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# ON THE HIERARCHY OF MESOSCALE VORTEXES IN THE TURBULENT MEDIA

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*Big whorls have little whorls,*
*Which feed on their velocity;*
*Little whorls have smaller whorls,*
*And so on unto viscosity.*
L. Richardson

**Abstract**. For mediums with developed turbulence it is obtained values of universal numbers of Reynolds, Prandtl, Richardson and Peclet: $\mathrm{Re}_t = ud/\nu_t = 1,\ \mathrm{Pr}_t = \nu_t/a_t = 1,\ \mathrm{Ri}_t = 1,\ \mathrm{Pe}_t = \mathrm{Re}_t \,\mathrm{Pr}_t = 1$, which allows us to generalize Kolmogorov's classic semi-empirical theory to the case of eddy (vortex) turbulence. It is suggested original method of the hierarchy turbulent vortexes classification in neutral and electroconductive environment having irregularities of various scales from thousand km to some cm. Between them are the mesoscale vortexes, for which formulas of the spectral density functions are obtained. For the inertial and viscous intervals respective recurrent formulas are obtained. It is shown that considered mesoscale turbulent vortexes hierarchy is in a good agreement with the experimental results obtained for ocean, upper stratosphere, mesosphere, lower thermosphere, E- and F-layers of ionosphere, magnetosheath, and numerical experiments modelling the process of the turbulent vortex structure evolution.

**Keywords and phrases**: Eddy turbulence, hierarchy, mesoscale, vortex, spectral function.

**AMS subject classification**: 76V05, 76F05, 76E20.

**1. Introduction.** The study and explanation of complex turbulent processes proceeding in significant parts of the Earth's atmosphere, oceans or inner liquid core is of interest to scientists from many disciplines, in particular, magneto-thermo-hydrodynamics [1]-[4].

The renewal interest to the buoyancy subrange theories is linked with the increasing number of observations showing energy spectra versus vertical wavenumbers with spectral slopes close to 3 at scales somehow larger than the turbulent isotropic inertial ones, in the atmosphere, and in the ocean as well; but in the mesosphere, low thermosphere, ionosphere and magnetosheath of the Earth the spectral slopes is changed in more wide interval than (3, 5/3) lying between 1.2 and 7 [5]-[14].

**2. Essential aspects of vortical turbulence.**

**2.1.** According to Kolmogorov's micro-scale turbulence theory the scale values of size $\eta = (\nu^3 \varepsilon^{-1})^{1/4}$ and velocity $\upsilon = (\nu\varepsilon)^{1/4}$ at the boundary of dissipation of turbulent energy between the inertial and viscous subranges the numbers of Reynolds, Prandtl, Richardson and Peclet are equal to unit, which we name the law of four universal units of the theory of isotropic turbulence [5],

$$\mathrm{Re} = \upsilon\eta/\nu = 1,\ \ \mathrm{Pr} = \nu/a = 1,\ \ \mathrm{Ri}_c = 1,\ \ \mathrm{Pe} = \mathrm{Re}\,\mathrm{Pr} = 1, \tag{1}$$

where $\nu$ is the kinematic viscosity coefficient, $a$ is the temperature conductivity coefficient.

**2.2.** For small values of the wavenumber $k$ a hierarchy of mesoscale turbulent vortexes may be classified similarly by using turbulent viscosity and temperature conductivity instead of kinematic ones and as scale values a diameter and rotational velocity of the turbulent vortex the law of four universal units of the hierarchy of mesoscale turbulent vortexes:

$$\mathrm{Re}_t = ud/\nu_t = 1, \ \ \mathrm{Pr}_t = \nu_t/a_t = 1, \ \ \mathrm{Ri}_t = 1, \ \ \mathrm{Pe}_t = \mathrm{Re}_t\mathrm{Pr}_t = 1, \tag{2}$$

as a basis for generalization of classical Kolmogorov's theory to the vortex turbulence. This law has been obtained for ocean and sea vortexes earlier in [5]. According to Chimonas (1974) and Erukhimov et al. (1974) (see [8]): electron fluctuations spectral function $\sim k^{-3.5}$, $\nu_t = 10^4$ m$^2$s$^{-1}$, $d =$ 10-40 m, at ionospheric D- and E-layers, and to Zimmerman et al. (1972) respective data about turbulence at levels of the upper atmosphere (see [9]): $\nu_t \approx 10^2$-$10^3$ m$^2$s$^{-1}$, $u \approx 100$ ms$^{-1}$, and $d \approx$ 10-100 m give similar to law (2) results.

