Reports of Enlarged Session of the
Seminar of I. Vekua Institute
of Applied Mathematics
Volume 25, 2011

NUMERICAL RESOLUTION OF ONE SYSTEM OF NONLINEAR PARTIAL DIFFERENTIAL EQUATIONS

Kiguradze Z., Nikolishvili M., Tabatadze B.

Abstract. One-dimensional analog of the system of nonlinear partial differential equations arising in process of vein formation of young leaves is considered. Numerical resolution of the initial-boundary value problems for this system is done by the finite difference schemes. Graphical illustrations of the tests experiments are given.

Keywords and phrases: System of nonlinear partial differential equations, vein formation model, finite difference schemes.

AMS subject classification: 35Q80, 65N06, 65Y99.

The purpose of this note is numerical resolution of following initial-boundary value problem by the finite difference scheme:

$$\begin{aligned}
&\frac{\partial U}{\partial t} = \frac{\partial}{\partial x}\left(V\frac{\partial U}{\partial x}\right) + f(x,t), && (x,t)\in\Omega\times(0,T),\\
&\frac{\partial V}{\partial t} = -V + g\left(V\frac{\partial U}{\partial x}\right) + \varepsilon\frac{\partial^2 V}{\partial x^2} + \varphi(x,t), && (x,t)\in\Omega\times(0,T),\\
&U(x,t) = V(x,t) = 0, && (x,t)\in\partial\Omega\times(0,T),\\
&U(x,0) = U_0(x), V(x,0) = V_0(x) \geq Const > 0, \quad x\in\bar{\Omega},
\end{aligned} \tag{1}$$

where g, U_0, V_0 are known sufficiently smooth functions, $g_0 \leq g(\xi) \leq G_0$; T, g_0, G_0, δ_0, ε are given positive constants; $\Omega = (0,1)$.

If $f \equiv \varphi \equiv 0$, $\varepsilon = 0$ then nonlinear equations considered in problem (1) is one-dimensional analogue of the model arising in process of vein formation of young leaves [1].

Many scientific works are devoted to this type models with $\varepsilon = 0$ (see, for example, [1]-[8] and references therein). Investigation and numerical solution to nonlinear parabolic type models to which belongs investigated problem (1) when $\varepsilon \neq 0$ are carried out in many works as well (see, for example, [9]-[11] and references therein).

On $[0,1]\times[0,T]$ let us introduce a net with mesh points denoted by $(x_i,t_j) = (ih, j\tau)$, where $i = 0,1,...,M$; $j = 0,1,...,N$ with $h = 1/M$, $\tau = T/N$. The discrete approximation at (x_i,t_j) is designed by u_i^j, v_i^j and the exact solution to the problem (1) by U_i^j, V_i^j.

Using the usual method of construction of discrete models [12] let us consider the following finite difference scheme:

$$\frac{u_i^{j+1} - u_i^j}{\tau} = \frac{1}{h^2}\left\{v_i^{j+1}u_{i-1}^{j+1} - \left(v_i^{j+1} + v_{i+1}^{j+1}\right)u_i^{j+1} + v_{i+1}^{j+1}u_{i+1}^{j+1}\right\} + f_i^{j+1}, \tag{2}$$

$$\frac{v_i^{j+1}-v_i^j}{\tau}=-v_i^{j+1}+g\left(v_i^j\frac{u_{i+1}^j-u_{i-1}^j}{2h}\right)+\varepsilon\frac{v_{i-1}^{j+1}-2v_i^{j+1}+v_{i+1}^{j+1}}{h^2}+\varphi_i^{j+1}, \tag{3}$$

$$u_0^j=u_M^j=v_0^j=v_M^j=0, \quad j=0,1,...,N, \tag{4}$$

$$u_j^0=U_{0,i} \quad v_j^0=V_{0,i}, \quad i=0,1...,M. \tag{5}$$

The following statement takes place.

Theorem. *The finite difference scheme (2)-(5) converges to the solution of problem (1) in the norm of the space C_h with the rate $O\left(\tau+h\right)$.*

Using simple transformations we give the following algorithm of solving the scheme (2)-(5): at first we solve system (2) by well known tridiagonal matrix algorithm and after we solve system (3) by the same algorithm, using in both cases suitable boundary and initial conditions from (4), (5). Numerous computer test experiments are done by using above-mentioned algorithm.

Problem (1) for the case $\varepsilon=0$ is also solved using this algorithm. It is clear that in this case we have not stated boundary conditions on the function V and suitable part of (2)-(5) is solved by explicit scheme (3) ($\varepsilon=0$) and then equation (2) is solved by above-mentioned algorithm with tridiagonal matrix.

The graphical illustrations of some part of these numerical results are given in Fig. 1.

The graphs in the Fig. 1 illustrate numerical results of problem (1) for the case $\varepsilon\neq0$. Here exact solutions are: $U(x,t)=\frac{1}{2}x(1-x)(1+t)$, $V(x,t)=x(1-x)(3+t^3)$.

