Reports of Enlarged Session of the
Seminar of I. Vekua Institute
of Applied Mathematics
Volume 24, 2010

ON A CERTAIN PROBLEM OF MAULDIN-PREISS-WEIZSACKER

Pantsulaia G.

Abstract. We show that Gardner transition kernel (see, [1], Example 1.4, p. 972) is such an example of a modified uniformly orthogonal transition kernel which is not modified completely orthogonal. This answers negatively to the Problem 2 posed in [3].

Keywords and phrases: Modified uniformly orthogonal transition kernel, modified completely orthogonal.

AMS subject classification: 03E35, 03E50, 11T99.

Let X and Y be Polish spaces.

Definition 1. A family $(\mu_x)_{x\in X}$ is called a transition kernel from X to Y if for x in X, $(\mu_x)_{x\in X}$ is a family of probability measure defined on the Borel subsets of Y and for each Borel subset E of Y the function $x \to \mu_x(E)$ is a Borel measurable map of X into $I = [0, 1]$.

Definition 2. The transition kernel $(\mu_x)_{x\in X}$ is said to be pairwise orthogonal provided that if $x \neq x^{'}$, then μ_x and $\mu_{x'}$ are mutually singular.

Definition 3. The transition kernel $(\mu_x)_{x\in X}$ is said to be uniformly orthogonal provided there is a Borel subset B of $X \times Y$ such that for each x in X, $\mu_x(B_x) = 1$ and if $x \neq x^{'}$, then $\mu_{x'}(B_x) = 0$; $(\mu_x)_{x\in X}$ is said to be modified uniformly orthogonal (m.u.o) if the set B is not required to be a Borel set.

Definition 4. The kernel $(\mu_x)_{x\in X}$ is said to be completely orthogonal provided there is a Borel subset B of $X \times Y$ such that for each x in X, $\mu_x(B_x) = 1$ and if $x \neq x^{'}$, then $B_x \cap B_{x'} = \emptyset$; The set B is said to be a completely separating the transition kernel $(\mu_x)_{x\in X}$. The kernel $(\mu_x)_{x\in X}$ is modified completely orthogonal (m.c.o) if the requirement that B be a Borel set is dropped.

R.D. Mauldin, D. Preiss and H.Y. Weizsacker stated the following

Problem 1. (see, [3], Problem 2, p. 2) Is a modified completely orthogonal kernel a completely orthogonal kernel?

We need the following auxiliary propositions.

Lemma 1. (Shoenfield [4]) *(MA) Let $(X_i)_{i\in I}$ be a family of Lebesgue null sets in $\mathbf{R}^n$ $(n \in N \setminus \{0\})$ with $card(I) < c$. Then, the $\cup_{i\in I} X_i$ is Lebesgue null in $\mathbf{R}^n$.*

Lemma 2. (see [4]) *Every analytic set in $\mathbf{R}$ is Lebesgue measurable.*

By using Lemmas 1-2, we get the validity of the following assertion answering negatively to the Problem 1.

Theorem. *(MA) Gardner transition kernel (see, [1], Example 1.4, p. 972) is such an example of a modified completely orthogonal kernel which is not completely orthogonal.*

REFERENCES

1. Gardner R.J. A note on conditional distributions and orthogonal measures. *Ann. Probab.*, **10**, 3 (1982), 877-878.

2. Mauldin R.D., Preiss D., Weizsacker H.Y. Orthogonal transition kernels. *Ann. Probab.*, **11**, 4 (1983), 970-988.

3. Mauldin R.D., Preiss D., Weizsacker H.Y. A survey of results and problems concerning orthogonal transition kernels, preprint (Research supported by NSF Grant MCS 81-015881).

4. Shoenfield J.R. Martin's axiom. *Amer. Math. Monthly.*, **82** (1975), 610-617.

5. Martin Donald A. Measurable cardinals and analytic games. *Fund. Math.*, **66** (1969/1970), 287-291.

Received 24.05.2010; revised 25.10.2010; accepted 12.11.2010.

Author's address:

G. Pantsulaia
Department of Mathematics
Georgian Technical University
77, M. Kostava St., Tbilisi 0175
Georgia

I. Vekua Institute of Applied Mathematics of
Iv. Javakhishvili Tbilisi State University
2, University St., Tbilisi 0186
Georgia
E-mail: gogipantsulaia@yahoo.com