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MODELING EXERCISE OF THE PLASMA CUTTING PROCESS

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Abstract. Plasma-arc cutting process is caused by a diversity of the related effects. The complexity of mathematical formulation of each, even separate effect is generally known.

In these circumstances significant simplifications can be achieved if in frames of the tentative assumptions one link is found by which correlation of the effects takes basically place and which is the result of this correlation. In the suggested model the geometrical surface of cutting represents such link.

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Plasma arc cutting process is caused by a diversity of the related effects (E-field radiation, gas and liquids flow, including electro conductive ones, heat transfer, phase changes, etc). The complexity of mathematical formulation of each, even separate effect is generally known.

In these circumstances significant simplifications can be achieved if in frames of the tentative assumptions one link is found by which correlation of the effects takes basically place and which is the result of this correlation. In the suggested model the geometrical surface of cutting represents such link. Let us take the Cartesian rectangular system of coordinates: $Oxyz$ (pic. 1: axis Oz is directed through the axis of the arc and axis Ox - in direction of the speed of cutting $\vec{\nu}_p$).

All processes and their corresponding physical fields in the system will be considered to be stationary. In this system the equation of the cutting surface (boundary surface between the air and the liquid or solid phase) will look like:

$$z = z(x, y). \tag{1}$$

Let us bind the cutting surface to the orthogonal curvilinear coordinates $\xi\eta\zeta$. Coordinate curves ξ and η lay on the cutting surface; axis ζ is directed perpendicularly to the surface. Assume that the transformations

$$(x, y) \Leftrightarrow (\xi, \eta) \tag{2}$$

are one-to-one and smooth enough.

Coordinate curves η will be considered to be collateral to the current line of particles of the liquid melt on the surface of the liquid film. Experimental data shows that movement of particles of the liquid film is conditioned by the high speed air flow (according to estimations, shearing stresses from the air flow exposing to the liquid film of the metal are 1-2 orders of magnitude higher than the gravity). As this takes place the speeds of the particles of the liquid film are much lower than the speed of particles of the air flow. Under these conditions coincidence of direction of the tangents η to the coordinate curves to directions of the speeds of the air flow near the boundary with liquid film can be considered to be a good approximation.

Thus the radius vector of the cutting surface point is - $\vec{R}\,(x(\xi\eta),\ z(\xi\eta),\ y(\xi\eta))$.

The ordinary basis vector of the axis ζ is:

$$\vec{e}_\eta = \frac{1}{\left|\dfrac{\partial\vec{R}}{\partial\xi} \times \dfrac{\partial\vec{R}}{\partial\eta}\right|} \frac{\partial\vec{R}}{\partial\xi} \times \frac{\partial\vec{R}}{\partial\eta}. \tag{3}$$

Consider the surface element on the surface of cutting at a point $E(\xi,\eta)$:

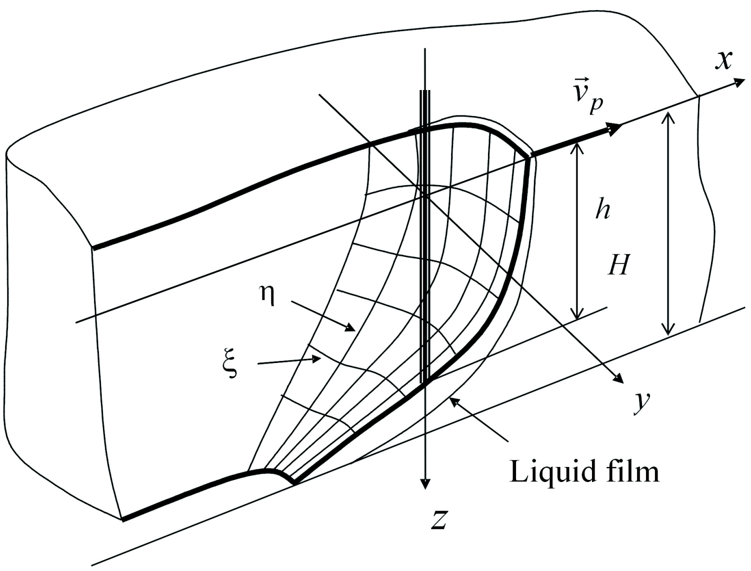
$$dA = \left|\frac{\partial\vec{R}}{\partial\xi} \times \frac{\partial\vec{R}}{\partial\eta}\right| d\xi d\eta \tag{4}$$

and relevant volume element of the liquid film

$$dV = \{(\xi,\eta,\zeta) : (\xi,\eta) \in dA, \zeta \in [0,\delta]\},$$

where $\delta = \delta(\xi\eta)$ - is the film thickness (pic.2).

