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A DISTRIBUTION OF A PERIODIC THERMAL WAVE TO A MESOSCALE BOUNDARY LAYER OF AN ATMOSPHERE

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Abstract. The problem about mesoscale boundary layer of an atmosphere above thermal "island" is put and solved at its periodic heating with take into account of humidity fields. It is received space-temporary distribution of meteorological fields (components of an air velocity, pressure, temperature, water-vapor- and liquid-water mixing ratious). The accent is done on process fog- and cloudformation.

Keywords and phrases: A mesoscale boundary layer of an atmosphere, a numerical simulation, a fog, a cloud, a local weather forecast.

AMS subject classification: 86A10, 37N10, 92D40.

Introduction. The object of our researches is numerical modelling of a mesoscale boundary layer of an atmosphere (MBLA) and meteoprocesses existing in it (clouds, fogs, spreading of an aerosol...). We are interested also in artificial influence on the some these processes.

The subject is really "Georgian" following from physico-geographical conditions of our republic. Its urgency is caused by such significant areas of a science, as local weather forecast, ecology, aviation, agro- and sea meteorology, etc.

We wish especially to concern humidity processes of an atmosphere, particularly, clouds and fogs. They are just responsible for radiating and water balance on the Earth. In their person the infinite stock of a pure water is had on the future; also their role is great in formation of an electromagnetic mode on our planet. Their research is rather actual and from the point of view of ecology; besides it is necessary to recollect and that fact, that artificial influence was conducted above these processes in our republic during several tens years [1].

Statement of a problem and its decision. We consider a two-dimensional non-stationary problem about MBLA in vertical $(x-z)$ planes. If we shall apply Bussinesc free convection simplification, cvasistatic approximation, constancy of turbulence coefficients, the method of taking into account of macrometeoprocesses on mesoprocesses, then the initial system of MBLA equation will have such view [2-6]:

$$
\begin{aligned}
&\frac{du}{dt} = -\frac{\partial \pi}{\partial x} + \Delta' u,\\
&\frac{\partial \pi}{\partial z} = \lambda \theta,\\
&\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0,\\
&\frac{d\theta}{dt} + Sw = \frac{L}{c_p}\Phi + \Delta' \theta,
\end{aligned}
$$

$$\frac{dq}{dt} + \gamma_q w = -\Phi + \Delta' q,$$
$$\frac{dv}{dt} = \Phi + \Delta' v,$$
$$\frac{d}{dt} = \frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + w\frac{\partial}{\partial z},$$
$$\Delta' = \mu\frac{\partial^2}{\partial x^2} + \nu\frac{\partial^2}{\partial z^2},$$

where u, w are horisontal and vertical components of an air velocity, respectively, π, θ, q - deviations of a pressure analog, a potential temperature and a water-vapor mixing ratio from their undisturbed fields, respectively, v - a liquid-water mixing ratio, λ, S - parameters of an atmospheric flotation and a stratification, respectively, γ_q - a vertical gradient of an undisturbed water-vapor mixing ratio, Φ - a rate of a water-vapor condensation, L -a latent heat of condensation, c_p - a specific heat of a dry air at a constant pressure, μ, ν - horisontal and vertical coefficients of turbulence, respectively.

Initial and boundary conditions in a general view can be written down as follows:

$$\begin{array}{llllll}
\text{at } z = 0 & u = 0, & w = 0, & \theta = F(x,t), & q = 0, & v = 0, \\
\text{at } z = Z & u = 0, & \pi = 0, & \theta = 0, & \dfrac{\partial q}{\partial z} = 0, & \dfrac{\partial v}{\partial z} = 0, \\
\text{at } x = 0, X & \dfrac{\partial u}{\partial x} = 0, & \dfrac{\partial \theta}{\partial x} = 0, & \dfrac{\partial q}{\partial x} = 0, & \dfrac{\partial v}{\partial x} = 0, & \\
\text{at } t = 0 & u = 0, & \theta = 0, & q = 0, & v = 0, &
\end{array}$$

where X, Z horizontal and vertical sizes of MBLA, and temperature of MBLA underlying surface which we take from meteoexperiments [4]:

$$F1(x,t) = \begin{cases} 0 & 0 \le x \le 32\text{km}, \qquad 48\text{km} < x \le 80\text{km}, \\ 5\sin\omega t & 32\text{km} \le x \le 48\text{km}, \end{cases}$$

here ω is an angular velocity of daily rotation of the Earth. Thus, we have set the problem about in the conditions of only temperature nonhomogeneity of a underlying surface. On the other hand our model can be considered also as distribution of a periodic thermal wave to atmosphere from an earth surface - this thought and has served us as an occasion to the present entitling of the given article. Problem statement in this plan has as well other loading - about what we mention in a final part of our work.

