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AN APPROXIMATE ALGORITHM FOR A KIRCHHOFF NONLINEAR DYNAMIC BEAM EQUATION

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Abstract. An initial boundary value problem is posed for the Kirchhoff type integro-differential equation, which describes the dynamic state of a beam. The solution is approximated with respect to a spatial and a time variables by the Galerkin method and a stabile difference scheme. The algorithm has been approved on tests and the results of recounts are represented in graphics.

Keywords and phrases: Kirchhoff type nonlinear dynamic beam equation, approximate algorithm, Galerkin's method, difference scheme, calculations results.

AMS subject classification (2000): 65M60.

1. Statement of the Problem. Let us consider the nonlinear integro-differential equation

$$\frac{\partial^2 u}{\partial t^2}(x,t)+\frac{\partial^4 u}{\partial x^4}(x,t)-\left(\alpha+\beta\int\limits_0^L\left(\frac{\partial u}{\partial x}(x,t)\right)^2dx\right)\frac{\partial^2 u}{\partial x^2}(x,t)=0, \tag{1.1}$$

$$0<x<L,\ \ 0<t\le T,$$

with the initial boundary conditions

$$u(x,0)=u^0(x),\ \ \frac{\partial u}{\partial t}(x,0)=u^1(x), \tag{1.2}$$

$$u(0,t)=u(L,t)=0,\ \frac{\partial^2 u}{\partial x^2}(0,t)=\frac{\partial^2 u}{\partial x^2}(L,t)=0, \tag{1.3}$$
$$0\le x\le L,\ \ 0\le t\le T,$$

where α,β,L and T are some positive constants, $u^0(x)$ and $u^1(x)$ are the given functions, and $u(x,t)$ is the function we want to find. Equation (1.1) describes beam oscillation and it is called a Kirchhoff type equation. It was obtained by S. Woinowsky-Kriger in 1950 [1]. Several authors were studying questions of resolution, uniqueness of the solution, asymptotic behavior, etc. for equation (1.1). See, for example works by J.M.Ball, P.Biler, E.H.Brito, L.A.Medeiros, D.C.Pereira. We discuss here one numerical algorithm of the solution of the problem (1.1)-(1.3). We use methods from works [2], [3].

2. The Algorithm. The algorithm consists of two parts.
The first part-the Galerkin method.
The solution of problem (1.1)-(1.3) is represented in the form

$$u_n(x,t) = \sum_{i=1}^{n} u_{ni}(t) \sin \frac{i\pi}{L} x, \tag{2.1}$$

where the coefficients $u_{ni}(t)$ satisfy the following system of differential equations

$$u''_{ni}(t) + \left(\frac{\pi i}{L}\right)^4 u_{ni}(t) + \left(\frac{\pi i}{L}\right)^2 \left(\alpha + \beta \frac{\pi^2}{2L} \sum_{j=1}^{n} j^2 u_{nj}^2(t)\right) u_{ni}(t) = 0, \tag{2.2}$$

$$i = 1, 2, \ldots, n, \;\; 0 < t \le T,$$

with the initial conditions

$$u_{ni}(0) = a_i^{(0)}, \quad u'_{ni}(0) = a_i^{(1)}, \quad i = 1, 2, \ldots, n. \tag{2.3}$$

where

$$a_i^{(p)} = \frac{2}{L} \int_0^L u^{(p)}(x) \sin \frac{i\pi}{L} x dx, \;\; p = 0, 1. \tag{2.4}$$

The second part - the difference scheme. We proceed to solve problem (2.2), (2.3) by means of the difference method. On the time interval $[0, T]$ let us introduce the grid $\{t_k | 0 = t_0 < t_1 < \cdots < t_m = T\}$, $t_k = t\tau$, , with a step $\tau = T/m$, $k = 0, 1, \ldots, m$. Approximate values of $u_{ni}(t)$ on the k-th time layer, i.e. for $t = t_k$, $k = 0, 1, \ldots, m$, denoted by u_{ni}^k is defined by the explicit symmetric scheme

$$\frac{u_{ni}^{k+1} - 2u_{ni}^k + u_{ni}^{k-1}}{\tau^2} + \left\{ \left(\frac{\pi i}{L}\right)^2 \left[\left(\frac{\pi i}{L}\right)^2 + \propto + \beta \frac{\pi^2}{2L} \sum_{j=1}^{n} j^2 (u_{nj}^k)^2 \right] \right\} \frac{u_{ni}^{k+1} + u_{ni}^{k-1}}{2} = 0, \tag{2.5}$$

$$k = 0, 1, \ldots, m-1.$$

On the first two levels let us use formulas

$$u_{ni}^1 = a_i^0 + \tau a_i^1 - \frac{\tau^2}{2} \left\{ \left(\frac{\pi i}{L}\right)^2 + \alpha + \beta \frac{\pi^2}{2L} \sum_{j=1}^{n} j^2 (a_i^0)^2 \right\} \left(\frac{\pi i}{L}\right)^2 (a_i^0),$$

$$i = 1, 2, \ldots, n.$$

3. Algorithms Realization. The algorithm proposed in subsection 2 enables us to find approximate solutions of problem (1.1)-(1.3). The approximate program has been designed in Turbo Pascal algorithm language and calculations have been done on the computer. The results obtained are good enough. The algorithm has been approved by tests and the results of recounts are represented in the graphics.

A problem (1.1)-(1.3) with the following data is discussed

$$\alpha = 1, \; \beta = 1, \; L = 1, \; T = 1, \; a_i^0 = 1/i, \; a_i^1 = 1/i^{1.5}, \; i = 1, 2, \ldots.$$

Approximate solutions according to formula (2.1) are compared for $n = 5$ (Fig. 1) and $n = 10$ (Fig. 2) in case of $\tau = 0.02$ step.

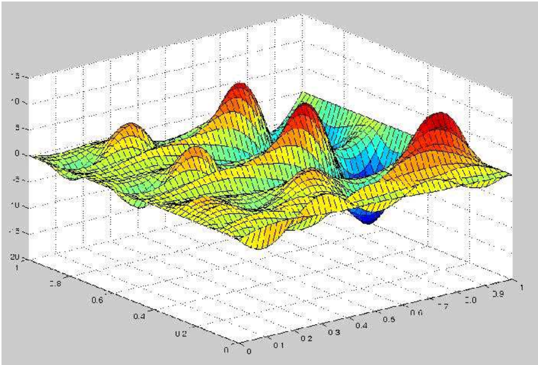

Fig.1.

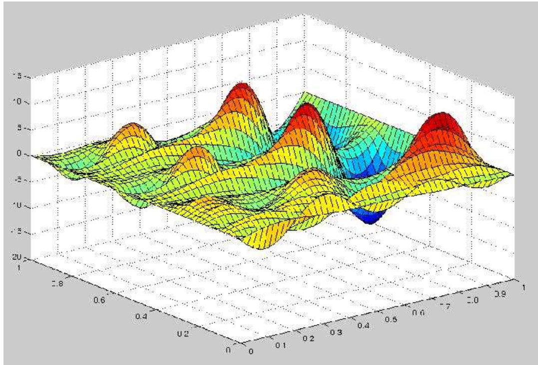

Fig.2.

The comparison of graphics shows that the increase of n parameter slightly changed the approximate solutions.

R E F E R E N C E S

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