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ON SOME NON-CLASSICAL TERMOELASTICITY PROBLEMS FOR A THREE-LAYER RECTANGULAR PARALLELEPIPED

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Abstract. In the present paper, in the Cartesian system of coordinates, thermoelastic equilibrium of a rectangular parallelepiped consisting of three homogeneous isotropic layers is considered. Either symmetry or antisymmetry conditions are defined on the lateral faces of the parallelepiped, while either rigid contact or sliding contact conditions are defined on the contact surfaces. The problem is to define disturbances on the upper and lower sides of the parallelepiped so that stresses and normal displacement or displacements and normal stress on some planes parallel to the upper and lower sides inside the body would take the prescribed values.

Keywords and phrases: Non-classical thermoelasticity problem, separation of variables.

AMS subject classification: 74B05.

Introduction. The following non-classical thermoelasticity problem are stated and analytically solved in the present paper.

In the Cartesian system of coordinates thermoelastic equilibrium of a rectangular parallelepiped consisting of three homogeneous isotropic layers is considered. Either symmetry or antisymmetry conditions are defined on the lateral faces of the parallelepiped, while either rigid contact or sliding contact conditions are defined on the contact surfaces.

The problem is to define disturbances on the upper and lower sides of the parallelepiped so that stresses and normal displacement or displacements and normal stress on some planes parallel to the upper and lower sides inside the body would take the prescribed values. Disturbances on the upper and lower faces of the parallelepiped mean that stresses and temperature, displacements and temperature or combinations of stresses, displacements and temperatures are defined. We should note that instead of temperature its normal derivative can be given.

Due to size restrictions the authors confine themselves to the statement just of the above-mentioned non-classical problems.

Problem statement. Let in a Cartesian system of coordinates x, y, z a defined elastic body consisting of three homogeneous isotropic layers occupy a domain $\Omega = \Omega_1 \bigcup \Omega_2 \bigcup \Omega_3$, where $\Omega_1 = \left\{0 < x < x_1,\ 0 < y < y_1,\ \ -z_1 < z < -z_2\right\}$, $\Omega_2 = \left\{0 < x < x_1,\ 0 < y < y_1,\ -z_2 < z < z_2\right\}, \Omega_3 = \left\{0 < x < x_1,\ 0 < y < y_1,\ z_2 < z < z_1\right\}$, where z_1, z_2, z_3, x_1, y_1 - are some positive constants.

The domains Ω_1 and Ω_3 are assumed to be filled with the same isotropic homogeneous material, while the domain Ω_2 is assumed to be filled with another material which is also isotropic and homogeneous which can be, in particular, incompressible.

On the lateral sides of the body the following conditions are defined:

$$\left.\begin{array}{l} \text{For}\quad x=x_j:\quad a)\quad \sigma_{xx}^{(k)}=0,\quad v^{(k)}=0,\quad w^{(k)}=0,\quad T^{(k)}=0,\\ \qquad\qquad\quad \text{or}\\ \qquad\qquad\quad b)\quad u^{(k)}=0,\quad \sigma_{xy}^{(k)}=0,\quad \sigma_{xz}^{(k)}=0,\quad \partial_x T^{(k)}=0. \end{array}\right\} \tag{1}$$

$$\left.\begin{array}{l} \text{For y} = \text{y}_\text{j}:\quad a)\quad \sigma_{yy}^{(k)}=0,\quad u^{(k)}=0,\quad w^{(k)}=0,\quad T^{(k)}=0,\\ \qquad\qquad\quad \text{or}\\ \qquad\qquad\quad b)\quad v^{(k)}=0,\quad \sigma_{yx}^{(k)}=0,\quad \sigma_{yz}^{(k)}=0,\quad \partial_y T^{(k)}=0, \end{array}\right\} \tag{2}$$

where $j=0,\quad 1,$ and $x_0=y_0=0$; $u^{(k)},\ v^{(k)},\ w^{(k)}$ are components of the displacement vector of the k-th layer along the axes $x,\ y,\ z$, respectively, $\sigma_{xx}^{(k)},\ \sigma_{yy}^{(k)},\ \sigma_{zz}^{(k)}$ are normal stresses $\sigma_{xy}^{(k)}=\sigma_{yx}^{(k)}$, $\sigma_{xz}^{(k)}=\sigma_{zx}^{(k)}$, and $\sigma_{yz}^{(k)}=\sigma_{zy}^{(k)}$ are tangential stresses of the k-th layer; $T^{(k)}$ is a change in the temperature of the k-th layer satisfying the following equation

$$\Delta T^{(k)}=0, \tag{3}$$

where $k=1,2,3$; $\Delta\equiv\partial_{xx}+\partial_{yy}+\partial_{zz},\qquad \partial_x\equiv\frac{\partial}{\partial x},\quad \partial_y\equiv\frac{\partial}{\partial y},\quad \partial_z\equiv\frac{\partial}{\partial z}.$

