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ON FINITE-DIFFERENCE METHOD FOR APPROXIMATE SOLUTION OF ANTIPLANE PROBLEMS OF ELASTICITY THEORY FOR COMPOSITE BODIES WEAKENED BY CRACKS

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Abstract. In the present article the solution of the composite (piece-wise homogeneous) bodies weakened by cracks using finite-difference method is studied. The plane is changed by a large square and the differential equations with boundary conditions are approximated by differential analogies. Such kind of statement of the problems gives opportunity to find numerical values of the stress functions in the grid points. The corresponding algorithms are composed and realized for the concrete practical tasks. The results of theoretical and numerical investigations are in a good conformation and are presented.

Keywords and phrases: Antiplane problems, cracks, finite-difference method.

AMS subject classification: 65M06, 65N06, 65M60, 65M70.

Introduction. Study of boundary value problems for the composite bodies weakened by cracks has a great practical significance. Initial approximation for study of boundary value problems for the composite bodies weakened by cracks my be used equations of the elasticity antiplane theory for piece-wise homogeneous orthotropic plane. In the articles [1]-[7] using integral equations method are studied a very interesting cases when cracks intersect an interface or penetrate it at rectangular angle or all sorts of angle. In the present article first of all the solution of the homogeneous bodies weakened by cracks using finite-difference method is studied and then we will study the composite (piece-wise homogeneous) bodies weakened by cracks using once again finite-difference method. Also our aim is to construct rapidly convergent algorithm and schemes.

Statement of the problem. Given a distorted harmonic equation, with $2n \times 2n$ size, in square $\overline{\Omega} = \Omega_1 \cup \Omega_2$ (Ω_1 and Ω_2 areas, (see Fig. 1)

$$\frac{\partial^2 w_k(x,y)}{\partial x^2} + \lambda_k^2 \frac{\partial^2 w_k(x,y)}{\partial y^2} = 0, \quad (x,y) \in \Omega_k, \quad k = 1,2. \tag{1}$$

a) On the curves of the crack L_x^+ and L_x^- tangent stresses are given (Fig. 1) while end points of the crack coherence conditions are given

$$\tau_{yz}^{(\pm)} = b_{44}^{(k)} \frac{\partial w_k(x,\pm 0)}{\partial y} = q_k^{(\pm)}(x), \quad x \in L_k, \ \ L_1 = [0;1], \quad L_2 = [-1;0], \tag{2}$$

$$w_2(-1,+0) = w_2(-1,-0), \ \ w_1(1,+0) = w_1(1,-0); \tag{3}$$

b) on the axis y (on the dividing line) the condition of continuity is fulfilled

$$w_1(0;y) = w_2(0;y), \quad y \in [-n,n], \ y \neq 0, \tag{4}$$

$$\tau_{xz}^{(1)} = \tau_{xz}^{(2)}, \quad or \quad b_{55}^{(1)} \frac{\partial w_1(0;y)}{\partial x} = b_{55}^{(2)} \frac{\partial w_2(0;y)}{\partial x}; \tag{5}$$

c) on the side pieces of the square $\overline{\Omega}$ we have

$$\begin{aligned} &w_2(-n,y) = 0 \quad and \quad w_1(n,y) = 0, \quad y \in [-n,n], \\ &w_2(x,\pm n) = 0, \quad x \in [-n,n], \quad and \quad w_1(x,\pm n) = 0, \quad x \in [0,n]. \end{aligned} \tag{6}$$

