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OPTIMIZATION OF COMPLEX TECHNICAL SYSTEMS WITH TWO SERVICE TYPES

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Abstract. The work considers standby technical system with two operation types (replacement and renewal) with priority of substitution. Mathematical model of the system and the solution of algebraic equations system in explicit form are given. The algorithm of determination of final probability states of technical system is constructed. The problem of system optimization according the proposed economical criteria is stated. The results of numerical realization of the stated optimization problem are written.

Keywords and phrases: priority system of queues, standby, replacement, renewal, explicit form of solution, optimization of the model.

AMS subject classification (2000): 93A30; 93E03; 60J05; 60K25.

Introduction. One of the basic objectives of construction of the model of any functioning system is the problem of analysis of system behavior, determination of its specific characteristics, such, for example, as dependability or economical efficiency. The problems of model construction, economical effectiveness criterion, also some special calculations of effectiveness for standby technical system with two types of operations are considered in works [1–3]. When the problem of optimization of technical systems is solved, it is advisable (and sometimes even essential) to formalize, in a general form, the process of construction of mathematical models of the studied technical system.

The given work proposes, in general case, the algorithm of determination of final probabilities of technical system states. Further, on the basis of the constructed algorithm the problem of optimization of technical system is solved. Algorithms of the given problems are realized with using MatLab packet.

The Main Part. Consider the standby technical system composed of m main and n standby elements. All elements are identical. For normal functioning of the system all m main elements are necessary to be maintained in operating condition. The system continues to function in case when main elements number is reduced, but the effectiveness of system functioning drops down, as a result it becomes necessary to turn to standby elements and to replace the failed main element with the standby one.

The main elements fail with intensity α and standby ones - with intensity β. The failed main element is replaced with an operating standby element at the first possibility, the replacement being done with intensity λ. If in the system there is no necessity to make replacement (all m main elements are in operating state) or if there is an organ that is not replaced (the number of replacement and renewal organs exceeds the number of failed main elements), the failed element is renewed. Renewal is done with intensity μ. In the process of replacement or renewal of one element only one organ takes place.

Accentuate some peculiarities of the described system: 1) momentary replacement of the failed main element with standby one is not assumed; 2) absolute priority of replacement is assumed, i.e., in case of the necessity replacement any renewal of element is discontinued and the main element is replaced with standby one. At the same time the discontinuous renewal begins all over again.

Note that the given work considers the case when the number of organs executing replacement or renewal is equal to one.

For the solution of different problems, for example, problem of optimization (by method of search of possible versions) of the considered technical system by economic criterion, it is necessary to calculate the constructed model for different n. In work [3] special cases the above described system are considered. Specifically, a model of the system with two main elements is constructed. The problem of determination of economical effectiveness of the mentioned system in the existence of standby elements in quantity $n = 0$ (system operates without standby), $n = 1, 2, 3$ (system operates with one, two, three standby elements, respectively) is considered. In connection to the solved problems the quantity of necessary special cases of the considered general problem, the models of which are to be considered, are not as a rule, known in advance.

The solution of the problem of automation of realization of mathematical model of the considered standby technical system for any quantities of m and n is of particular interest.

For construction of a mathematical model of the considered system, introduce the notion of system state. We say that system is in state $s(i, j)$ ($i = \overline{0, m}$, $j = \overline{i, n + i}$), if the number of deficit main elements is i, and the number of non-operating elements (main and standby) is j. The process of the system functioning is described by the functions $p(i, j, t)$, which implies the probability that at the moment of time t the system is in the state $s(i, j)$.

In work [4], for determination of final probabilities ($t \to \infty$) of system state $p(i, j)$, a system of linear algebraic equations is received:

$$\begin{aligned}
&\big(\alpha(m-i) + \beta(n-(j-i)) + v_{i,j}\,\lambda + u_{i,j}\,\mu\big)\,p(i,j) = \\
&\qquad = \alpha(m-i+1)\,p(i-1,j-1) + \beta(n-j+i+1)\,p(i,j-1) + \\
&\qquad + \lambda\,p(i+1,j) + u_{i,j+1}\mu\,p(i,j+1), \qquad i=\overline{0,m}, \quad j=\overline{i,n+i}, \qquad (1)\\
&\sum_{i=0}^{m}\sum_{j=i}^{n+i} p(i,j) = 1
\end{aligned}$$

where

$v_{i,j} = 0$, if $i = 0$ or $j = n + i$, otherwise $v_{i,j} = 1$.