On surfaces $z=-z_2$ and $z=z_2$ the following type contact conditions are defined

$$\begin{array}{l} \text{For}\ \ z-(-1)^j\,z_2:\quad u^{(j)}=u^{(j+1)},\ \ v^{(j)}=v^{(j+1)},\ \ w^{(j)}=w^{(j+1)},\ \ T^{(j)}=T^{(j+1)},\\ \qquad\qquad \sigma_{zx}^{(j)}=\sigma_{zx}^{(j+1)},\ \ \sigma_{zy}^{(j)}=\sigma_{zy}^{(j+1)},\ \ \sigma_{zz}^{(j)}=\sigma_{zz}^{(j+1)},\ \ \partial_z T^{(j)}=\partial_z T^{(j+1)}, \end{array} \tag{4}$$

or

$$\begin{array}{l} \sigma_{zz}^{(j)}=\sigma_{zz}^{(j+1)},\ \ w^{(j)}=w^{(j+1)},\ \ \sigma_{zx}^{(j)}=0,\quad \sigma_{zx}^{(j+1)}=0,\\ \sigma_{zy}^{(j)}=0,\quad \sigma_{zy}^{(j+1)}=0,\ \ T^{(j)}=T^{(j+1)},\ \ \partial_z T^{(j)}=\partial_z T^{(j+1)}, \end{array} \tag{5}$$

where j=1,2.

Boundary conditions (1) and (2) are antisymmetry boundary conditions while boundary conditions (1b) and (2b) are symmetry boundary conditions [1,2]. Conditions (4) are rigid contact conditions, while conditions (5) are sliding contact conditions.

The problem consists in that on sides $z=-z_1$ and $z=z_1$ to find such stresses $\sigma_{zz}^{(j)}$, $\sigma_{zx}^{(j)}$, $\sigma_{zy}^{(j)}$ and change of temperature $T^{(j)}$, or displacements $u^{(j)}$, $v^{(j)}$, $w^{(j)}$ and $T^{(j)}$ or $\sigma_{zz}^{(j)}$, $u^{(j)}$, $v^{(j)}$, and $T^{(j)}$, or $\sigma_{zx}^{(j)}$, $\sigma_{zy}^{(j)}$, $w^{(j)}$ and $T^{(j)}$ (instead of function $T^{(j)}$ it may be required its normal derivative $\partial_z T^{(j)}$; when $z=-z_1$, then j =1, and when $z=z_1$, then j =3) so that following conditions could be satisfied

$$\text{For } z = \mp z_3: \quad \sigma_{zz}^{(k)} = f_3^{\mp}(x, y);$$

$$\left.\begin{array}{l} \text{For } z = \mp z_4 : a) \quad \sigma_{zx}^{(l)} = f_1^{\mp}(x, y), \quad \sigma_{zy}^{(l)} = f_2^{\mp}(x, y), \\ \qquad\qquad \text{or} \\ \qquad\qquad b) \quad u^{(l)} = g_1^{\mp}(x, y), \quad v^{(l)} = g_2^{\mp}(x, y); \end{array}\right\}$$

$$\text{For } z = \mp z_5: \quad w^{(p)} = g_3^{\mp}(x, y),$$

where indices k, l and p take values 1 or 2 in the case $z = -z_q$ and values 2 or 3 in the case $z = z_q$,$q = 3, 4, 5$; $f^{\mp}(x, y)$, $g^{\mp}(x, y)$, $j = 1, 2, 3$ are defined analytical functions in the domain $\overline{\omega} = \{0 \leq x \leq x_1, 0 \leq y \leq y_1\}$. z_q are constants and $0 \leq z_q \leq z_1$. In the case of a sliding contact on the planes $z = \pm z_2$, $z_q \neq z_2$.

Hence, the problem is to find disturbances of the sides $z = -z_1$ and $z = z_1$, satisfy the given values of normal stresses on the planes $z = \mp z_3$, to satisfy the given values of tangential stresses or tangential displacement on the planes $z = \mp z_4$ and satisfy the given values of normal displacements on the planes $z = \mp z_5$ (see also [3]).

