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ANALYSIS OF COMPOSITE ORTHOGONAL PLATES

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Abstract. It is possible to consider as classical example of composite materials the reinforced concrete constructions which the mankind applies already almost two centuries. Nowadays the large application was received by two versions of the reinforced fibers: Kevlar-29 and Kevlar-49. Physic mechanical properties of these fibers essentially differ from usual organic fibers. The material is characterized by high strength. the linear dependence between strain and stress takes place until of fibers break that from the mathematical point of view simplifies the problem solution within the linear theory. These materials differ also by high chemical stability. From composite materials are created typical load-bearing units - various plates and shells.

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To the analysis problem of shells and plates with discontinuous parameters are devoted the works of many known scientists in which the basic accent is made on the possible account of caused jumps by cuts and apertures.

One of powerful mathematical methods of the given problem general solution is developed by Shalva and Merab Mikeladze the general theory of design of discontinuous solutions [1, 2]. By basing on this theory are considered such plate's analysis problems, which geometrical and physical characteristics undergo the ordinary discontinuity in one or several points.

Is considered the rectangular plate with sides length of 2 and b. The plate consists from separate orthogonal elements which are connected with each other by ideal joints. The plate undergoes influence of the distributed loading.

Is the work is considered the rectangular plate with sides length of $2a$ and b. The plate consists from separate orthogonal elements which are connected with each other by ideal joints. The plate undergoes influence of the distributed loading. The coordinates (x_1, y_1, z_1) are selected as it is shown on Fig. 1. The differential equation of plate bending has the following form [3]

$$\frac{\partial^4 W}{\partial x^4} + 2\eta^2 \frac{\partial^4 W}{\partial x^2 \partial y^2} + \eta^4 \frac{\partial^4 W}{\partial y^4} = \frac{q(x,y)a^4}{D_n}\,, \tag{1}$$

where W is the value of plate deflection, $D = \frac{E_n h^3}{12(1-v_n^2)}$ is the cylindrical rigidity, $q(x_1, y_1)$ is the distributed load intensity, E_n is the separate elements modulus of elasticity, v_n is the Poisson's ratio, and h is the plates thickness, which also can stepwise changes and x and y are the dimensionless quantity

$$x = \frac{x_1}{a}\,,\quad -1 \le x < +1, \quad \eta = \frac{a}{b}\,,\quad y = \frac{y_1}{b}\,,\quad 0 \le y < 1.$$

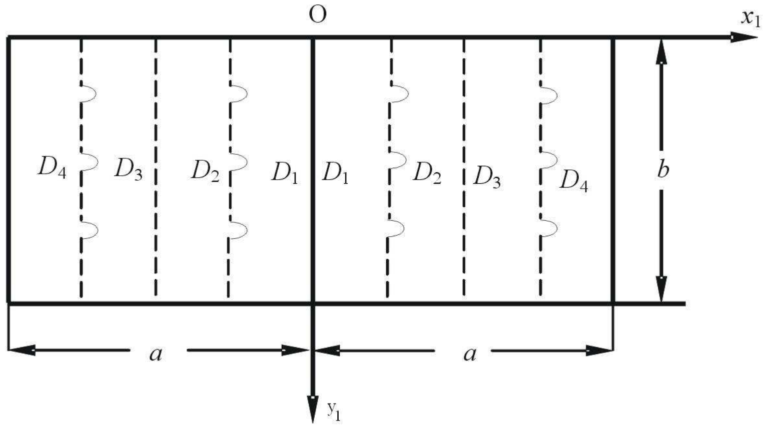

Fig.1.

