^2} + D_i \frac{\partial^2 S^k}{\partial z^2}, \quad i = 1, n, \tag{1}$$

where t-is time, x and z-are coordinate axes (line Ox is located on the surface of soil and an axis Oz is directed vertically downwards, $S_i = W_i - W_0$ are concentrations of soil by oil (i.e. saturation of unit volumes of different layers of soil by oil, W_i-are saturations of the soil by liquid (oil), W_0 is a comparative volume related with water, n and k are describing degree of nonlinear character of filtration and diffusion processes, consequently. K_i and D_i are filtration and diffusion coefficients, of the soil layers i, consequently, following the studies [1, 3], K_i and D_i can be described by the following expressions:

$$K_i = \frac{-KK}{(\sigma_i - W_0)^n} \cdot \frac{y_W}{y_{oil}}, \qquad D_i = \frac{KK\alpha_i}{(n+m+1)(\sigma_i - W_0)^m},$$

where KK is a coefficients of water filtration in the i layer of soil, σ_i are the porosities of the soil in the i layer of soil, $\alpha_i = \dfrac{P_0 y_W}{(\sigma_i - W_0) y_{oil}}$, y_W, y_{oil} are kinematic coefficients of viscosity of water and oil, respectively, P_0 is a liquid pressure at full saturation of vapors, i.e. when $W = \sigma$. We suppose that above the soil the accidental spilled oil has rectangular form with height H and its value is changed by the evaporation and infiltration processes which we are described by the following expression:

$$H(t) = h_0 \left(1 - \frac{t}{t_0} e^{-(t_0 - t)}\right), \quad 0 \le t \le t_0, \tag{2}$$

where h_0 is the initial height of the rectangular form spilled oil, t_0 is the final moment of time, when the spilled oil is fully disappeared from the earth surface. The value of t_0 is defined by the data of experimental investigations and its value is dependent on the kind of oil, volume of oil, meteorological conditions, kind of soil, etc. In this article for clearness, evidence and simplicity of the further discussions we are limited by the case when soil contains only two layers (Fig.1).

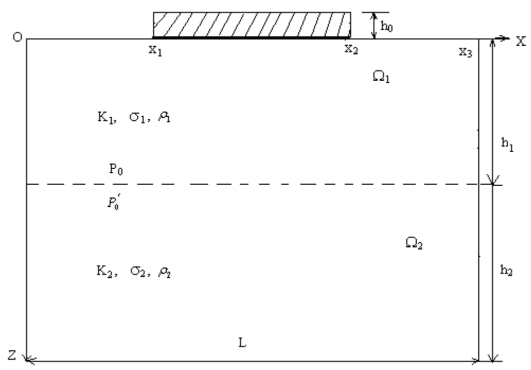

Fig. 1. Chart of the soil constructed by two different kinds of layers with spilt oil

An accidental oil spill on the horizontal soil constructed by the two different kinds of layers.

So equation (1) is solved in the rectangular area $\Omega_i, i = 1, 2$ with the following initial conditions:

$$S_i(O, x, z) = S_i^0(x, z), \ \text{ at } \ t = 0, i = 1, 2. \tag{3}$$

Let us consider the boundary conditions for the areas Ω_1 and Ω_2 separately. Following the denotations marked in Fig. 1 the boundary conditions for the areas Ω_1 and Ω_2 are as follows: on the upper border $z = 0$, of the area Ω_1 we have

$$\begin{aligned} &S(t,x,0) = S^0(0,x,0) \approx 0, && \text{if} \quad x \in [0, x_1[\cup]x_2, x_1], \\ &S(t,x,0) = \sigma_1 - W_0, && \text{if} \quad x \in [x_1, x_2], \ \text{ and } \ H(t) > 0, \\ &S(t,x,0) = S^0(0,x,0), && \text{if} \quad x \in [x_1, x_2], \ \text{ and } \ H(t) = 0. \end{aligned} \tag{4}$$

At the inside fracture site (i.e. at the inner border line) $z = h$, we proceed from the equation of continuity, from the point of view that the movement at the border line of two, different kinds of soils is uninterrupted, unbroken. Otherwise for the nearby areas of inner borderline we have $V_1|_{x=h_1-\Delta x} \approx V_2|_{z=h_1+\Delta z}$ and $P_1|_{x=h_1-\Delta x} \approx P_2|_{z=h_1+\Delta z}$ where V_1, P_1 and V_2, P_2 are values of the velocity and pressure in the areas Ω_1 and Ω_2, consequently and Δz is small along of the axis Oz. Taking into account the above said at the inner border line $z = h$, we have

$$\left(\frac{\partial S}{\partial z}\right)_{z=h-\Delta z} + S|_{z=h_1} = S|_{z=h-\Delta z_1} + \alpha \left.\frac{\partial S}{\partial z}\right|_{z=h}. \tag{5}$$