$u_{i,j} = 1 - v_{i,j}$, but $u_{0,0} = 1$;

$p(i, j) = 0$ if $i < 0$ or $i > m$ or $i > j$ or $j > n + i$; $i = \overline{0, m}$, $j = \overline{i, n + i}$,

Solution (1) at additional condition of standardization

$$\sum_{i=0}^{m}\sum_{j=i}^{n+i} p(i,j) = 1 \qquad (2)$$

is received in explicit form, particularly:

$$p(0,0) = \left(\sum_{i=0}^{m} \sum_{j=0}^{n} q(i,j) \right)^{-1}, \quad p(i,i+j) = q(j,i)\, p(0,0), \quad i = \overline{0,m}, \quad j = \overline{0,n},$$

where

$$\begin{aligned}
&\text{(I)} \qquad q(0,i) = \frac{\alpha^i\, m!}{(m-i)!} \left(\prod_{k=1}^{i} \big(\alpha(m-k) + \beta n + \lambda\big) \right)^{-1}, \\
&\qquad\qquad i = \overline{1,m}, \quad q(0,0) = 1, \\
&\text{(II)} \qquad q(j,i) = k_{ij}\big(\alpha(m-i+1)\, q(j,i-1) + \beta(n-j+1)\, q(j-1,i) \\
&\qquad\qquad + \lambda\, q(j-1,i+1)\big), \quad j = \overline{1,n-2}, \quad i = \overline{1,m}, \\
&\qquad\qquad \text{where} \quad k_{ij} = (\alpha(m-i) + \beta(n-j) + \lambda)^{-1}, \quad q(i,m+1) = 0, \\
&\text{(III)} \qquad q(j,0) = \mu^{-1}\big((\alpha\, m + \beta(n-j+1) + \mu)\, q(j-1,0) \\
&\qquad\qquad - \beta(n-j+2)\, q(j-2,0) - \lambda\, q(j-2,1)\big), \quad j = \overline{2,n}, \\
&\qquad\qquad q(1,0) = \mu^{-1}(\alpha m + \beta n), \\
&\text{(IV)} \qquad q(n-1,i) = \lambda^{-1}\big((\alpha(m-i+1) + \mu)\, q(n,i-1) \\
&\qquad\qquad - w\, \alpha(m-i+2)\, q(n,i-2) - \beta\, q(n-1,i-1)\big), \quad i = \overline{1,m},
\end{aligned}$$

here $w = 0$ if $i = 1$ otherwise $w = 1$.

$$\begin{aligned}
q(n,i) = &\mu^{-1}\big((\alpha(m-i) + \beta + \lambda)\, q(n-1,i) - \alpha(m-i+1)\, q(n-1,i-1) \\
&- 2\beta\, q(n-2,i) - \lambda\, q(n-2,i+1)\big), \quad i = \overline{1,m},
\end{aligned}$$

where $q(-1,i) = 0$.

In Fig. 1, in the form of block-diagram, the algorithm of determination of solution of system (1–2) is presented in explicit form for the given quantity of main (m) and standby (n) elements. (In the given block-diagram, for the convenience of registering, there are given references on expressions (I, II, III, IV) by which the quantities are calculated).

The proposed algorithm and respective program realization enable to computerize the process of retrieving numerical characteristics of explicit cases of the system described in this paper. This, in its turn, is the precondition for solution of different applied problems (such, for example, as problems of dependability or optimization of technical systems).

Consider the index of economical effectiveness is introduced as an effectiveness function:

