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HEAT TRANSFER IN A CIRCULAR MAGNETOHYDRODYNAMIC CHANNEL FOR FINITE VALUES OF MAGNETIC REYNOLDS NUMBER

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Abstract. The results of calculations of heat transfer in a circular magnetohydrodynamic (MHD) channel under the action of an inhomogeneous magnetic field with regard for the magnetic field induced by electric currents in the fluid, are obtained.

Keywords and phrases: Fluid, flow, magnetohydrodynamic channel, heat transfer.

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At present, the works dealing with the processes of heat transfer in MHD channels under the action of inhomogeneous magnetic fields are, practically, not available. As usual, we consider completely developed regimes of heat transfer; in some works concerning the development of heat transfer the problem is being solved either for a given velocity profile (homogeneous, parabolic, Hartmanean), or together with the flow development [1,2]. Simultaneous development of temperature and velocity profiles have been studied in [3,4]. Calculations performed in the above-mentioned works show that temperature profiles in the channels possess, despite velocity changes along the flow, no considerable qualitative singularities. Two-dimensionality of the flow affects only characteristics of heat transfer [1, p.337]. It should also be noted that in [3,4] variation of the velocity profile along the flow has no singularities.

As is shown in [3-8], the character of the flow in an inhomogeneous magnetic field is defined by the size and type of its inhomogeneity Depending on the character of magnetic field inhomogeneity there may occur plane Hartmanean structures of flows, as well as those with a considerable inhomogeneity in the velocity profile. The existence of complex velocity structures in some types of inhomogeneous magnetic fields allows us to presuppose that processes of heat transfer have singularities.

In the present work we present the results of calculations for the heat transform in a circular MHD channel under the action of the inhomogeneous magnetic field with regard for the magnetic field induced by electric currents flowing in the fluid (for finite values of magnetic Reynolds number). As is known, cylindrical surfaces are the most wide-spread heat transferring surfaces. An exterior magnetic field arises by a cylindrical two-sided ferromagnetic inductor whose outer magnetowire has current load $|z < c_1|$, $r = r_2 + d$. Magnetic field of such a system is axial-symmetric, inhomogeneous with respect to radius and to z, with alternating signs (when passing through the cross-section it changes its sign $z = 0$). In the working space of the inductor there is a circular channel with isolated walls. Velocity structures of the MHD flow in the circular channel of such an inductor for finite values of magnetic Reynolds number (Rm) are given in [4]. Although physical properties of the medium (density, electric conductivity, coefficients of viscosity and heat conductivity) depend strongly on its temperature, in order to single out the influence of magnetic field inhomogeneity on the heat transfer it is desirable at the first stage to neglect this dependence. Under such an assumption, the magnetohydrodynamic and heat parts of the problem separate, and using the variable scalar Ψ and vector A magnetic potentials, vorticity ω, current function ψ and temperature Θ, we can write a system of equations in cylindrical coordinates in the form

$$\Delta\Psi = 0, \tag{1}$$

$$\frac{1}{r}\frac{D(\psi,\omega)}{D(z,r)} - \frac{1}{r^2}\frac{\partial\psi}{\partial z}\omega = \frac{1}{\mathrm{Re}}\Big(\Delta\omega - \frac{\omega}{r^2}\Big) + NF(\psi, A, \Psi),$$

$$\Delta\psi = r,$$

$$\Delta A - \frac{A}{r^2} + \frac{Rm}{r}\,\theta(r-r_2)\theta(r_1-r)\Big(\frac{D(\psi,A)}{D(r,z)} + \frac{\partial\psi}{\partial r}\frac{\partial\Psi}{\partial r} - \frac{\partial\psi}{\partial z}\frac{\partial\Psi}{\partial z}\Big) = 0,$$

$$\frac{1}{r}\frac{D(\psi,\theta)}{D(z,r)} = \frac{1}{Pe}\Delta\theta + \frac{Ec}{\mathrm{Re}}\Phi + EcNI. \tag{2}$$

Here

$$\Delta = \frac{1}{r}\frac{\partial}{\partial r}\Big(\frac{\partial}{\partial r}\Big) + \frac{\partial^2}{\partial z^2}; \quad \frac{D(u,v)}{D(x,y)} = \frac{\partial u}{\partial x}\frac{\partial v}{\partial y} - \frac{\partial u}{\partial y}\frac{\partial v}{\partial x},$$

$$\theta(r) = \begin{cases} 0, & r \geq 0; \\ 1, & r < 0; \quad F = \mathrm{rot}_\varphi[(\vec{V}\times\vec{b})\times\vec{b}], \end{cases}$$

$\Phi = 2\Big[\Big(\frac{\partial v_r}{\partial r}\Big)^2 + \Big(\frac{v_r}{r}\Big)^2 + \Big(\frac{\partial v_z}{\partial z}\Big)^2 + \frac{1}{2}\Big(\frac{\partial v_z}{\partial r} + \frac{\partial v_r}{\partial z}\Big)^2\Big]$ is the Joule's heat release; A is the unit vector-potential component of magnetic field.

