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ON ONE MIXED BOUNDARY VALUE PROBLEM FOR THE NON-SHALLOW SPHERICAL SHELLS WHEN THE COMPONENTS OF AN EXTERNAL FORCE ARE CONSTANTS

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Abstract. In the present paper the non-shallow spherical bodies of shell type are considered, when the displacement vector is independent from the thickness coordinate x_3 and an external force $\mathbf{\Phi}$ is equal to constant. The plane deformation analogous model for the spherical bodies of shell type has been obtained. Mixed boundary value problem for the spherical segment has been solved.

Keywords and phrases: Non-shallow shell, displacement vector, stress vector.

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Let us suppose that the displacement vector is independent from the thickness coordinate x_3

$$\boldsymbol{u}(x^1,x^2,x^3)=\boldsymbol{u}(x^1,x^2).$$

It is known, that the equilibrium equations and stress-strain relations (Hook's Law) have the following complex form in the system of isometric coordinates [1]

$$\begin{cases} \dfrac{1}{\Lambda}\dfrac{\partial}{\partial z}(T_{11}-T_{22}+2iT_{12})+\dfrac{\partial}{\partial \bar z}T_\alpha^\alpha+\dfrac{2}{\rho}T_+ +F_+=0, \\ \dfrac{1}{\Lambda}\left(\dfrac{\partial T_+}{\partial z}+\dfrac{\partial \overline{T}_+}{\partial \bar z}\right)+\dfrac{1}{\rho}\left(T^{33}-T_\alpha^\alpha\right)+F_3=0. \end{cases} \tag{1}$$

$$\begin{cases} T_{11}-T_{22}+2iT_{12}=4\mu\Lambda\dfrac{\partial u^+}{\partial \bar z}, \quad \theta=\dfrac{1}{\lambda}\left(\dfrac{\partial u_+}{\partial z}+\dfrac{\partial \overline{u_+}}{\partial \bar z}\right), \\ T_\alpha^\alpha=2(\lambda+\mu)\left(\theta+\dfrac{2}{\rho}u_3\right), \quad T_{33}=\dfrac{\lambda}{2(\lambda+\mu)}T_\alpha^\alpha, \\ T_+=T_{13}+iT_{23}=T_{31}+iT_{32}=\mu\left(2\dfrac{\partial u_3}{\partial \bar z}-\dfrac{1}{\rho}u_+\right), \\ F_+=F_1+iF_2, \quad u_+=u_1+iu_2, \quad u^+=u^1+iu^2, \end{cases} \tag{2}$$

where $x^1=tg\dfrac{\theta}{2}cos\varphi$, $x^2=tg\dfrac{\theta}{2}sin\varphi$, $\left(z=x^1+ix^2, \Lambda=\dfrac{4\rho^2}{(1+z\bar z)^2}\right)$, are the isometric coordinates on the shell mid-surface. Here we use the notations

$$\boldsymbol{T^\alpha}(x^1,x^2)=\left(1+\frac{x_3}{\rho}\right)^2\boldsymbol{\sigma}^\alpha(x^1,x^2,x^3),$$

$$\boldsymbol{T^3}(x^1,x^2)=\left(1+\frac{x_3}{\rho}\right)\boldsymbol{\sigma}^3(x^1,x^2,x^3),$$

$$\boldsymbol{F}(x^1,x^2)=\left(1+\frac{x_3}{\rho}\right)^2\boldsymbol{\Phi}(x^1,x^2,x^3).$$

$\boldsymbol{\sigma}^i$ are contravariant stress vectors, $\boldsymbol{\Phi}$ an external force, $\boldsymbol{u}$ the displacement vector, λ and μ are Lame's constants, ρ is a radius of sphere.

Let us consider the components of an external force $\boldsymbol{F}$ are equal to constants

$$F_+=P_+=const,\quad F_3=P_3=const.$$

The solutions of the system (1), (2) have the following form [2]:

$$\begin{aligned}
u_+ &= 2\rho\left[\frac{\partial\chi}{\partial\bar z}-\frac{z}{1+z\bar z}\overline{\varphi(z)}-\overline{\psi(z)}\right]+\frac{\rho^2}{\mu}\left[\frac{\ln(1+z\bar z)}{\bar z^2}-\frac{z}{\bar z(1+z\bar z)}\right]\bar P_+\\
&+\frac{2\rho^3}{\mu}\frac{z}{1+z\bar z}P_3,\\
u_3 &= \chi(z,\bar z)+\frac{\lambda+3\mu}{4(\lambda+2\mu)}\left[\varphi(z)+\overline{\varphi(z)}-\frac{\rho}{2\mu}(P_+\bar z+\bar P_+z)\right]-\frac{\rho^2}{2\mu}P_3,
\end{aligned}$$

where $\varphi(z)$ and $\psi(z)$ are holomorphic functions of z and $\chi(z,\bar z)$ is a solution of the equation $\nabla^2\chi+\frac{2}{\rho^2}\chi=0$ [3], which is expressed with the help of holomorphic function $f(z)$ by formula

$$\chi(z,\bar z)=2Re\left[f(z)-\frac{2\bar z}{1+z\bar z}\int_0^z f(t)dt\right].$$

Let us consider the mixed boundary value problem for the non-shallow spherical shells. We have to find the elasticity balance, when the stresses are marked on the some part of the boundary points and the displacements are on the remainder.