At the lateral borders we have

$$\frac{\partial S}{\partial x}, \quad \text{when} \quad x = 0, \wedge x = L. \tag{6}$$

At the low border we have

$$\frac{\partial S}{\partial z} = 0. \tag{7}$$

Now set boundary conditions for the area Ω_2. At the upper border of the area Ω_2 we use the same boundary conditions that we have set for the lower border of the area Ω_1, i.e. we transfer that conditions to $z = h_1$

$$\left.\frac{\partial S}{\partial z}\right|_{z=h_1} = \left.\frac{\partial S}{\partial z}\right|_{z=h_1-\Delta z} \quad \text{i, e.} \quad S|_{z=h_1} = S|_{z=h_1-\Delta z} + \alpha \left.\frac{\partial S}{\partial z}\right|_{z=h_1-\Delta z}, \tag{8}$$

where α is a constant depending on the infiltration coefficient at the soil in Ω_2. At the other bounders of the area Ω_2 we have

$$\frac{\partial S}{\partial x} = 0, \quad \text{when} \quad x = 0 \quad \text{and} \quad x = L, \tag{9}$$

$$\frac{\partial S}{\partial z} = 0, \quad \text{when} \quad z = h_1 + h_2. \tag{10}$$

After (definition) of solutions of equation (1) in the areas and we are setting boundary condition at the inside line of divider of different soils. We have a new boundary condition

$$S|_{z=h_1} = \frac{1}{2}\left(S|_{z=h-\Delta z} + \alpha_1 \left.\frac{\partial S}{\partial z}\right|_{z=h_1-\Delta z} + S|_{z=h_1+\Delta z} + \alpha_2 \left.\frac{\partial S}{\partial z}\right|_{z=h_1+\Delta z}\right). \tag{11}$$

For numerical solving of problem (13)-(21) let us introduce denotations:

$x_i = ih_i,\ y_i = jh_2,\ z_i = kh_3,\ i = 0, \pm 1, \pm 2, \cdots,\ k = 0, \pm 1, \pm 2, \cdots,$

$h_1 > 0,\ h_2 > 0,\ h_3 > 0,\ t_l = t_{l-1} + t,\ l = 1, 2, \cdots, N,\ t_0 = 0,\ \tau = T/N,\ S^l_{i,j,k}$- is a net function.

Equation (1), with initial and boundary conditions (3) - (11) with respect to time is approximated by the following Adams-Beshfort scheme

$$S^{(l+1)^*} = S^{(l)} + \frac{\Delta t}{2} f^{(l)}, \quad S^{(l+1)} = S^{(l)} + \Delta t \left[\frac{3}{2} f^{(l+1)*} - \frac{1}{2} f^{(l)}\right], \tag{12}$$

(12) is a scheme of the second order accuracy with respect to time. With respect to space (1) is approximated according to Schumann scheme, which also is of the second order accuracy. We have carried out some numerical experiments connected with spilled on the ground oil penetration into nonhomogeneous soil composite by two different kinds of soils (sandy and brown soils) with σ_1 and σ_2 porosities, consequently. Fig.1 shows oil penetration into non-homogeneous soil composite by sandy and brown soils.

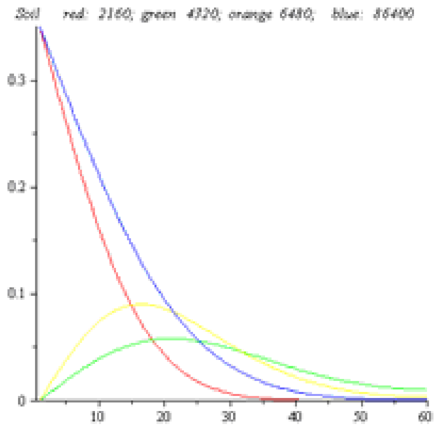

Fig. 2. An accidental spilt oil penetration into the soil with two different kinds of layers

The numerical calculation showed, that the process of oil infiltration in all layers of the considered soils proceeds qualitatively equally i.e. in all considered soils it is possible to distinguish a stage of absorption of the oil in the soil and a stage of distribution of the oil to depth and width of soils. Fig. 2 shows that the process of infiltration in the sandy soil is the most intensively and the maximal values of concentration at $t = 96$, 192 hours reaches on the depths $Z = 36$, 50 cm respectively. The process of infiltration in the brown soil is the least intensively, but non-local boundary conditions have proved to be a good worker.

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Received 18.05.2014; revised 19.10.2014; accepted 22.12.2014.

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