**3. Hierarchy of mesoscale vortexes.**

**3.1.** Using Heisenberg's formula for the viscous subrange $E_v(k) = 1/4\alpha^2\varepsilon^2\nu^4k^{-7}$ and Kolmogorov's one $E(k) = \alpha\varepsilon^{2/3}k^{-5/3}$ for the inertial subrange, one can seek a hierarchy of viscous mesoscale vortexes, the spectral function of which is geometric mean of mentioned spectral functions:

$$E(k) = \alpha\varepsilon^{2/3}k^{-5/3}, E_{vh}(k) = \frac{1}{2}\alpha^{3/2}\varepsilon^{4/3}\nu^2k^{-13/3}, \ \ E_v(k) = \frac{1}{4}\alpha^2\varepsilon^2\nu^4k^{-7}. \tag{3}$$

Experimental measurements of the power spectra in the middle and upper atmosphere show distinct groupings of the spectral type with discreet indices both for the inertial and for the viscous subranges [10-14]. This idea is also prompted independently and naturally by the well-known theoretical results. Both circumstances allow us to use them as support of introducing of the series of hierarchy of mesoscale vortexes, by the method of geometric mean of respective spectral functions, both in the inertial and the viscous subranges. After finding the first order, central mesoscale vortexes, to the right and to the left of it between the central mesoscale vortex and above mentioned asymptotic values next the second order mesoscale vortexes hierarchy is obtained, etc.

**3.2.** For the inertial subrange, the indices of the power spectra of the kinetic energy density of the mesoscale vortexes between ($k^-3$ and $k^{-5/3}$ ) denote as $p = -3$ and $m/n = -5/3$,

$$E(k) = \alpha\varepsilon^{2/3}k^{-5/3}, \ \ E_h(k) = \alpha_h\omega_B\varepsilon^{1/3}k^{-7/3}, \ \ E_\bullet(k) = \alpha\omega_B^2k^{-3}, \tag{4}$$

and for the viscous subrange the mesoscale vortexes indices, denoted $m/n = -5/3$ and $q = -7$ , we get, respectively:

(a) the inertial subrange:

$$
\begin{array}{r}
\frac{1}{2}(p+m/n), \\
\frac{1}{2^2}(3p+m/n),\ \frac{1}{2^2}(p+3m/n), \\
\frac{1}{2^3}(7p+m/n),\ \frac{1}{2^3}(5p+3m/n),\ \ \frac{1}{2^3}(3p+5m/n),\ \ \frac{1}{2^3}(p+7m/n), \\
\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots;
\end{array}
\tag{5}
$$

the indices' numerical values of the power spectra of the hierarchy of mesoscale vortexes are:

$$3,\quad 17/6,\quad 8/3,\quad 5/2,\quad 7/3,\quad 13/6,\quad 2,\quad 11/6,\quad 5/3; \tag{5a}$$

(b) the viscous subrange:

$$
\begin{array}{r}
\frac{1}{2}(m/n+q), \\
\frac{1}{2^2}(3m/n+q),\ \frac{1}{2^2}(m/n+3q), \\
\frac{1}{2^3}(7m/n+q),\ \frac{1}{2^3}(5m/n+3q),\ \ \frac{1}{2^3}(3m/n+5q),\ \ \frac{1}{2^3}(m/n+7q), \\
\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots;
\end{array}
\tag{6}
$$

the indices' numerical values of the power spectra of the hierarchy of viscous mesoscale vortexes are:

$$5/3,\quad 7/3,\quad 3,\quad 11/3,\quad 13/3,\quad 5,\quad 17/3,\quad 19/3,\quad 7. \tag{6a}$$

