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ON ONE NONLINEAR AVERAGED INTEGRO-DIFFERENTIAL SYSTEM WITH SOURCE TERMS

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Abstract. One nonlinear averaged integro-differential system with source terms is considered. The model arises on mathematical simulation of the process of penetration of a magnetic field into a substance. Semi-discrete difference scheme is studied.

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One type nonlinear integro-differential model arises on mathematical simulation of the process of penetration of a magnetic field into a substance. This model were introduced after reduction of well known nonlinear Maxwell's differential system [1] to the integro-differential form [2]. In [3] some generalization of such type models is given. One-dimensional simple analog called by averaged integro-differential model by author describing the same physical process has the following form:

$$\begin{aligned} &\frac{\partial U}{\partial t} - a\left(\int_0^t\int_0^1\left[\left(\frac{\partial U}{\partial x}\right)^2+\left(\frac{\partial V}{\partial x}\right)^2\right]dxd\tau\right)\frac{\partial^2 U}{\partial x^2}=0,\\ &\frac{\partial V}{\partial t} - a\left(\int_0^t\int_0^1\left[\left(\frac{\partial U}{\partial x}\right)^2+\left(\frac{\partial V}{\partial x}\right)^2\right]dxd\tau\right)\frac{\partial^2 V}{\partial x^2}=0,\end{aligned} \qquad (1)$$

where $a = a(S) \geq a_0 = Const > 0$ is a given function of its argument.

The investigation and numerical resolution of the integro-differential models described in [2] and [3] are given in many works. Especially, in [2]-[9] solvability and uniqueness of the initial-boundary value problems for these type models are studied. At first the investigation of (1) type averaged equations was carried out in [7]. Asymptotic behavior of solutions as $t \to \infty$ is investigated in many works also (see, for example, [7]-[15]). Numerical resolution by finite difference scheme and finite element method is given in works [11]-[13], [16], [17] and in a number of other works as well. To investigation and approximation solution of (1) kind systems are devoted the following works [9]-[15], [17].

The aim of this note is to study approximate solution constructed by semi-discrete difference scheme for one generalization of the system of type (1) by adding monotonic, power like nonlinear terms. This system has the form:

$$\begin{aligned} &\frac{\partial U}{\partial t}-\left(1+\int_0^t\int_0^1\left[\left(\frac{\partial U}{\partial x}\right)^2+\left(\frac{\partial V}{\partial x}\right)^2\right]dxd\tau\right)^p\frac{\partial^2 U}{\partial x^2}+|U|^{q-2}U=0,\\ &\frac{\partial V}{\partial t}-\left(1+\int_0^t\int_0^1\left[\left(\frac{\partial U}{\partial x}\right)^2+\left(\frac{\partial V}{\partial x}\right)^2\right]dxd\tau\right)^p\frac{\partial^2 V}{\partial x^2}+|V|^{q-2}V=0, \end{aligned} \tag{2}$$

where $0 < p \leq 1$, $q \geq 2$.

In the $[0,1]\times[0,T]$ let us consider following initial-boundary value problem:

$$\begin{aligned} &U(0,t)=U(1,t)=V(0,t)=V(1,t)=0,\\ &U(x,0)=U_0(x),\quad V(x,0)=V_0(x), \end{aligned} \tag{3}$$

where U_0 and V_0 are given functions.

On [0,1] let us introduce a net with mesh points denoted by $x_i = ih$, $i = 0, 1, \ldots, M$, with $h = 1/M$. The boundaries are specified by $i = 0$ and $i = M$. The semi-discrete approximation at (x_i, t) is designed by $u_i = u_i(t)$ and $v_i = v_i(t)$. The exact solution to the problem at (x_i, t) is denoted by $U_i = U_i(t)$ and $V_i = V_i(t)$. At points $i = 1, 2, \ldots, M-1$, the integro-differential equation will be replaced by approximation of the space derivatives by a forward and backward differences. We will use the following known notations:

$$r_{x,i}(t)=\frac{r_{i+1}(t)-r_i(t)}{h},\quad r_{\bar{x},i}(t)=\frac{r_i(t)-r_{i-1}(t)}{h}.$$

Using usual methods of construction of discrete analogs (see, for example, [18]) let us construct the following semi-discrete scheme for problem (2),(3):

$$\begin{aligned} &\frac{du_i}{dt}-\left(1+h\sum_{i=1}^M\int_0^t\left[(u_{\bar{x},i})^2+(v_{\bar{x},i})^2\right]d\tau\right)^p u_{\bar{x}x,i}+|u_i|^{q-2}u_i=0,\\ &\frac{dv_i}{dt}-\left(1+h\sum_{i=1}^M\int_0^t\left[(u_{\bar{x},i})^2+(v_{\bar{x},i})^2\right]d\tau\right)^p v_{\bar{x}x,i}+|v_i|^{q-2}v_i=0,\\ &\qquad i=1,2,\ldots,M-1, \end{aligned} \tag{4}$$

$$u_0(t)=u_M(t)=v_0(t)=v_M(t)=0, \tag{5}$$

$$u_i(0)=U_{0,i},\quad v_i(0)=U_{0,i},\quad i=0,1,\ldots,M. \tag{6}$$

The following statement takes place.

Theorem. *If* $0 < p \leq 1$, $q \geq 2$ *and the initial-boundary value problem (2),(3) has the sufficiently smooth solution* $U = U(x,t)$, $V = V(x,t)$, *then the semi-discrete scheme (4)-(6) converges and the following estimate is true*

$$\|u(t) - U(t)\| + \|v(t) - V(t)\| \leq Ch.$$

Here $\|\cdot\|$ is a discrete analog of the norm of the space $L_2(0,1)$ and C is a positive constant independent of h.

Note that for solving the corresponding to (4) finite difference scheme we use an algorithm analogical to [12]. So, it is necessary to use Newton iterative process [19]. According to this method the great numbers of numerical experiments are carried out. These experiments agree with the theoretical result.

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