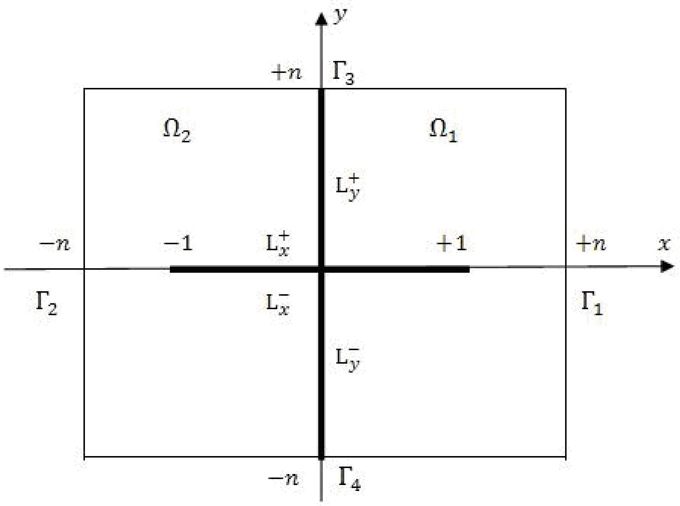

Fig. 1.

In the above mentioned equations $\lambda_k^2 = \dfrac{b_{44}^{(k)}}{b_{55}^{(k)}}$, $b_{44}^{(k)}$, $b_{55}^{(k)}$, are elastic constants, which have been taken from the Hooke's law, $q_k^{(\pm)}(x)$ is a function of Holder's class, In particular, if we have isotropic case $b_{44}^{(k)} = b_{55}^{(k)} = \mu_k$, $\lambda_k = 1$, numerical parameter $\lambda_k = 1$, where μ_k is module of displacement, $k = 1,2$;

Finite-difference method. Primarily suppose that $n, N \in \mathbb{N}$ ($2n$ is a length of the side pieces of the square, $2N$ is a number of dividing points on the crack line), $h_1 = h_2 = h = 1/N$, $N \in \mathbb{N}$. steps in the directions x and y are equal, that is there is a regular quadratic grid $\Omega_h = \{(x_i, y_i),\ x_i = ih,\ y_j = jh,\ i,j = [-nN, nN]\}$, For boundary value problem (1)-(6) a different scheme is the following: differential operator in the basic (1) equation is approximated by five point template with $O(h^2)$ accuracy, while differential operator in (2) and (5) equations are approximated by five point template with $O(h)$ accuracy $W_{k,i,j}$, For finding grid function W the following iteration method is used:

$$W_{k,i,j}^{(m+1)} = \frac{1}{2(1+\lambda_k^2)} \left[W_{k,i+1,j}^{(m)} + W_{k,i-1,j}^{(m)} + W_{k,i,j+1}^{(m)} + W_{k,i,j-1}^{(m)} \right];$$

a) everywhere, excepting crake's line and dividing line (border) we have

$$j \neq 0, \ \text{then} \ i = -(nN-1), -(nN-2), \cdots, (-1), 0, 1, \cdots, (nN-2), (nN-1),$$

$$j = 0, \ \text{then} \ i = -(nN-1), -(nN-2), \cdots, -(N+2), -(N+1),$$

$$i=(N+1),(N+2),\cdots,(nN-2),(nN-1).$$

b) On the lines of cracks

$$W_{k,i,(+0)}^{(m+1)}=W_{k,i,(+1)}^{(m)}-\frac{h}{b_{44}^{(k)}}q_{k,i}^{(+)}\ \text{ and }\ W_{k,i,(-0)}^{(m+1)}=W_{k,i,(-1)}^{(m)}-\frac{h}{b_{44}^{(k)}}q_{k,i}^{(-)},$$

$$i=-N,-(N-1),\cdots,(-1),0,1,\cdots,(N-1),N;$$

Also at the ending points of the crack it is necessary to take into consideration the following fitting condition (matched condition)

$$q_N^{(+)}=q_N^{(-)},\quad q_{(-N)}^{(+)}=q_{(-N)}^{(-)};$$

and in the point (x_0,y_0) condition of consistency

$$\frac{q_{(1)}^{(+)}(0)}{b_{44}^{(1)}}\equiv\frac{q_{(2)}^{(+)}(0)}{b_{44}^{(2)}},\quad \frac{q_{(1)}^{(-)}(0)}{b_{44}^{(1)}}\equiv\frac{q_{(2)}^{(-)}(0)}{b_{44}^{(2)}},$$