We do not stop in detail as on physical, and numerical methods of the decision of a problem; they are given in our early works [5, 6]. Let's present values of physical constants and those parameters of a problem which did not change in discussed numerical experiments : $\lambda = 0.033\frac{m^2}{sec.grad}$, $S = 0.004\frac{grad}{m}$, $L = 600\frac{cal}{g}$, $c_p = 0.24\frac{cal}{g.grad}$, $\mu = 10^4\frac{m^2}{sec}$, $\nu = 10\frac{m^2}{sec}$, relative humidity $f = 0.95$, $X = 80$km, $Z = 2$km.

Discussion of results. As a result of the problem decision a space-time distribution of meteofields $(u, w, \pi, \theta, q, v)$ is received. Because of the limited volume of the article only liquid-water mixing ratio isolines of the simulated cloud ($t = 9h.$, $v_{max} = 0.05\frac{g}{kg}$) and fog ($t = 21h.$, $v_{max} = 0.08\frac{g}{kg}$) by way of illustration are resulted, Fig. 1 and Fig. 2, respectively. These humidity processes had the maximum capacity just during these moments. The received results qualitatively quite well describe investigated meteoprocess [4].

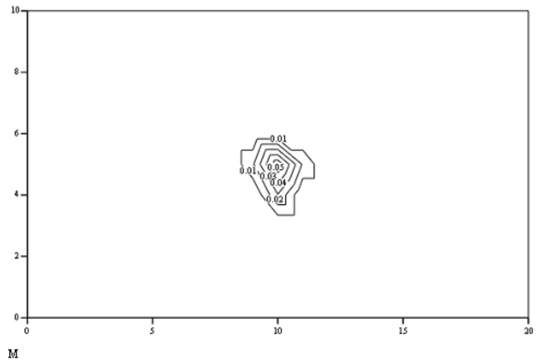

Fig.1.

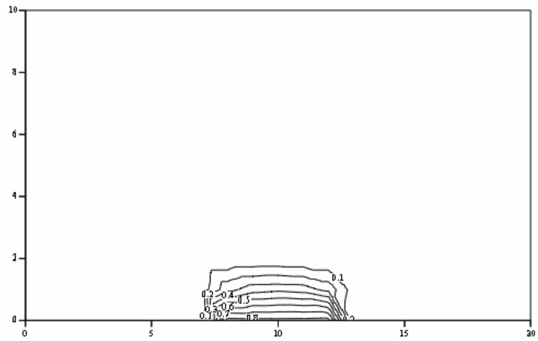

Fig.2.

At the decision of mesometeorological problems (System of the nonlinear differential equations with private derivatives) we cannot prove its correctness (existence, uniqueness, stability of the decision) - it is an Achilles' heel of our speciality workers. Therefore we should spend different physical and mathematical tests with the purpose of check of the numerical results received by us. In our case (periodic heating of an underlying surface) one of such tests is reception of the periodic decision, that really takes place in the nature. With this purpose we have started a problem on the account within several, ten days.

We have received rather satisfactory periodicity of meteoprocess. It is remarkable, that in due course (the eighth, ninth, tenth days) periodicity improves - probably, it is caused by the best adaptation of meteofields. This fact (obvious periodicity of process) has not only applied value (local weather forecast), but testifies to correct statement of the problem. Certainly, the case in point requires further, more detailed development to what we shall return to the near future.

Apparently from the aforesaid, investigated object periodic. It has strongly pronounced signs of self-oscillation (straight lines and feedback between the meteorological variables, well organised steady system...). It gives possibility to assume all, that further our problem can be considered from the point of view of such rather modern sphere of a science, as synergetrics [7].

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