Let's make separation of the equation (1). With this purpose we will present operating on plate load $q(x, y)$ as the following trigonometric series:

$$q(x, y) = \sum_{m=1}^{\infty} q_m(x) \sin m\pi y, \tag{2}$$

where

$$q(x) = 2 \int_0^1 q(x, y) \sin m\pi y \, dy. \tag{3}$$

In such case is reasonable deflection of the plate $W(x, y)$ represents as similar series

$$W(x, y) = \sum_{m=1}^{\infty} W_m(x) \sin m\pi y. \tag{4}$$

If we will mean, that plate with sides of $y = 0$ and $y = 1$ is freely-supported, then defined by (2), (3) deflection function satisfy to the boundary conditions $y = 0$ and $y = 1$ along the edges. In the case of uniformly distributed load (2) and (3) equalities gives:

$$\begin{aligned} q(x) &= 2 \int_0^1 q(x, y) \sin m\pi y \, dy = \frac{4q}{m\pi}, \quad m = 1, 3, 5, \ldots, \\ q(x, y) &= \frac{4q}{m\pi} \sum_{m=1,3,5} \frac{1}{m} \sin m\pi y. \end{aligned} \tag{5}$$

By taking into account (4) and (5), (1) gives

$$W_m^{iv}(x) + 2\alpha_m^2 W_m''(x) + \alpha_m^4 W_m(x) = \frac{q_m a^4}{D_m}, \quad \alpha_m = \eta m\pi, \quad q_m = \frac{4q}{m\pi}. \tag{6}$$

According to Hookes law and by taking into account (4) gives:

$$
\begin{aligned}
M_x &= -\frac{D_n}{a^2}\left(W_m''(x) - v_n\alpha_m^2 W_m(x)\right)\sin m\pi y,\\
M_y &= -\frac{D_n}{a^2}\left(v_n W_m''(x) - \alpha_m^2 W_m(x)\right)\sin m\pi y,\\
H &= -\frac{D_n}{a^2}(1-v_n)\alpha_m W_m'(x)\cos m\pi y,\\
Q_x &= -\frac{D_n}{a^2}\left(W_m''(x) - \alpha_m^2 W_m(x)\right)\sin m\pi y,\\
Q_y &= -\frac{D_n}{a^2}\left(\alpha_m W_m''(x) - \alpha_m^3 W_m(x)\right)\cos m\pi y.
\end{aligned}
$$

(6) represents the ordinary differential quartic equation which right part and derivatives of desired $W_m(x)$ function undergoes discontinuity of the first kind in points of interfaces. We will apply generalized by Sh. Mikeladze Maclaurin series without remainder term also we will make discontinuous solution of (6) differential equation. Lets generate the discontinuous solution

$$
W_m(x) = \sum_{k=0}^{n}\frac{x^k}{k!}W_m^{(k)}(0) + \sum_{k=0}^{n}\frac{x^k}{k!}\sum_{\rho=1}^{z}\delta_\rho^{(k)}(x-x_\rho),
$$

where x_ρ and $\delta_\rho^{(k)}$ accordingly denotes the break points and $W_m^{(k)}$ the function jump, n is the remained in series terms number, and r is the number of break points.

As in general the jump value $\delta_\rho^{(k)}$ in the $x = x_\rho$ break points k-th derivative $W_m^{(k)}(x)$ of $W_m(x)$ function jump value $\delta_\rho^{(k)}$ in the $x = x_\rho$ break points is defined by formula

$$
\delta_\rho^{(k)} = W_m^{(k)}(x_\rho + 0) + W_m^{(k)}(x_\rho - 0),
$$

where $W_m^{(k)}(x_\rho{+}0)$ and $W_m^{(k)}(x_\rho{-}0)$ accordingly represents the values of $W_m(x)$ function when we approximate to x_ρ point from the right and then from the left.

