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AN APPROXIMATE SOLUTION OF ONE SYSTEM OF THE SINGULAR INTEGRAL EQUATIONS

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Abstract. In the present work it is investigated questions of the approached decision of one system (pair) of the singular integral equations. The study of boundary value problems for the composite bodies weakened by cracks has a great practical significance. The system of the singular integral equations is solved by a collocation method, in particular, a method discrete singular in cases both uniform, and non-uniformly located knots.

Keywords and phrases: Singular integral equations, method integral equations, antiplane problems, cracks, collocation method.

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1. Statement of the problem. Let's consider system the singular integral equations containing an immovable singularity (see [1])

$$\begin{aligned} &\int_0^1 \left[\frac{1}{t-x} - \frac{a_1}{t+x}\right] \rho_1(t)dt + b_1 \int_{-1}^0 \frac{\rho_2(t)dt}{t-x} = 2\pi f_1(x), \quad x \in (0;1), \\ &b_2 \int_0^1 \frac{\rho_1(t)dt}{t-x} + \int_{-1}^0 \left[\frac{1}{t-x} - \frac{a_2}{t+x}\right] \rho_2(t)dt = 2\pi f_2(x), \quad x \in (-1;0), \end{aligned} \tag{1}$$

where $\rho_k(x)$ is unknown and $f_k(x)$ is given real functions, a_k, b_k are constants, $f_k(x) \in H$, $\rho_k(x) \in H^*$, $k = 1, 2$.

2. Collocation method. The system (1) of the singular integral equations is solved by a collocation method, in particular, a method discrete singular (see [2]) in cases both uniform, and non-uniformly located knots.

A. Algorithm of uniform division.

Decisions of equations system (1) such view (see [1])

$$\rho_1(t) = \frac{\rho_1^*(t)}{\sqrt{1-t}}, \qquad \rho_2(t) = \frac{\rho_2^*(t)}{\sqrt{1+t}}, \tag{1}$$

where $\rho_k^*(t) \in H$, $k = 1, 2$.

Let's enter such distribution of knots for variables of integration and account points accordingly

$$t_{1i} = 0 + ih, \quad t_{2i} = -1 + ih, \qquad i = 1, 2, \cdots, n;$$

$$x_{1j} = t_{1j} - h/2, \quad x_{2j} = t_{2j} + h/2, \quad j = 1, 2, \cdots, n;$$

$$h = \frac{1}{n+1}.$$

The pair of the equations (1) can be presented as follows with the help of quadrature formulas (see [2])

$$\sum_{i=1}^{n}\left(\frac{h}{t_{1i}-x_{1j}}-\frac{a_1 h}{t_{1i}+x_{1j}}\right)\rho_1(t_{1i})+b_1\sum_{i=1}^{n}\left(\frac{h}{t_{2i}-x_{1j}}\right)\rho_2(t_{2i})=2\pi f_1(x_{1j}),$$
$$j=1,2,\cdots,n;$$
$$b_2\sum_{i=1}^{n}\left(\frac{h}{t_{1i}-x_{2j}}\right)\rho_1(t_{1i})+\sum_{i=1}^{n}\left(\frac{h}{t_{2i}-x_{2j}}-\frac{a_2 h}{t_{2i}+x_{2j}}\right)\rho_2(t_{2i})=2\pi f_2(x_{2j}),$$
$$j=1,2,\cdots,n; \tag{2}$$

Thus, we have $2n$ equations with $2n$ unknowns. The received system of the linear equations it is possible to solve with the help to one of direct method, for example, by Gauss modified method.

