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ON ONE METHOD OF APPROXIMATE SOLUTION OF ANTIPLANE PROBLEMS OF ELASTICITY THEORY FOR COMPOSITE BODIES WEAKNED BY CRACKS

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Abstract. In the present article finite-difference solution of antiplane problems of elasticity theory for composite (piece-wise homogeneous) bodies weakened by cracks is presented. The differential equation with corresponding initial boundary conditions is approximated by finite-differential analogies in the rectangular quadratic area. Such kind set of the problem gives opportunity to find directly numerical values of shift functions in the grid points. The suggested calculation algorithms have been tested for the concrete practical tasks. The results of numerical calculations are in a good approach with the results of theoretical investigations.

Keywords and phrases: Antiplane problems, cracks, finite-difference method.

AMS subject classification: 65M06, 65N06, 65M60, 65M70.

Introduction. Study of boundary value problems for the composite bodies weakened by cracks has a great practical significance. Mathematical model investigated boundary value problems for the composite bodies weakened by cracks in the first approximation can be based on the equations of antiplane approach of elasticity theory for composite (piece-wise homogeneous) bodies. When cracks intersect an interface or penetrate it at all sorts of angle on the base of the integral equations method is studied in the works [1]-[7]. In the present article the problems for composite (piece-wise homogeneous) orthotropic bodies weakened by cracks is studied by finite-difference method. At the initial stage the problem is studied for homogenous body and further for composite (piece-wise homogeneous) bodies. Besides theoretical research of the above mentioned tasks our aim is to construct and develop rapidly convergent algorithm and numerical method.

Statement of the problem. Given a distorted harmonic equation, with $2n \times 2n$ size, in square $\overline{\Omega} = \Omega_1 \cup \Omega_2$ (Ω_1 and Ω_2 areas, see Fig. 1)

$$\frac{\partial^2 w_k(x,y)}{\partial x^2} + \lambda_k^2 \frac{\partial^2 w_k(x,y)}{\partial y^2} = 0, \quad (x,y) \in \Omega_k, \quad k = 1,2. \tag{1}$$

a) On the curves of the crack L_x^+ and L_x^- tangent stresses are given (Fig. 1) while end points of the crack coherence conditions are given

$$\tau_{yz}^{(\pm)} = b_{44}^{(k)} \frac{\partial w_k(x, \pm 0)}{\partial y} = q_k^{(\pm)}(x), \quad x \in L_k, \ \ L_1 = [0;1], \quad L_2 = [-1;0], \tag{2}$$

$$w_2(-1,+0) = w_2(-1,-0), \ \ w_1(1,+0) = w_1(1,-0); \tag{3}$$

b) on the axis y (on the dividing line) the condition of continuity is fulfilled

$$w_1(0;y) = w_2(0;y), \quad y \in [-n,n],\ y \neq 0, \tag{4}$$

$$\tau_{xz}^{(1)} = \tau_{xz}^{(2)}, \quad or \quad b_{55}^{(1)} \frac{\partial w_1(0;y)}{\partial x} = b_{55}^{(2)} \frac{\partial w_2(0;y)}{\partial x}; \tag{5}$$

c) on the side pieces of the square $\overline{\Omega}$ we have

$$\begin{gathered} w_2(-n,y) = 0 \quad and \quad w_1(n,y) = 0, \quad y \in [-n,n], \\ w_2(x,\pm n) = 0, \quad x \in [-n,n], \quad and \quad w_1(x,\pm n) = 0, \quad x \in [0,n]. \end{gathered} \tag{6}$$