By taking into account the obtained equalities in the generalized Maclaurin formula we finally obtain:

$$
\begin{aligned}
W_m(x) &= W_m(0)\left[1 - \sum_{i=1}^{n/2}(i-1)a_m^{2i}\frac{x^{2i}}{(2i)!}\right]\\
&+ W_m(0)\left[\frac{x^2}{2!} + \sum_{i=1}^{n/2} i a_m^{2i-2}\frac{x^{2i}}{(2i)!}\right] + \frac{q_m a^4}{D_\rho}\sum_{i=1}^{n/2}(i-1)a_m^{2i-4}\frac{x^{2i}}{(2i)!}\\
&+ \sum_{\rho=1}^{r}\delta_\rho^{(1)}\left[(x-x_\rho) - \sum_{i=1}^{n/2}(i-1)a_m^{2i}\frac{(x-x_\rho)^{2i+1}}{(2i+1)!}\right]\\
&+ \sum_{\rho=1}^{r}\delta_\rho^{(3)}\left[\frac{(x-x_\rho)^{-3}}{3!} + \sum_{i=1}^{n/2} i a_m^{2i}\frac{(x-x_\rho)^{2i+1}}{(2i+1)!}\right]
\end{aligned}
$$

$$+\sum_{\rho=1}^{r}\delta_\rho^{(4)}\left[\sum_{i=1}^{n/2}(i-1)a_m^{2i-1}\frac{(x-x_\rho)^{2i}}{(2i)!}\right].$$

In regard to parameters $W_m(0)$, $W_m''(0)$, $\delta_\rho^{(1)}$ and $\delta_\rho^{(3)}$ by purpose of its definition on the $M_x x = \pm 1$ edges we have the boundary conditions, and in their hinges condition of equality to zero the bending moments M_x.

Thus we obtain the simultaneous equations by which we define all force factors and components of deformation with following equalities

$$W = W^0\frac{q_m}{D_1}\sin m\pi\eta,\quad M_x = M_x^0 q_m\sin m\pi\eta,\quad M_y = M_y^0 q_m\sin m\pi\eta,$$
$$H = H^0 q_m\cos m\pi\eta,\quad Q_x = Q_x^0 q_m\sin m\pi\eta,\quad Q_y = Q_y^0 q_m\cos m\pi\eta.$$

In that particular case when plate consists from five orthogonal elements with different rigidity and along the $x = 0.2$ and $x = 0.6$ straight lines we have hinges (hinged connections) and in the series are remained two terms $m = 1$ and $m = 3$ and at the same time $\frac{D_1}{D_2} = \frac{D_2}{D_3} = 0.8$ and $v = 0,3$. For the W^0, M_x^y, M_y^0, H, Q_x^0, Q_y^0 values when the $x = \pm 1$ sides are rigidly attached in the some points of plate for their values we have following table

x^0	W^0	M_x^y	M_y^0	H	Q_x^0	Q_y^0
0.0	10.034	0.334	2.730	0.000	0.000	1.130
0.1	10.876	0.223	2.728	−0.100	−0.205	1.116
0.2	10.967	0	2.647	−2.240	−0.420	1.027
				1.870		
0.3	8.365	0.213	2.642	1.710	−0.410	1.149
0.4	7.016	0.305	2.005	1.625	−0.220	0.987
0.5	5.012	0.232	1.537	1.570	−1.117	0.710
0.6	3.105	0	0.940	1.527	−1.410	0.380
				1.256		
0.7	2.187	−0.525	0.613	1.160	−1.860	0.039
0.8	1.056	−1.420	−0.080	0.970	−2.466	−0.530
0.9	0.445	−2.308	−0.640	0.60	−3.168	−1.270
1.0	0	−4.439	−1.270	0.000	−4.140	−2.220

In work is raised and solved analytically the question of made from composite material rectangular plate calculation when the plate is presented from various rigidity elements which at the same time are connected with each other by cylindrical hinges. The question is solved by developed by Sh. Mikeladze for discontinuous functions Maclaurin generalized series.

R E F E R E N C E S

1. Mikeladze Sh.E. Some Problems of Structural Mechanics. *Moscow–Leningrad*, 1948.
2. Mikeladze M.Sh. Elasticity and Plasticity of Structures and Machines Elements. *Metsniereba*, 1976.
3. Timoshenko S.P., Voinovsky-Kriger S. Plates and shells. *Publishing of Physical and Mathematical Literature, Moscow*, 1963.

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