The boundary conditions for the components of the stresses and displacement vector are expressed with the help of holomorphic functions $\varphi(z)$, $\psi(z)$, $f(z)$ by formulas

$$\begin{aligned}
(T_{(ll)}+iT_{(ls)})_{\vartheta=\vartheta_0} &= \frac{\mu}{2\rho}\frac{\lambda+\mu}{\lambda+2\mu}\left[\varphi(z)+\overline{\varphi(z)}\right]-\frac{\lambda+\mu}{2(\lambda+2\mu)}\left[P_+\bar z+\overline{P}_+z\right]\\
&-4\mu\rho\left\{\overline{f''(z)}-\frac{z}{1+z\bar z}\left[\overline{\varphi'(z)}+\frac{z}{1+z\bar z}\overline{\varphi(z)}\right]-\left[\overline{\psi'(z)}+\frac{2z}{1+z\bar z}\overline{\psi(z)}\right]\right.\\
&+\frac{\rho}{\mu}\frac{1}{\bar z^2(1+z\bar z)}\left[\frac{z(2+z\bar z)}{2(1+z\bar z)}-\frac{\ln(1+z\bar z)}{\bar z}\right]\overline{P}_+\\
&\left.+\frac{\rho^2}{\mu}\frac{z^2}{(1+z\bar z)^2}P_3\right\}\left(\frac{d\bar z}{ds}\right)^2,\\
(u_3)_{\vartheta=\vartheta_0} &= \chi(z,\bar z)+\frac{\lambda+3\mu}{4(\lambda+2\mu)}\left[\varphi(z)+\overline{\varphi(z)}-\frac{\rho}{2\mu}(P_+\bar z+\bar P_+z)\right]-\frac{\rho^2}{2\mu}P_3.
\end{aligned}$$

Let us the boundary conditions are equal to zero on the boundary points [4]

$$\begin{cases} T_{\vartheta\vartheta}+iT_{\vartheta\varphi}=0, & r=r_0,\\ u_{(3)}=0, & r=r_0.\end{cases}\tag{3}$$

If functions $\varphi(z)$, $\psi(z)$, $f(z)$ are introduced by series

$$\varphi(z)=\sum_{n=0}^{\infty}a_nz^n,\quad \psi(z)=\sum_{n=0}^{\infty}b_nz^n,\quad f(z)=\sum_{n=0}^{\infty}c_nz^n,$$

then solutions of this system (3) have the following forms:

$$\begin{aligned}
&a_0=-\frac{\rho^2r_0^2}{\mu}\frac{\lambda+2\mu}{\lambda+\mu-(\lambda+2\mu)r_0^2(1+r_0^2)^2}P_3,\\
&a_1=\frac{2\rho}{\mu}\frac{\lambda+2\mu}{\lambda+\mu}\ln K_0\bar{P}_+,\\
&a_n=0,\quad n\geq 2,\\
&b_0=\frac{\rho}{r_0\mu}\frac{1}{1+r_0^2}\left[\ln K_0\left(\frac{1}{(1+r_0^2)^2}+\frac{(1+2r_0^2)(\lambda+2\mu)}{\lambda+\mu}\right)-\frac{\lambda+\mu}{\lambda+2\mu}\frac{1}{2(1+r_0^2)^2}\right]\bar{P}_+,\\
&b_1=0,\quad n\geq 1,\\
&c_0=\frac{\rho^2}{4\mu}\frac{1+r_0^2}{1-r_0^2}\left[1+\frac{(\lambda+3\mu)r_0^2}{\lambda+\mu-(\lambda+2\mu)r_0^2(1+r_0^2)^2}\right]P_3,\\
&c_1=\frac{\rho}{2\mu}(1+r_0^2)(\lambda+3\mu)\left[\frac{1}{4(\lambda+2\mu)}-\frac{1}{\lambda+\mu}\ln K_0\right]\bar{P}_+.\\
&c_2=0,\quad n\geq 2,
\end{aligned}$$

where

$$\ln K_0=\frac{(1+r_0^2)\ln(1+r_0^2)}{r_0^4}-\frac{1}{r_0^2}-\frac{\lambda+3\mu}{4(\lambda+2\mu)}.$$

R E F E R E N C E S

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