Magnetic field b can be defined by summing the exterior (b_e) and the induced (b_i) field: $b_e = -\Delta\Psi$, $b_i\nabla\times A$ and the velocity components can be found by differentiating the current functions; $v_r = \frac{1}{r}\frac{\partial\psi}{\partial z}$; $v_z = -\frac{1}{r}\frac{\partial\psi}{\partial r}$. The dimensionless criteria such as Reynolds number $\mathrm{Re} = V_0h/v$, magnetic Reynolds number $Rm = \mu\sigma V_0 L$, parameter of MHD interaction $N = \sigma B_0^2 h/\rho V_0$, Péclet $Pe = V_0 h/\alpha$ and Eckert's $Ec = V_0^2/c_p(T_2 - T_1)$ numbers, appearing in the system of equations (1), contain the following base values: V_0, the flow rate in the channel; h, the channel height; L, inductor length; B_0, magnetic field size on the channel axis under the ferromagnetic (currentless load); $T_2 - T_1$, difference of wall temperatures.

When writing the boundary conditions, we have used the property of fluid sticking to walls, magnetic field orthogonality of ferromagnetic surface ($\mu = \infty$) and its vanishing on the perephery: on $\Gamma_1\Psi = 0$, $\partial A/\partial r = 0$; on Γ_2 and Γ_5, $\partial\Psi/\partial r = 0$; $\partial A/\partial r = 0$; $\partial A/\partial z = 0$ on $\Gamma_3\partial\Psi/\partial r = 0$, $A = 0$; on $\Gamma_4\Psi = -\delta$, for $z_1 < -c_1$; $\Psi = \delta z/c_1$ for $|z| < c_1$; and $\Psi = \delta$ for $z > c_1$; $\partial A/\partial n = 0$.

For the inlet cross-section of the channel $\omega = \text{const}$, $\psi = \text{const}$; these values were defined with regard for the velocity profile in the developed flow. On the outlet boundary of the calculated area we have put "soft" boundary conditions: $\partial\omega/\partial z = 0$ and $\partial\psi/\partial r = 0$, on the channel walls ($r = r_1$ and $r = r_2$) $\psi = \text{const}$, approximation of the boundary conditions for vorticity was of the form

$$\omega_{i,j_0} = 2(\psi_{i,j_0+1} - \psi_{i,j_0})/r_1/(\Delta r)^2, \quad \omega_{i,j_M} = 2(\psi_{i,j_M-1} - \psi_{i,j_M})/r_2/(\Delta r)^2,$$

where j_0 and j_M are, respectively, the wall points on the interior and exterior channel walls, Δr is the mesh step with respect to radius.

For temperature on the channel walls we prescribed the following first kind conditions: $\Theta = 1$ for a hot wall and $\Theta = 0$ on a cooler wall. On the magnetic field input we prescribed a completely stabilized temperature field, and on its outlet, $\partial\Theta/\partial z = 0$.

The method of solution of the magnetohydrodynamic part of the problem and the results of calculations can be found in [4]. The obtained distributions of hydrodynamic and electromagnetic variables have been organized into a collection of data which later on were used for the solution of heat problem. The solution of the equation of hear influx (2) is performed numerically by the method of establishment. We used the explicit two-layer scheme with the values of variables on the current iteration. Since the length of the heat stabilization for large values Pe is more than hydrodynamic one, the calculated region increased with respect to z (in some cases, doubly). The numerically obtained temperature distribution in the stabilized area coincided with that obtained analytically [4,6,7].

The character of temperature distribution in a medium depends essentially on the Péclet number. For small values Pe, temperature distribution corresponds to the stabilized heat transfer in a circular channel, i.e., it does not, practically, depend on the flow velocity of the fluid. For $Pe = 100$, there dominates the convective mechanism of heat transfer in a medium, and the temperature distribution is defined by the structure of the flow. The intensity of heat transfer between the wall and the medium varies along the channel due to the non-uniform MHD flow in the inhomogeneous magnetic field [5]. For example, for $Rm = 0$, near the cross-section $z = 0$, the convective heat transfer from the hot outer surface to the fluid decreases significantly, whereas on the internal cylindrical surface of the channel the heat transfer processes in the cross-section become stronger. The increase of intensity in heat transfer processes in the internal wall can be explained by the existence of jet flow flowing around it on the entire segment of the channel. In the same cross-section, the fluid discharge in the external wall is minimal which leads to decreasing the convective heat removal. Mixing by extremum of the function $Nu(z)$ for small values $(Rm = 2)$ is due to the shift of a part of jet flow [5]. For sufficiently large values of the magnetic Reynolds number $(Rm) = 6 \cdots 20$, the velocity maximum shiftsf from the lower to the upper wall [5,8], and hence the local Nusselt's number on both walls varies.

On the whole, the non-uniform MHD flow strengthens heat transfer between the wall and the fluid in an inhomogeneous magnetic field.

When Eckert's number is small for metals in the liquid state, in most cases we may neglect dissipative processes occurring in them. In the case under consideration, we have defined the values for the viscous and Joule's energy dissipation by, respectively, the combinations Ec/RRe and $Ec \cdot N$ (2). Since the parameter of MHD interaction in power units with metallic heat-transfer agents in the liquid state may achieve $\sim 10^4$ [5], the contribution of the Joulean energy dissipation may turn out to be very essential.

Thus the non-uniformity of MHD flow in an inhomogeneous magnetic field stipulates the existence of the convective mechanism of heat transfer. Decreasing of convective heat transfer on some parts of the channel should be taken into account to avoid overheating and subsequent destruction of the system under high temperatures. On the other hand, the non-uniform MHD flow in an inhomogeneous magnetic field can be used for cooling those areas with high-temperature regime and for heat distribution over the whole cold agent.

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