B. Algorithm of non-uniformly division.

All terms of system (1) of the singular integral equations we will transfer in a interval $(-1,1)$. Let's apply following transformation of variables

$$x=\frac{x_1+1}{2},\quad x_1=2x-1,\quad x\in[0;1],\quad x_1\in[-1;+1];$$
$$t=\frac{t_1+1}{2},\quad t_1=2t-1,\quad t\in[0;1],\quad t_1\in[-1;+1];$$
$$t=\frac{t_2-1}{2},\quad t_2=2t+1,\quad t\in[-1;0],\quad t_2\in[-1;+1];$$
$$x=\frac{x_2-1}{2},\quad x_2=2x+1,\quad x\in[-1;0],\quad x_2\in[-1;+1];$$

As a result of the above stated transformation of variables the system of the integral equations (1) assumes the following view

$$\int_{-1}^{1}\left[\frac{1}{t_1-x_1}-\frac{a_1}{t_1+x_1+2}\right]\rho_1\left(\frac{t_1+1}{2}\right)dt_1$$
$$+\int_{-1}^{1}\frac{b_1}{t_2-x_1-2}\rho_2\left(\frac{t_2-1}{2}\right)dt_2=2\pi f_1\left(\frac{x_1+1}{2}\right),\quad x_1\in(-1;1),$$
$$\int_{-1}^{1}\left[\frac{1}{t_2-x_2}-\frac{a_2}{t_2+x_2-2}\right]\rho_2\left(\frac{t_2-1}{2}\right)dt_2$$
$$+\int_{-1}^{1}\frac{b_2}{t_1-x_2+2}\rho_1\left(\frac{t_1+1}{2}\right)dt_1=2\pi f_2\left(\frac{x_2-1}{2}\right),\quad x_2\in(-1;1).$$

To a finding of unknown functions $\rho_1\left(\frac{t_1+1}{2}\right)$, $\rho_2\left(\frac{t_2-1}{2}\right)$ we will apply quadrature formulas of a following view (see [2])

$$
\begin{aligned}
&\sum_{i=1}^{n}\left(\frac{1}{t_{1i}-x_{1j}}-\frac{a_1}{t_{1i}+x_{1j}+2}\right)A1_i\rho_1\left(\frac{t_{1i}+1}{2}\right)\\
&\quad+\sum_{i=1}^{n}\left(\frac{b_1}{t_{2i}-x_{1j}-2}\right)A2_i\rho_2\left(\frac{t_{2i}-1}{2}\right)=2\pi f_1\left(\frac{x_{1j}+1}{2}\right),\\
&\qquad\qquad\qquad\qquad\qquad\qquad j=1,2,\cdots,n;\\
&\sum_{i=1}^{n}\left(\frac{b_2}{t_{1i}-x_{2j}+2}\right)A1_i\rho_1\left(\frac{t_{1i}+1}{2}\right)\\
&\quad+\sum_{i=1}^{n}\left(\frac{1}{t_{2i}-x_{2j}}-\frac{a_2}{t_{2i}+x_{2j}-2}\right)A2_i\rho_2\left(\frac{t_{2i}-1}{2}\right)=2\pi f_2\left(\frac{x_{2j}-1}{2}\right),\\
&\qquad\qquad\qquad\qquad\qquad\qquad j=1,2,\cdots,n;
\end{aligned}
\tag{3}
$$

where

$$
\begin{aligned}
t_{1i}&=\cos\frac{2i-1}{2n+1}\pi, && i=1,2,\cdots,n;\\
t_{2i}&=\cos\frac{2i}{2n+1}\pi, && i=1,2,\cdots,n;\\
x_{1j}&=\cos\frac{2j}{2n+1}\pi, && j=1,2,\cdots,n;\\
x_{2j}&=\cos\frac{2j-1}{2n+1}\pi, && j=1,2,\cdots,n;\\
A1_i&=\frac{4\pi}{2n+1}\sin^2\frac{i}{2n+1}\pi, && i=1,2,\cdots,n;\\
A2_i&=\frac{4\pi}{2n+1}\sin^2\frac{n+i}{2n+1}\pi, && i=1,2,\cdots,n;
\end{aligned}
$$

Thus we have $2n$ equations with $2n$ unknowns as well as at uniform division.

3. Numerical experiments. As we have noted above, for definition of value of unknown functions in knots we receive system of the equations of order $2n$ with $2n$ unknown variable. (2), (3) problems is solved by program system Mathcad. To the solve of system of the linear algebraic equations it is applied procedure lsolve. Corresponding graphics of the approached decisions of various specific problems are constructed.

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R E F E R E N C E S

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