**3.3.** Constructing the series of hierarchy of mesoscale vortexes for the inertial subrange, one can choose suitable one at comparison with results of observations:

TABLE (a). The inertial subrange, comparison [11] with experimental data [10]

| $h$, **km** | 67.5 | 70.5 | 77.0 | 79.5 | 81.2 |
|---|---|---|---|---|---|
| **[10]** | $k^{-1.9}$ | $k^{-1.92}$ | $k^{-1.76}$ | $k^{-2.27}$ | $k^{-2.09}$ |
| **theor.** | $k^{-15/8}$ | $k^{-23/12}$ | $k^{-7/4}$ | $k^{-7/3}$ | $k^{-2}$ |

Constructing the theoretic series of hierarchy of mesoscale vortexes in the viscous subrange, one can choose suitable one at comparison with results of observations:

TABLE (b). The viscous subrange, comparison [14] with experimental data [10]

| $h$, **km** | 67.5 | 70.5 | 77.0 | 79.5 | 81.2 |
|---|---|---|---|---|---|
| **[10]** | $k^{-1.36}$ | $k^{-2.64}$ | $k^{-4.27}$ | $k^{-4.42}$ | $k^{-6.38}$ |
| **theor.** | $k^{-4/3}$ | $k^{-8/3}$ | $k^{-13/3}$ | $k^{-53/12}$ | $k^{-19/3}$ |

It is evident that results of observation and theoretical calculations have rather close values.

## R E F E R E N C E S


1. Moffat H.K. Magnetic Field Generation in Electrically Conducting Fluids. *M.: Mir*, 1980, 339 p.

2. Parker E.N. Cosmical Magnetic Fields. **1, 2**. *M.: Mir*, 1982, 608 p., 479 p.

3. Handbook of Turbulence, /Eds Walter Frost and Trevor H. Moulden. *M.: Mir*, **1** (1980), 142-163.

4. Polovin R.V., Demutsky V.P. Principles of Magnetohydrodynamics. *M.: Energoatomizdat*, 1987, 206 p.

5. Gvelesiani A. Characteristics of the turbulent mesosphere and lower thermosphere. *J. Georgian Geophys. Soc.*, **2B** (1997), 51-60.

6. Gvelesiani A. The energy spectral density of the turbulent mesosphere and lower thermosphere. *J. Georgian Geophys. Soc.*, **6B** (2001), 68-75.

7. Gvelesiani A. The generalized semiempirical model of the turbulent mesosphere and lower thermosphere. *J. Georgian Geophys. Soc.,* **6B** (2001), 76-83.

8. Gershman B.N. Ionospheric irregularities. *Itogi Nauki i Tekhniki: Geomagnetizm and high layers of the atmosphere. M.: VINITI,* **3** (1976), 62-87.

9. Ivanov-Kholodny G.S. Composition and structure of the Earth's upper atmosphere. *Itogi Nauki i Tekhniki: Geomagnetizm and high layers of the atmosphere. M.: VINITI*, **3** (1976), 7-61.

10. Sinha H.S.S. Plasma density irregularities in the equatorial $D$-region produced by neutral turbulence. *J. Atmos. Terr. Phys.,* **54**, 1 (1992), 49-61.

11. Thrane E.V. et al. Neutral air turbulence in the upper atmosphere observed during the Energy Budget Campaign. *J. Atmos. Terr. Phys.,* **47** (1985), 243-264.

12. Sinha H.S.S., Raizada S. Some new features of ionospheric plasma depletions over the Indian zone using all sky optical imaging. *Earth Planets Space*, **52** (2000), 549-559.

13. Raizada S., Sinha H.S.S. Some new features of ionospheric plasma depletions over the Indian zone using all sky optical imaging. *Ann. Geophysicae,* **18** (2000), 141-151.

14. Sinha H.S.S., Raizada S. First in situ measurement of electric field fluctuations during strong spread F in the Indian zone. *Ann. Geophysicae*, **18** (2000), 523-531.


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