Since, layers and planes, for which the conditions are given are located symmetrically to the plane Oxy, each of the stated problem can be reduced to the solution of two non-classical problems for the domain $\Omega^* = \Omega_1^* \bigcup \Omega_3$ where $\Omega_1^* = \{0 < x < x_1,\ 0 < y < y_1,\ 0 < z < z_2\}$. In the first problem we have

$$\text{For } z = z_3: \quad \sigma_{zz}^{(k)} = \frac{1}{2}\left(f_3^{+} + f_3^{-}\right);$$

$$\left.\begin{array}{l} \text{For } z = z_4 : a) \quad \sigma_{zx}^{(l)} = \frac{1}{2}\left(f_1^{+} - f_1^{-}\right), \quad \sigma_{zy}^{(l)} = \frac{1}{2}\left(f_2^{+} - f_2^{-}\right), \\ \qquad \text{or} \\ \qquad\qquad b) \quad u^{(l)} = \frac{1}{2}\left(g_1^{+} + g_1^{-}\right) \quad v^{(l)} = \frac{1}{2}\left(g_2^{+} + g_2^{-}\right); \end{array}\right\}$$

$$\text{For } z = z_5: \quad w^{(p)} = \frac{1}{2}\left(g_3^{+} - g_3^{-}\right),$$

$$\text{For } z = 0: \quad w^{(2)} = 0, \quad \sigma_{zx}^{(2)} = 0, \quad \sigma_{zy}^{(2)} = 0, \quad \partial_z T^{(2)} = 0.$$

And in the second problem we have

$$\text{For } z = z_3: \quad \sigma_{zz}^{(k)} = \frac{1}{2}\left(f_3^{+} - f_3^{-}\right);$$

$$\left.\begin{array}{l} \text{For } z = z_4 : a) \quad \sigma_{zx}^{(l)} = \frac{1}{2}\left(f_1^{+} + f_1^{-}\right), \quad \sigma_{zy}^{(l)} = \frac{1}{2}\left(f_2^{+} + f_2^{-}\right), \\ \qquad \text{or} \\ \qquad\qquad b) \quad u^{(l)} = \frac{1}{2}\left(g_1^{+} - g_1^{-}\right) \quad v^{(l)} = \frac{1}{2}\left(g_2^{+} - g_2^{-}\right); \end{array}\right\}$$

$$\text{For } z = z_5: \quad w^{(p)} = \frac{1}{2}\left(g_3^{+} + g_3^{-}\right),$$

$$\text{For } z = 0: \quad \sigma_{zz}^{(2)} = 0, \quad u^{(2)} = 0, \quad w^{(2)} = 0, \quad T^{(2)} = 0,$$

where k, l and q take the fixed values 2 or 3.

The stated non-classical problems are solved analytically by the method of separation of variables and applying a general solution of a system of equations in three-dimensional thermoelasticity using harmonic functions according to formulas [1]

$$2\mu_k u^{(k)} = -z\partial_x \Phi^{(k)} + \partial_y \Psi^{(k)} + \partial_x G^{(k)} - 2\mu_k(1+\nu_k)\beta_k \partial_x \tilde{T}^{(k)},$$

$$2\mu_k v^{(k)} = -z\partial_y \Phi^{(k)} - \partial_x \Psi^{(k)} + \partial_y G^{(k)} - 2\mu_k(1+\nu_k)\beta_k \partial_y \tilde{T}^{(k)},$$

$$2\mu_k w^{(k)} = (3-4\nu_k)\Phi^{(k)} - z\partial_z \Phi^{(k)} + \partial_z G^{(k)} + 2\mu_k(1+\nu_k)\beta_k \partial_z \tilde{T}^{(k)},$$

where μ_k is a displacement module of the k-th layer, ν_k is Poisson's coefficient of the k-th layer,β_k is a coefficient of linear thermal expansion of the k-th layer , $\Phi^{(k)}$, $\Psi^{(k)}$ and $G^{(k)}$ are arbitrary harmonic functions in the domain Ω_k, and $\tilde{T}^{(k)}$ is also a harmonic function associated with the changes in the temperature $T^{(k)}$ by means of the following relation $T^{(k)} = \partial_{zz}\tilde{T}^{(k)}$.

Problems studied in the given paper differ from the other known non-classical problems studied by different authors [4-9] and are of great practical importance.

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