Reports of Enlarged Session of the
Seminar of I. Vekua Institute
of Applied Mathematics
Volume 25, 2011

ON ONE MODEL OF CONDENSATE ORIGINATION AND DETERMINATION OF ITS PLACEMENT

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Abstract. In the present paper the problem of prediction of possible points of hydrates origin in the main pipelines taking into consideration gas non-stationary flow and heat exchange with medium is studied. For solving the problem the system of partial differential equations is investigated. Numerical calculations have shown efficiency of the suggested method.

Keywords and phrases: Hydrates origin, pipeline, gas non-stationary flow, mathematical modelling.

AMS subject classification: 76N15.

1. Problem formulation. One of the main reasons of pipeline obstruction is generation of hydrates. To take timely steps against generating of hydrates, it is necessary to study humidity and distribution of pressure and temperature. It is well-known that favorable conditions for generating hydrates are along the main pipeline, where dew point lies. Dew point is obtained at intersection of equilibrium temperature plot of hydrate generation and corresponding curve of gas temperature. Equilibrium temperature T_{hdr} is calculated as follows [1]:

$$T_{hdr} = s \ \lg P - u, \tag{1}$$

where P is pressure, s and u are constants, which are defined from experiment for gas with particular content and specific weight (special graphs exist) [1]. Therefore, to define the initial point of possible generation zone of hydrate, it is necessary to find out pressure and temperature. Problems of such type are discussed and investigated under the condition of stationary flow [1]-[7]. We will consider the problem of defining of possible generation point of condensate in the pipeline under the conditions of non-stationary flow. So investigation the problem of disclosing of the location and amount of accidental gas escape from the main gas pipeline is one of the urgent task of the present days.

Mathematical statement of the problem. It is well-known that non-stationary, non-isothermal flow of gas in the main pipeline is described by the following system of equations [1]:

$$\frac{\partial \omega}{\partial t} = \frac{1}{\rho_0}\frac{\partial P}{\partial x} + \nu_0 \cdot \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \omega}{\partial r}\right), \tag{2}$$

$$\frac{\partial P}{\partial t} = \rho_0 C_p \left(1\frac{C_p}{C_\nu}\right)\frac{\partial T}{\partial t} - \frac{\rho_0 C^2 C_\nu}{C_p}\frac{\partial \omega}{\partial x}, \tag{3}$$

$$\frac{\partial \rho}{\partial t} = \rho_0 \frac{\partial \omega}{\partial x}, \tag{4}$$

$$\frac{\partial T}{\partial t} = -\frac{1}{\rho C_p}\frac{\partial P}{\partial t} + a \cdot \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial x}\right), \tag{5}$$

where $\omega(x, r.t)$ is a velocity of gas flow, $P(x, r.t)$ - gas pressure, $T(x, r.t)$ - temperature (absolute), $\rho(x, r.t)-$ gas density, ρ_0 - is gas density in normal conditions, α - heat conduction coefficient of gas, ν_0 - viscosity in normal conditions, C_p - heat capacity under the constant pressure, C_ν- heat capacity under the constant volume, C - speed of sound propagation in gas. We consider initial and boundary conditions below. r is a distance from point of the circle obtained by cross-section to the center. Using functions $P(x, r.t)$ and $T(x, r.t)$, obtained as a result of solution of problem (2)-(5), on the basis of equality (1), we construct an inequality:

$$T(x, r.t) < s \lg P(x, r.t) - u,$$

with the additional constraints: $0 \leq x < L$, $0 \leq r < R$, $t \geq 0$.

Here R is a radius of pipeline cross-section; L - length of the main pipeline. In the expression for equilibrium temperature T_{hdr} of possible generation of hydrates, to define numerical values of parameters s and we will use relation between equilibrium temperature T_{hdr} and pressure of possible generation of hydrates, which is different for various gases [3]-[5]. For example, points (10^0C; $6 \cdot 10^6$ Пa) and (20^0C; $2.5 \cdot 10^7$ Пa) lie on one graph and from the graph we have:

$$s\ \lg(6 \cdot 10^6) - u = 10 \quad s = 16,1,$$

$$s\ \lg(2.5 \cdot 10^7) - u = 20 \quad u = 99,1,$$

$$T_{hdr} = 16,1 \lg P - 99,1.$$

Problem solution. To use conveniently numerical methods, let us rewrite system (2)-(5). Namely, if we solve equations (3)-(5) with regards to derivatives $\dfrac{\partial P}{\partial t}$ and $\dfrac{\partial T}{\partial t}$, system (2)-(5) takes the form:

$$\frac{\partial \omega}{\partial t} = \frac{1}{\rho_0}\frac{\partial P}{\partial x} + \nu_0 \cdot \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \omega}{\partial r}\right), \tag{6}$$