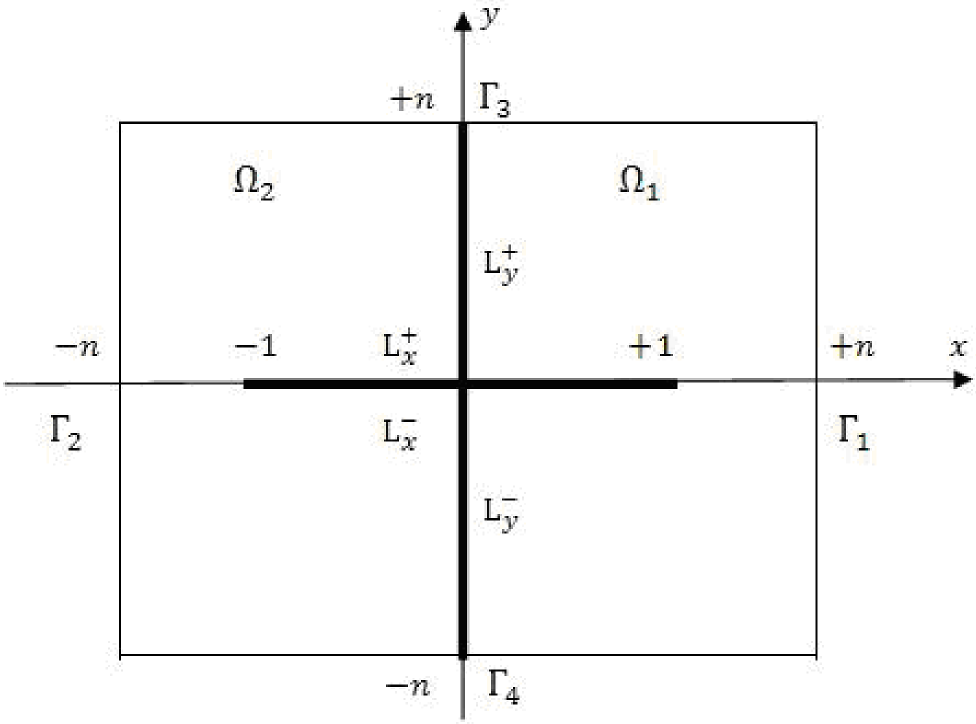

Fig. 1.

In the above mentioned equations $\lambda_k^2 = \frac{b_{44}^{(k)}}{b_{55}^{(k)}}$, $b_{44}^{(k)}$, $b_{55}^{(k)}$, are elastic constants, which have been taken from the Hooke's law, $q_k^{(\pm)}(x)$ is a function of Holder's class, In particular, if we have isotropic case $b_{44}^{(k)} = b_{55}^{(k)} = \mu_k$, $\lambda_k = 1$, numerical parameter $\lambda_k = 1$, where μ_k is module of displacement, $k = 1,2$;

Finite-difference method. Primarily suppose that $n, N \in \mathbb{N}$ ($2n$ is a length of the side pieces of the square, $2N$ is a number of dividing points on the crack line), $h_1 = h_2 = h = 1/N$, $N \in \mathbb{N}$. steps in the directions x and y are equal, that is there is a regular quadratic grid $\Omega_h = \{(x_i, y_i),\ x_i = ih,\ y_j = jh,\ i,j = [-nN, nN]\}$, For boundary value problem (1)-(6) a different scheme is the following: differential operator in the basic (1) equation is approximated by five point template with $O(h^2)$ accuracy, while differential operator in (2) and (5) equations are approximated by five point template with $O(h)$ accuracy $W_{k,i,j}$, For finding grid function W the following iteration method is used:

$$W_{k,i,j}^{(m+1)}=\frac{1}{2(1+\lambda_k^2)}\left[W_{k,i+1,j}^{(m)}+W_{k,i-1,j}^{(m)}+W_{k,i,j+1}^{(m)}+W_{k,i,j-1}^{(m)}\right];$$

a) everywhere, excepting crake's line and dividing line (border) we have

$$j\neq 0,\ \text{ then }\ i=-(nN-1),-(nN-2),\cdots,(-1),0,1,\cdots,(nN-2),(nN-1),$$

$$j=0,\ \text{ then }\ i=-(nN-1),-(nN-2),\cdots,-(N+2),-(N+1),$$
$$i=(N+1),(N+2),\cdots,(nN-2),(nN-1).$$

b) On the lines of cracks

$$W_{k,i,(+0)}^{(m+1)}=W_{k,i,(+1)}^{(m)}-\frac{h}{b_{44}^{(k)}}q_{k,i}^{(+)}\ \ and\ \ W_{k,i,(-0)}^{(m+1)}=W_{k,i,(-1)}^{(m)}-\frac{h}{b_{44}^{(k)}}q_{k,i}^{(-)},$$

$$i=-N,-(N-1),\cdots,(-1),0,1,\cdots,(N-1),N;$$

Also at the ending points of the crack it is necessary to take into consideration the following fitting condition (matched condition)

$$q_N^{(+)}=q_N^{(-)},\quad q_{(-N)}^{(+)}=q_{(-N)}^{(-)};$$

and in the point (x_0,y_0) condition of consistency

$$\frac{q_{(1)}^{(+)}(0)}{b_{44}^{(1)}}\equiv\frac{q_{(2)}^{(+)}(0)}{b_{44}^{(2)}},\quad \frac{q_{(1)}^{(-)}(0)}{b_{44}^{(1)}}\equiv\frac{q_{(2)}^{(-)}(0)}{b_{44}^{(2)}},$$

c) on the dividing line (border)

$$W_{1,0,j}^{(m+1)}=W_{2,0,j}^{(m+1)}=\frac{b_{55}^{(2)}W_{2,-1,j}^{(m)}+b_{55}^{(1)}W_{1,1,j}^{(m)}}{b_{55}^{(2)}+b_{55}^{(1)}};$$

$$j=-N,-(N-1),\cdots,(-1),1,\cdots,(N-1),N;$$
$$W_{k,i,j}^{(0)}=0,\quad m=0,1,2,\cdots.$$

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