Reports of Enlarged Session of the
Seminar of I. Vekua Institute
of Applied Mathematics
Volume 23, 2009

STEADY MOTION OF VISCOUS INCOMPRESSIBLE CONDUCTING FLUID IN PLANE PIPE WITH EXTERNAL MAGNETIC FIELD

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Abstract. In this paper steady motion of viscous incompressible conducting fluid in plane pipe with external magnetic field is given.

Keywords and phrases: Magnetic field, plane pipe, temperature, Hartman and Reynolds numbers, velocity, suction, incompressible field.

AMS subject classification (2000): 76W05.

We consider steady motion of viscous incompressible conductive fluid in plane pipe, which border is $z = \pm a$ infinite itinerant plates in the presence of non-homogeneous external magnetic field $\vec{B}^e \left\{0,\ -B_0 \dfrac{y}{a}\beta,\ -B_0\left(\dfrac{z}{a}\beta + \alpha\right)\right\}$

Let us velocity of fluid: s dependent only z, $\vec{u}\left\{u(z);\ 0;\ v_0(1-\beta)\right\}$, where $v_0 =$ constant is the suction velocity, α and β parameters resile 1 or 0 value. When $\alpha = 1$ and $\beta = 0$ we get homogeneous external magnetic field. If $\alpha = 0$ and $\beta = 1$ magnetic field are non-homogeneous.

Let us effect pressure dependent all three coordinate

$$P^* = \frac{B_0^2}{4\pi a^2}\left[\left(y^2 + z^2\right)\beta + \alpha\right] - Px + \text{const}.$$

In this case equation of motion, magnetic field and energy, including viscous and Joule dissipation are

$$\rho v_0(1-\beta)\frac{\partial u}{\partial z} = \mu\frac{\partial^2 u}{\partial z^2} + \frac{B_0}{4\pi}\left(\frac{z}{a}\beta + \alpha\right)\frac{\partial b}{\partial z} + P,$$

$$v_0(1-\beta)\frac{\partial b}{\partial z} = \nu_m\frac{\partial^2 b}{\partial z^2} + B_0\left(\frac{z}{a}\beta + \alpha\right)\frac{\partial u}{\partial z},$$

$$\rho c_\tau v_0(1-\beta)\frac{\partial T}{\partial z} = k\frac{\partial^2 T}{\partial z^2} + \frac{\nu_m}{4\pi}\left(\frac{\partial b}{\partial z}\right)^2 + \mu\left(\frac{\partial u}{\partial z}\right)^2.$$

With boundary conditions:

$$u(-a) = w_1,\quad u(a) = w_2,$$

$$b(-a) = 0,\quad b(a) = 0,$$

$$T(-a) = T_1,\quad T(a) = T_2,$$

where ρ is the density, ν and μ - the cinematic and dynamic viscosity, ν_m - magnetic viscosity $\left(\nu_m = \dfrac{c^2}{4\pi\sigma}\right)$, k is the thermal conductivity, c_τ is the specific heat of fluid, when volume is constant. w_1 and w_2 is the walls velocity, $T_1 = const$ and $T_2 = const$ is the temperature at top and lower walls.

If we pass on non-dimensional quantities, take cinematic, magnetic and temperature Hartman and Reynolds numbers and let $Re = Re_m = Re_\tau = R_0$, when we obtain:

$$u'' - R_0(1-\beta)u' + Ha(z\beta+\alpha)b' = -1; \tag{1}$$

$$b'' - R_0(1-\beta)b' + Ha(z\beta+\alpha)u' = 0; \tag{2}$$

$$T'' - R_0(1-\beta)T' = -\left(b'^2 + u'^2\right). \tag{3}$$

Boundary conditions:

$$\begin{aligned} &u(-1) = w_1, \quad u(1) = w_2, \\ &b(-1) = 0, \quad\; b(1) = 0, \\ &T(-1) = T_1, \quad T(1) = T_2. \end{aligned} \tag{4}$$

As dynamic and magnetic field does not depend on the temperature field, we can solve the magnetohydrodynamic problem and result employ to define temperature field.