$$\frac{\partial T}{\partial t} = -\frac{\rho_0 C^2}{\varkappa^2 \cdot A} \cdot \frac{\partial \omega}{\partial x} + \frac{aC_\nu p}{A} \cdot \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right), \tag{7}$$

$$\frac{\partial \rho}{\partial t} = -\rho_0\frac{\partial \omega}{\partial x}, \tag{8}$$

$$\frac{\partial P}{\partial t} = -\frac{\rho\rho_0 C^2 C_\nu^2}{\varkappa \cdot A} \cdot \frac{\partial \omega}{\partial x} - \frac{\rho a \rho_0 C_p R_g}{A} \cdot \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right), \tag{9}$$

where $\varkappa = \dfrac{C_p}{C_\nu}$, $R_g = C_p - C_\nu$, $A = \rho \cdot C_\nu - \rho_0 C_\nu + \rho_0 C_p$.

The obtained system (6)-(9) with corresponding initial-boundary conditions represents mathematical model which can be effectively solved numerically using explicit or implicit schemes. However, in fact, it is not always possible to obtain data characterize initial distribution of velocity of gas flow, gas pressure, temperature and gas density in the pipeline.

Case when gas flow is adiabatic. The system (2)-(5) can be simplified. Let us consider the case when gas flow is adiabatic. It means that there is not heat exchange between pipeline and surrounding along the pipeline. In this case there is a simple relationship between pressure, density and sound velocity in gas, which can be described by the following relation [1], [3]-[5]:

$$\frac{dP}{d\rho} = C^2. \tag{10}$$

From (10) we have $P = c^2 \cdot \rho + P_0 - c^2\rho_0$, where P_0 is gas pressure in normal conditions.

From the last equality we have $\frac{\partial \rho}{\partial t} = \frac{1}{C^2}\frac{\partial P}{\partial t}$.

And equations (5) and (4) take the form:

$$\frac{\partial T}{\partial t} = -\frac{C^2}{C_p} \cdot \frac{1}{P} \cdot \frac{\partial P}{\partial t} + a \cdot \frac{1}{r} \cdot \frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right),$$

$$\frac{\partial P}{\partial t} = -C^2\rho_0\frac{\partial \omega}{\partial x}. \tag{11}$$

Substituting equation (11) in the equation (5) gives :

$$\frac{\partial T}{\partial t} = \frac{C^2\rho_0}{C_p\rho} \cdot \frac{\partial \omega}{\partial x} + a \cdot \frac{1}{r} \cdot \frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right).$$

If instead of equation (5), we consider equation (10) than as a result, we obtain the following system of equations:

$$\frac{\partial \omega}{\partial t} = \frac{1}{\rho_0}\frac{\partial P}{\partial x} + \nu_0 \cdot \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \omega}{\partial r}\right), \tag{12}$$

$$\frac{\partial T}{\partial t} = -\frac{C^2\rho_0}{C_p} \cdot \frac{1}{\rho} \cdot \frac{\partial \omega}{\partial x} + a \cdot \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right), \tag{13}$$

$$\frac{\partial P}{\partial t} = -C^2\rho_0 \cdot \frac{\partial \omega}{\partial x}, \tag{14}$$

$$P = C^2\rho + p_0 - c^2\rho_0. \tag{15}$$

If we neglect the change of speed along the radius, equation (12) will take the form:

$$\frac{\partial \omega}{\partial t} = -\frac{1}{\rho_0}\frac{\partial P}{\partial x}. \tag{16}$$

Equations (14) and (16) represent a system of two-variable equations with regards to functions $P(x,t)$ and $\omega(x,t)$, and from them we obtain the following hyperbolic equation

$$\frac{\partial^2 P}{\partial t^2} = C^2\frac{\partial^2 P}{\partial x^2}. \tag{17}$$

The equation (13) is solved by the following initial

$$P(x,0) = \varphi(x), \quad 0 \le x \le L, \tag{18}$$

$$\frac{\partial P}{\partial t}|_{t=0} = \psi(x), \quad 0 \le x \le L, \tag{19}$$

and boundary conditions

$$P(0,t) = \mu_1(t), \quad t \ge 0, \tag{20}$$

$$P(L,t) = \mu_2(t), \quad t \ge 0. \tag{21}$$

The method for finding the solution to problem (17)-(21) is known [2]. After finding the function $P(x,r,t)$, we will find the functions $\omega(x,r,t)$, $T(x,r,t)$, $\rho(x,r,t)$, from equations (16), (13) and (15), consequently.

Acknowledgement. The research has been funded by the Grant of the Georgian National Science Foundation #GNSF/ST09-614/5-210.

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Received 21.06.2011; revised 12.09.2011; accepted 29.10.2011.

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