c) on the dividing line (border)

$$W_{1,0,j}^{(m+1)}=W_{2,0,j}^{(m+1)}=\frac{b_{55}^{(2)}W_{2,-1,j}^{(m)}+b_{55}^{(1)}W_{1,1,j}^{(m)}}{b_{55}^{(2)}+b_{55}^{(1)}};$$

$$j=-N,-(N-1),\cdots,(-1),1,\cdots,(N-1),N;$$

$$W_{k,i,j}^{(0)}=0,\quad m=0,1,2,\cdots.$$

Numerical experiments and results of calculations. At the initial approximation numerical calculations have been performed for homogenous bodies. Below are presented some results of calculations (see Fig. 2, Fig. 3)

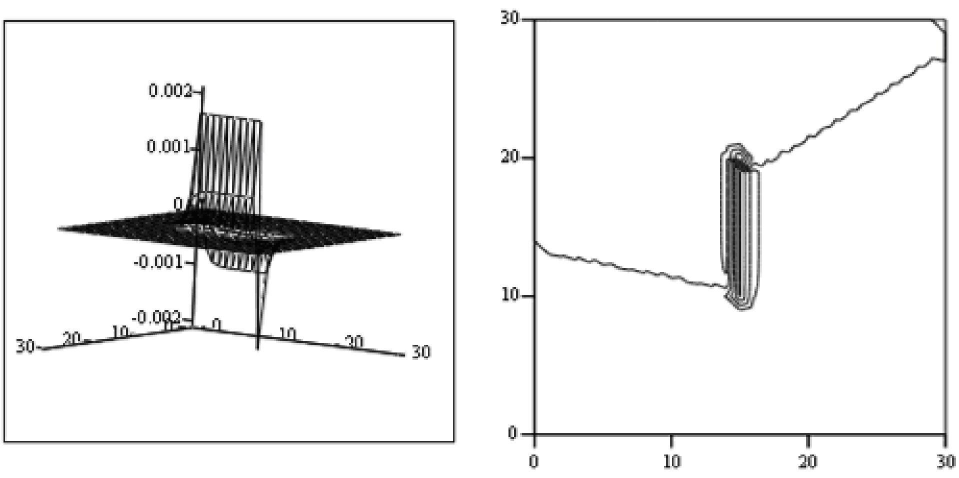

Fig. 2.

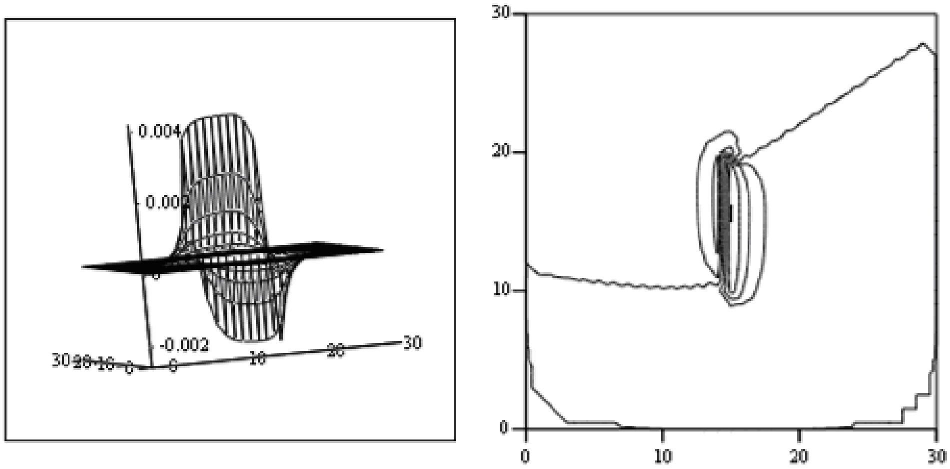

Fig. 3.

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