$$F(m,n) = (r_1 - c_1)E_1 + (r_2 - c_2)E_2 - \sum_{k=3}^{6} c_k E_k. \tag{3}$$

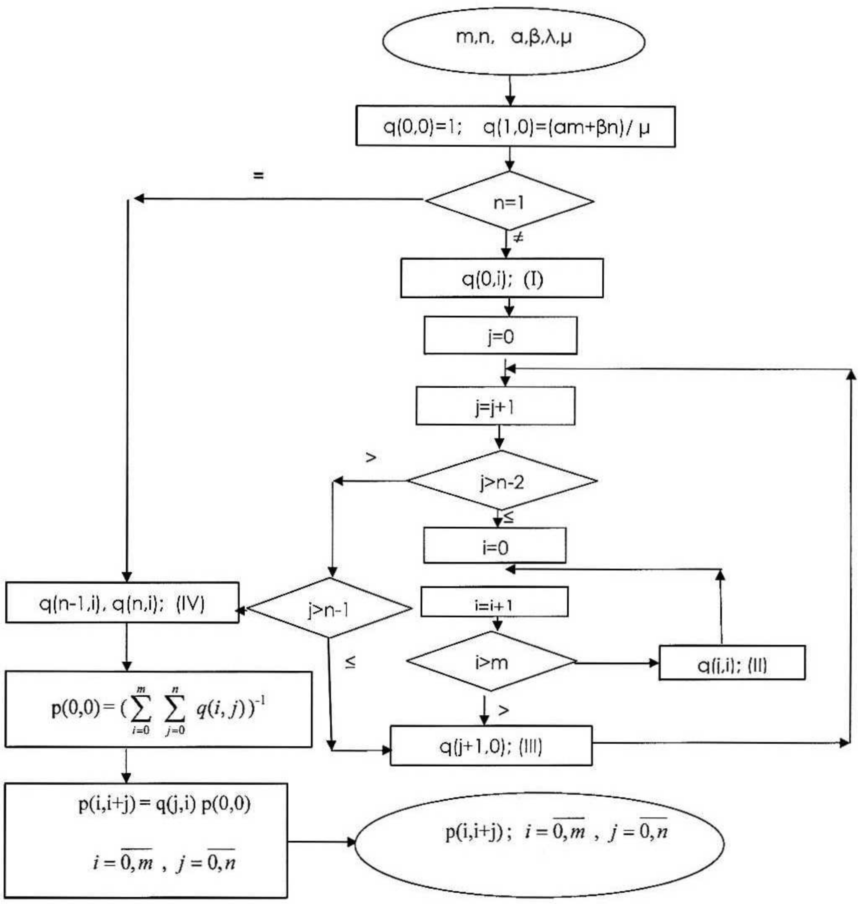

Fig. 1. Block-diagram of the algorithm of determination of final probabilities of technical system states.

Here: r_1 is the income in time unit from one main element; r_2 is the income in time unit from one operating standby element; c_1 is expenses in time unit from one main element; c_2 is expenses in time unit for one operating standby element; c_3 is expenses in time unit from one non-operating element; c_4 is expenses in time unit for one operating renewal organ; c_5 is expenses in time unit for one operating replacement organ; c_6 is expenses in time unit for one non-operating replacement and renewal organ. Denote $E_i = E_i(m,n)$, where E_1 is a mean number of main elements; E_2 is a mean number of standby elements; E_3 is a mean number of non-operating element; E_4 is a mean number of operating renewal organs; E_5 is a mean number of operating replacement organs; E_6 is a mean number of non-operating replacement and renewal organs.

Within the theory of queuing and dependability the initial economical characteristics r_i $(i = 1, 2)$ and c_i $(i = 1, \ldots, 6)$ are considered preset. Solving the system of linear algebraic equations (1), define probability characteristics $p_{i,j}$ and further, according to the given formulas [2] calculate the values of averages E_i $(i = 1, \ldots, 6)$. In these conditions the problem of mathematical programming (optimization problem) is formulated.

Namely, in case of fixed m function F, given by formula (3), depends on one argument n. Therefore, the problem is stated: for the given value m the value n is to be found which gives function F the maximum value

$$\max_{n \in N} F(m, n).$$

Consider as an example the following task:

For the given values of the parameters of technical system $\alpha = 0.01$, $\beta = 0.001$, $\lambda = 5$, $\mu = 2$ determine the number of necessary standby elements to achieve maximum economical effectiveness (3) of system functioning in condition that the system consists of five main elements.

For the solution of the above given problem of optimization use the algorithm proposed in the article. Then, as a result of realization of the given algorithm (Packet MatLab) we receive optimum number of standby elements $n = 2$. Economical effectiveness in this case is

$$\max_{n \in N} F(m, n) = F(5, 2) = 44.2771.$$

In conclusion note that the constructed algorithm is common for calculation of technical system considered in the article. And calculation of optimum quantity of standby elements is the demonstration of those problems the solution of which becomes available relying on the algorithm proposed in the given article.

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