Let us now introduce a new functions

$$\varphi_1 = u + b, \quad \varphi_2 = u - b.$$

Then from (1), (2) we get:

$$\varphi_1'' + \left[Ha(z\beta+\alpha) - R_0(1-\beta)\right]\varphi_1' = -1,$$

$$\varphi_2'' - \left[Ha(z\beta+\alpha) - R_0(1-\beta)\right]\varphi_2' = -1,$$

$$\varphi_1(-1) = w_1, \quad \varphi_1(1) = w_1,$$

$$\varphi_2(-1) = w_1, \quad \varphi_2(1) = w_2.$$

The solutions of these problem are

$$\begin{aligned} \varphi_1(z) = &\left[w_2 - w_1 + \int\limits_{-1}^{1}\int\limits_{-1}^{x} e^{Ha\left[\frac{s^2-x^2}{2}\beta+\alpha(s-x)\right]-R_0(s-x)(1-\beta)}dsdx\right] \\ &\times\frac{\int\limits_{-1}^{z} e^{-Ha\left(\frac{x^2}{2}\beta+\alpha x\right)+R_0(1-\beta)x}dx}{\int\limits_{-1}^{1} e^{-Ha\left(\frac{x^2}{2}\beta+\alpha x\right)+R_0(1-\beta)x}dx} \\ &-\int\limits_{-1}^{1}\int\limits_{-1}^{x} e^{Ha\left[\frac{s^2-x^2}{2}\beta+\alpha(s-x)\right]-R_0(s-x)(1-\beta)}dsdx + w_1 \end{aligned} \tag{5}$$

$$\varphi_2(z) = \left[w_2 - w_1 + \int\limits_{-1}^{1}\int\limits_{-1}^{x} e^{-Ha\left[\frac{s^2-x^2}{2}\beta+\alpha(s-x)\right]-R_0(s-x)(1-\beta)}dsdx\right]$$

$$\times\frac{\int\limits_{-1}^{z} e^{Ha\left(\frac{x^2}{2}\beta+\alpha x\right)+R_0(1-\beta)x}dx}{\int\limits_{-1}^{1} e^{Ha\left(\frac{x^2}{2}\beta+\alpha x\right)+R_0(1-\beta)x}dx}$$

$$-\int\limits_{-1}^{1}\int\limits_{-1}^{x} e^{-Ha\left[\frac{s^2-x^2}{2}\beta+\alpha(s-x)\right]-R_0(s-x)(1-\beta)}dsdx+w_1.$$

The velocity of fluid and magnetic field are:

$$u=\frac{1}{2}\left(\varphi_1+\varphi_2\right);\quad b=\frac{1}{2}\left(\varphi_1-\varphi_2\right).$$

Now we can solve (3), (4) problem. Integrate the (3) equation and use function φ we get:

$$T'+(1-\beta)R_0T=-\varphi_1+\text{const}. \tag{6}$$

If $\beta=0$, then from (6) obtain

$$T'+R_0T=-\varphi_1+\text{const},$$

and

$$T(z)=T_1e^{-R_0(z+1)}-\int\limits_{-1}^{z}\varphi_1e^{R_0(x-z)}dz+\frac{1-e^{-R_0(z+1)}}{1-e^{-2R_0}}\left[T_2-T_1e^{-2R_0}+\int\limits_{-1}^{1}\varphi_1e^{R_0(x-z)}dx\right],$$

where φ_1 define from (5).

If $\beta=0$, then we obtain

$$T(z)=-\int\limits_{-1}^{z}\varphi_1dx+\frac{z+1}{2}\left[T_2-T_1+\int\limits_{-1}^{1}\varphi_1dx\right]+T_1.$$

Now we can calculate the skin friction, heat flux at the plates (in the non-dimensional quantities).

$$\tau=\mu\left.\frac{\partial u}{\partial z}\right|_{z=\pm1}=\left.\frac{\mu}{2}\frac{\partial}{\partial z}\left(\varphi_1+\varphi_2\right)\right|_{z=\pm1}.$$

If $\beta=1$, $\alpha=0$,

$$q|_{z=-1}=k\left\{\frac{T_2-T_1}{2}+\frac{1}{2}\int\limits_{-1}^{1}\varphi_1dx-w_1\right\},$$

$$q|_{z=1}=k\left\{\frac{T_2-T_1}{2}+\frac{1}{2}\int\limits_{-1}^{1}\varphi_1dx-w_2\right\},$$

If $\beta = 0$, $\alpha = 1$,

$$q|_{z=-1} = k\left\{-w_1 + \frac{R_0}{1+e^{-2R_0}}\left[T_2 - T_1 + \int_{-1}^{1} \varphi_1 e^{R_0(x+1)} dx\right]\right\},$$

$$q|_{z=1} = k\left\{-w_2 + \frac{R_0}{1+e^{-2R_0}}\left[T_2 - T_1 e^{-2R_0} + \left(2 + e^{-2R_0}\right)\int_{-1}^{1} \varphi_1 e^{R_0(x-1)} dx\right]\right\}.$$

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Received 17.04.2009; revised 25.09.2009; accepted 29.11.2009.

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