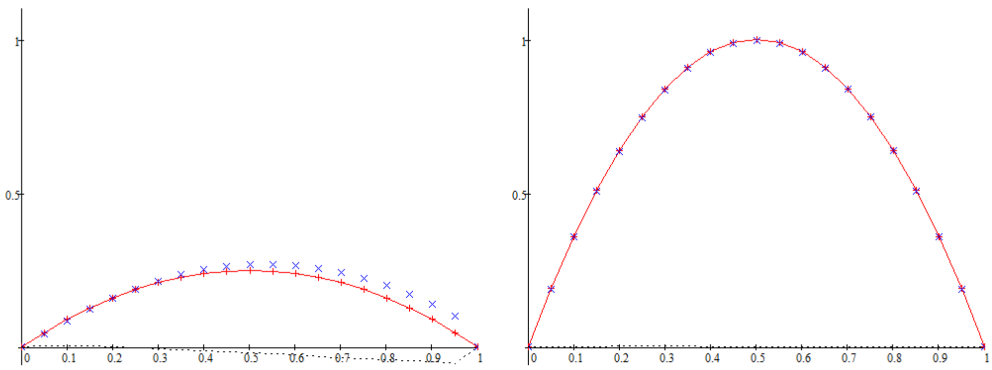

Fig. 1. Exact (solid line) and numerical (marked with $\times$) solutions and differences between exact and numerical solutions (marked with $\bullet$).

The graphs below (see Fig. 2) illustrate numerical results of problem (1) for the case $\varepsilon=0$.

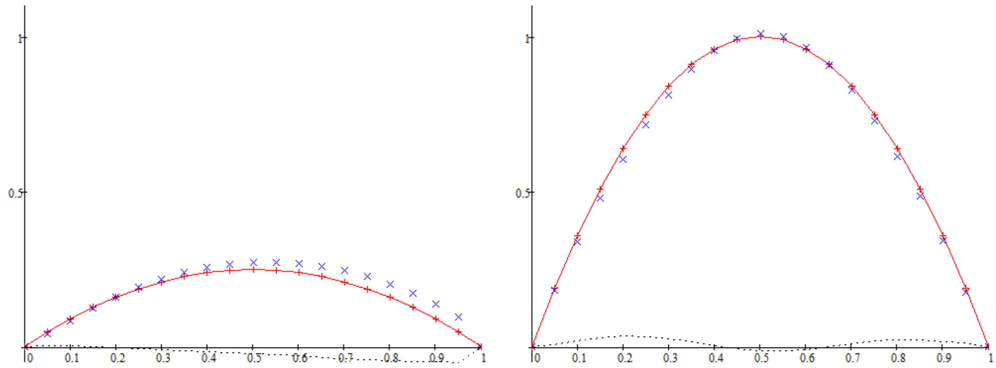

Fig. 2. Exact (solid line) and numerical (marked with $\times$) solutions and differences between exact and numerical solutions (marked with $\bullet$).

REFERENCES

1. Mitchison G.J. A model for vein formation in higher plants. *Proc. R. Soc. Lond. B.,* **207**, 1166 (1980), 79-109.

2. Bell J., Cosner C., Bertiger W. Solutions for a flux-dependent diffusion model. *SIAM J. Math. Anal.,* **13**, 5 (1982), 758-769.

3. Dzhangveladze T.A. Averaged model of sum approximation for a system of nonlinear partial differential equations. (Russian) *Proc. I. Vekua Inst. Appl. Math.,* **19** (1987), 60-73.

4. Dzhangveladze T.A., Tagvarelia T.G. Convergence of a difference scheme for a system of nonlinear partial differential equations, that arise in biology. (Russian) *Proc. I. Vekua Inst. Appl. Math.*, **40** (1990), 77-83.

5. Jangveladze T.A. Investigation and numerical solution of some systems of nonlinear partial differential equations. *Rep. Enlarged Sess. Semin. I. Vekua Appl. Math.,* **6**, 1 (1991), 25-28.

6. Jangveladze T.A., Tagvarelia T.G. The difference scheme of the type of variable directions for one system of nonlinear partial differential equations, arising in biology. *Rep. Enlarged Sess. Semin. I. Vekua Appl. Math.,* **8**, 3 (1993), 74-75.

7. Jangveladze T., Kiguradze Z., Nikolishvili M. On investigation and numerical solution of one nonlinear biological model. *Rep. Enlarged Sess. Semin. I. Vekua Appl. Math.,* **22** (2008), 46-50.

8. Jangveladze T., Kiguradze Z., Nikolishvili M. On aproximate solution of one nonlinear two-dimensional diffusion system. *Rep. Enlarged Sess. Semin. I. Vekua Appl. Math.,* **23** (2009), 42-45.

9. Friedman A. Partial Differential Equations of Parabolic Type. *Prentice-Hall,* 1964.

10. Ladyzhenskaya O.A, Solonnikov V.A, Ural'ceva N.N. Linear and Quasilinear Equations of Parabolic Type. (Russian) *Moscow*, 1968.

11. Lions J.-L., Quelques Mthodes de Rsolution des Problmes aux Limites Non Linaires. *Dunod, Gauthier-Villars, (French) Paris,* 1969.

12. Samarskii A.A. The Theory of Difference Schemes. (Russian) *Moscow,* 1977.

Received 12.05.2010; accepted 28.09.2010.

Authors' address:

Z. Kiguradze
I. Vekua Institute of Applied Mathematics
Iv. Javakhishvili Tbilisi State University
2, University St., Tbilisi 0186
Georgia
E-mail: zkigur@yahoo.com

M. Nikolishvili
Iv. Javakhishvili Tbilisi State University
2, University St., Tbilisi 0186
Georgia
E-mail: maianikolishvili@yahoo.com

B. Tabatadze
Iv. Javakhishvili Tbilisi State University
2, University St., Tbilisi 0186
Georgia
E-mail: besotabatadze84@gmail.com