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ON LEAK DETECTION IN THE MAIN PIPELINE

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Abstract. The leaks caused by damage of pipelines are usually very dangerous. Intensive leaks can stimulate explosions, fires and environment pollution, which can lead to the ecological catastrophe. In the present paper determine the location and amount of accidental gas escape from the main gas pipe-line is studied. For solving the problem it has been discussed early-made method, reason is that the exact analytical method has not been existed. The analytical expressions which are able to find location and expenditure of accidental gas escape in the main gas pipe-line with branches are obtained We have created quite general test, the manner of the solution has been known in advance. Comparison has shown us the affectivity of the suggested method.

Keywords and phrases: Leak detection, gas pipeline, mathematical modeling.

AMS subject classification: 49J20, 76N25.

At present natural gas consumption is very high in the world and it is expected that will increased on 50 % in nearest 20 years [1,2]. at the same time the gas delivery infrastructure is quickly deteriorate and nowadays in general it's not in a good condition [1-4]. The probability of leaks may occur at any time and location, therefore leak detection in transmission pipelines is important for the safe operation of pipelines and for economical and environmental point of views too. Today, there are several methods and many available modern technologies for leak detection [1-14]. For instance: Acoustic sensors (detects leaks based on acoustic emission); Gas sampling (flame Ionization detector. used to detect natural gas); Soil monitoring (detects tracer chemicals added to gas pipe line); Flow monitoring. (monitor either pressure change or mass flow); Dynamic modeling (monitored flow parameters modeled); Diode laser absorption (absorption of diode lasers monitored); Thermal imaging (passive monitoring of thermal gradients); Backscatter imaging (natural gas illuminated with CO2 laser); Multi-spectral imaging (passive monitoring using multi-wavelength infrared imaging); Millimeter wave radar systems (radar signature obtained above pipe lines). etc.

The solution of the problem of disclosing the location of an accidental gas escape from the main pipe-line is known not only for simple pipe-line but also for the complicated one. In conditions of stationary usage of gas even flow is stationary in the main pipe-line. But from the moment of accidental gas escape non-stationary process is in progress. After some periods new stationary situation is formed. That is way it's important to know (detect) the location and intensity of accidental gas escape even in non-stationary flow, with the purpose to reduce lack loss of gas and in ecological way too. Let's consider a linear mathematical model linear of non-stationary flow in gas pipe-line, which is described by the following parabolic partial differential equation and proper corresponding initial and boundary conditions:

$$\frac{\partial u}{\partial t} = a^2 \frac{\partial^2 u}{\partial x^2} + \sum_{k=1}^{m} M_k \delta(x - x_k) + q\delta(x - x^*)\sigma(t - t_0), \;\; 0 < x < L, \; t > 0, \qquad (1)$$

$$u(x,0) = Q(x), \quad 0 \le x \le L, \qquad (2)$$

$$\frac{\partial u}{\partial x} = g_1, \quad \text{when } \; x = 0, \;\; t > 0, \qquad (3)$$

$$\frac{\partial u}{\partial x} = g_2, \quad \text{when } \; x = L, \;\; t > 0. \qquad (4)$$

Here a^2 is a constant, m is number of equations, $M_k (k = 1 \cdots m)$ are expenses of gas in the off-shoots, $x_k (k = 1 \cdots m)$ are coordinates of points in off-shoots, q is intensity of the accidental gas escape. x^* is coordinate of the beginning of the escape, $\delta(\cdot)$ is Dirac function, $\sigma(\cdot)$ is Heaviside function, L is length of the main pipe-line, $u = u(x,t)$ is function which describes distribution of the pressure along the pipe-line, $Q(x)$ is function which describes distribution of the initial pressure and it may be represented as:

$$Q(x) = \frac{S - R}{L} x + R, \qquad (5)$$

where R and S are values of the pressures at the entrance and at the ending of the main gas pipe-line. With the help of Green function it is possible to write (find) solution of the problem (1)-(4) which will be based (depend) on x^* and q, as parameters. For x^* and q, parameters it will be enough to determine two extra conditions. For instance suppose that as the result of measure it has been found out that in moment T equalities are existed:

$$u(0,T) = \overline{R}\bar{R}, \quad u(L,T) = \bar{S}.$$

If we leave only the first two members from the sequence in Green function we will get: (constant and one trigonometric member):

$$q = \frac{L(\bar{R} + \bar{S} - R - S) + 2a^2 T(g_1 - g_2) - 2T \sum\limits_{i=1}^{m} M_i}{2(t - t_0)},$$

$$x^* = \frac{L}{\pi} \arccos \left\{ \frac{1}{2q(1 - e^{-p(T - t_0)})} \left[P(T - t_0)q + \frac{Lp}{2}(R + S) - Lp\bar{S} \right.\right.$$
$$+ \frac{4Lp(R - S)}{\pi^2} e^{-pT} + pT \sum_{i=1}^{m} M_i + pa^2 T(g_2 - g_1)$$
$$\left.\left. + 2\left(a^2 g_1 + a^2 g_2 - \sum_{i=1}^{m} M_i \cos \frac{\pi x_1}{L} \right)(1 - e^{-pT}) \right] \right\},$$

where $p = \frac{a^2 \pi^2}{L^2}$. There was represented one example to learn the affectivity of the method [5]. There will be created one more general test by us. We mean that the ending of the pipe are locked $g_1 = g_2 = 0_1$; The pressures at the ending points are equal $S = R$, and off-shoots are locked $M_j = 0$, $(j = 1 \cdots m)$. After time T, from the moment of gas escape, between the endings of pressure we have the following relationship $\bar{S} = a\bar{R}$. In these conditions, we will receive

$$q = \frac{L}{2T}[(a+1)/\bar{R} - 2R].$$

Let's note that in the equation (1) $q < 0$. In case of escape from the pipe, it means that:

$$(a+1)/\bar{R} - 2R < 0. \tag{6}$$

In above mentioned conditions we will have:

$$x^* = \frac{L}{\pi}\arccos\left\{\frac{p\bar{R}T}{2[(a+1)\bar{R} - 2R](1 - e^{-pT})}(1-a)\right\}.$$

It's clear that:

$$1 - e^{-pT} > 0. \tag{7}$$

(6) Let's put a sign note

$$K = \frac{\bar{R}p\bar{T}}{2[(a+1)\bar{R} - 2R](1 - e^{-pT})}. \tag{8}$$

We will receive $x^* = \frac{L}{\pi}\arccos[K(1-a)]$, where because of (5) and (6), $K < 0$.

Let's consider the following cases:

1) when $a = 1$, then $x^* = \frac{L}{2}$ place of escape is the middle point, it is like real fact;

2) when $0 < a < 1$, then $k(1-a) < 0$ and $\frac{\pi}{2} < \arccos[K(1-a)] < \pi$.

So $x^* > \frac{L}{2}$, which is reality, in these conditions $\bar{S} < \bar{R}$ and the point of escape is between the centre and the right ending;

3) when $a > 1$, then $\bar{S} > \bar{R}, K(1-a)] > 0, 0 < \arccos[K(1-a)] < \frac{\pi}{2}$ and that's why $x^* < \frac{L}{2}$. This fact describes reality, in this case the point of escape is between the centre and the left ending.

Obtained results can be summarized as follows:

Conclusions "To determine the location and amount of accidental gas escape from the main gas (oil) pipe-line has economical and ecological meaning." For solving the problem it has been discussed early-made method, reason is that the exact analytical method has not been existed." We have created quite general test, the manner of the solution has been known in advance. Comparison has shown us the affectivity of the following method.

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