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SEMI-DISCRETE SCHEME FOR ONE SYSTEM OF NONLINEAR AVERAGED INTEGRO-DIFFERENTIAL EQUATIONS

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Abstract. One nonlinear integro-differential system is considered. The model describes penetration of a magnetic field into a substance. Semi-discrete difference scheme with respect to space variable is studied.

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In [1] one kind of nonlinear integro-differential system is presented. This model arises on mathematical simulation of the process of penetration of a magnetic field into a substance and were introduced after reduction nonlinear Maxwell's system [2] to the integro-differential form. In [3] some generalization of such type models is given. One-dimensional simple analog called by author as averaged model has the following integro-differential form:

$$\begin{aligned}
&\frac{\partial U}{\partial t} - a\left(\int_0^t\int_0^1\left[\left(\frac{\partial U}{\partial x}\right)^2+\left(\frac{\partial V}{\partial x}\right)^2\right]dx d\tau\right)\frac{\partial^2 U}{\partial x^2}=0,\\
&\frac{\partial V}{\partial t} - a\left(\int_0^t\int_0^1\left[\left(\frac{\partial U}{\partial x}\right)^2+\left(\frac{\partial V}{\partial x}\right)^2\right]dx d\tau\right)\frac{\partial^2 V}{\partial x^2}=0,
\end{aligned}\tag{1}$$

where $a = a(S) \geq Const > 0$ is a given function of its argument.

Various works are dedicated to the investigation and numerical resolution of the integro-differential models described in [1] and [3]. Many authors study solvability, uniqueness (see, for example, [1], [3]-[10] and references therein) and asymptotic behavior as $t \to \infty$ of the initial-boundary value problems for these type models (see, for example, [8],[10]-[24] and references therein). Numerical resolution by finite difference scheme and finite element method are given in works [10], [14], [16]-[19], [21], [22], [24] and in a number of other works as well.

The purpose of this note is to construct and investigate semi-discrete difference approximation for the system (1) with special nonlinearity. This system has the form:

$$\begin{aligned}
&\frac{\partial U}{\partial t} - \left(1+\int_0^t\int_0^1\left[\left(\frac{\partial U}{\partial x}\right)^2+\left(\frac{\partial V}{\partial x}\right)^2\right]dx d\tau\right)^p\frac{\partial^2 U}{\partial x^2}=0,\\
&\frac{\partial V}{\partial t} - \left(1+\int_0^t\int_0^1\left[\left(\frac{\partial U}{\partial x}\right)^2+\left(\frac{\partial V}{\partial x}\right)^2\right]dx d\tau\right)^p\frac{\partial^2 V}{\partial x^2}=0.
\end{aligned}\tag{2}$$

Let us note that such kind investigations for the equation and system of type (1) are studied in [10], [14], [16]-[19], [21], [24] and in a number of other works as well. In the works [14], [16], [18], [19] fully-discrete finite difference schemes are also studied for such kind models for the case p=1.

In the $[0,1]\times[0,T]$ let us consider the following initial-boundary value problem:

$$\begin{gathered} U(0,t)=U(1,t)=V(0,t)=V(1,t)=0, \\ U(x,0)=U_0(x), \quad V(x,0)=V_0(x), \end{gathered} \tag{3}$$

where $U_0=U_0(x)$ and $V_0=V_0(x)$ are given functions.

On [0,1] let us introduce a net with mesh points denoted by $x_i=ih$, $i=0,1,\dots,M$, with $h=1/M$. The boundaries are specified by $i=0$ and $i=M$. The semi-discrete approximation at (x_i,t) is designed by $u_i=u_i(t)$ and $v_i=v_i(t)$. The exact solution of the problem (2), (3) at (x_i,t) is denoted by $U_i=U_i(t)$ and $V_i=V_i(t)$.

Using known notations [23] let us construct the following semi-discrete scheme for problem (2), (3):

$$\begin{gathered} \frac{du_i}{dt}=\left(1+h\sum_{i=1}^{M}\int_0^t\left[(u_{\bar{x},i})^2+(v_{\bar{x},i})^2\right]d\tau\right)^p u_{\bar{x}x,i}, \\ \frac{dv_i}{dt}=\left(1+h\sum_{i=1}^{M}\int_0^t\left[(u_{\bar{x},i})^2+(v_{\bar{x},i})^2\right]d\tau\right)^p v_{\bar{x}x,i}, \\ i=1,2,\dots,M-1, \end{gathered} \tag{4}$$

$$u_0(t)=u_M(t)=v_0(t)=v_M(t)=0, \tag{5}$$

$$u_i(0)=U_{0,i}, \quad v_i(0)=U_{0,i}, \quad i=0,1,\dots,M. \tag{6}$$

The following statement takes place.

Theorem. *If $0<p\le 1$ and the initial-boundary value problem (2), (3) has the sufficiently smooth solution $U=U(x,t)$, $V=V(x,t)$, then the semi-discrete scheme (4) - (6) converges and the following estimate is true*

$$\|u(t)-U(t)\|+\|v(t)-V(t)\|\le Ch.$$

Here $\|\cdot\|$ is a discrete analog of the norm of the space $L_2(0,1)$ and C is a positive constant independent of h.

Note that for (2) type equation with source term and with same nonlinearity result analogical to this Theorem is received in [24]. The existence, uniqueness and asymptotic behavior of solution are studied in [24] as well. Note also that the initial-boundary value problem (2), (3) for the case $p=1$ is studied in [22] by finite element method. Investigated semi-discrete scheme (4) - (6) is using for numerical solution of the problem (2), (3) by natural discretisation of time derivative and integral as it is given for example in [21]. Solving the obtaining finite difference scheme we use a algorithm analogical to [19] for the case $p=1$. So, it is necessary to use Newton iterative process [25].

According to this method the great numbers of numerical experiments are carried out. These experiments agree with the theoretical results.

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