On this small area the element of arc of the length of do at the point of L radiates in the unit time the power flow:

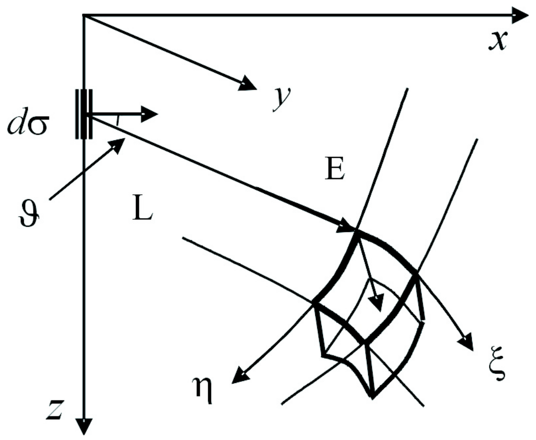


Pic.1. Geometry of the cutting

$$dQ_l = B \cdot D \cdot d\sigma \cdot \cos\vartheta \cdot \frac{dA \cdot \cos\psi}{LE^2}, \tag{5}$$

where B is intensity of radiation; D - arc diameter; ϑ - an angle between the normal to the radiate area and direction $\vec{LE}$ towards the area dA; ψ - an angle between the direction of radiation $\vec{LE}$ and perpendicular to the irradiated area dA - $\vec{e}_\zeta$. Cumulative flow of the arc radiation to the area dA will be received by integration of (5) by the whole arc length h:

$$dQ = B \cdot D \cdot dA \cdot \int_0^h \frac{\cos\vartheta \cos\psi}{LE^2} d\sigma. \tag{6}$$



Pic.2. Liquid film element and arc element

Taking into consideration the smallness of the magnitudes $\delta(\xi, \eta)$ let us consider the thickness of films of the linear approximation of velocity and temperature fields of the molten metal:

$$\begin{aligned} u(\xi,\eta,\zeta) &= u_u(\xi,\eta) \cdot \left(1 - \frac{\zeta}{\delta}\right), \\ T(\xi,\eta,\zeta) &= T_n \cdot \frac{\zeta}{\delta} + T_u(\xi,\eta) \cdot \left(1 - \frac{\zeta}{\delta}\right), \end{aligned} \tag{7}$$

where u_n and T_n are speed and temperature on the film surface; T_n - metal fusing temperature. In that case average through thickness speed and temperature

$$\begin{aligned} u_m(\xi,\eta) &= \frac{1}{2} u_u(\xi,\eta), \\ T_m(\xi,\eta) &= \frac{1}{2}\left(T_n + T_u(\xi,\eta)\right), \end{aligned} \tag{8}$$

If $\rho(T)$ is a density of the molten metal, mass balance equation in the volume dV will look in the following way:

$$\frac{\partial}{\partial \eta}\left(\rho\left(T_m\right) u_m \delta \sqrt{g_{\xi\xi}}\right) = \rho^n\left(T_n\right) \nu_p \left(\vec{e}_\zeta \cdot \vec{i}\right) \left|\frac{\partial \vec{R}}{\partial \xi} \times \frac{\partial \vec{R}}{\partial \eta}\right|. \tag{9}$$

The scalar product $\vec{e}_\zeta \cdot \vec{i}$ defines the cosine of the angle between speed of cut and perpendicular to the area dA. Under the fusion temperature both the phases can exist:

magnitudes relating to the liquid phase will be marked with one touch and to solid phase - with two touches.

Energy balance equation for the volume dV can be presented in the following way:

$$\frac{dQ}{dA} = \frac{\partial}{\partial \eta}\left(c_p\left(T_m\right) \cdot T_m \cdot \rho\left(T_m\right) \cdot u_m \cdot \delta \cdot \sqrt{g_{\xi\xi}}\right) + \lambda'\left(T_n\right)\frac{T_u - T_n}{\delta}. \tag{10}$$

On the boundary of liquid and solid phases the energy balance equation will look like:

$$\lambda'\left(T_n\right)\frac{T_u - T_n}{\delta} = r \cdot \rho''\left(T_n\right) \cdot \nu_p\left(\vec{e}_\zeta \cdot \vec{i}\right) - \lambda''\left(T_n\right)\left.\frac{\partial T}{\partial \eta}\right|_{\zeta=\delta^+}. \tag{11}$$

Equations (10) and (11) are written under the assumption of the small angles between the area dA and approximate area on the fusion surface, i. e. small gradients of the film thickness.

If the liquid metal is considered as a Newtonian liquid, taking into account (8) shearing stress on the film surface are connected to the average velocity:

$$\tau = \mu\left(T_m\right)\frac{2u_m}{\delta}, \tag{12}$$

where $\mu\left(T_m\right)$ is an absolute viscosity coefficient.

On the ground of the suggested approach let us consider the simplified model problem. Assume that the task of cutting of a metal sheet by the radiating arc is a plane one (pic. 3). Coordinate curve lays in the sub space Oxz and the natural parameter $\eta = s$ (length of arc) can be taken as a coordinate. The balance equations (9), (10), (11) will be accordingly rewritten:

$$\frac{d}{ds}\left(\rho\left(T_m\right)u_m\delta\right) = \rho''\left(T_n\right)\nu_p\frac{dz}{ds},$$

$$\rho\left(T_m\right)u_m\delta = \left(\rho\left(T_m\right)u_m\delta\right)_{z=0} + \rho''\left(T_n\right)\nu_p z, \tag{9'}$$

$$\frac{dQ}{ds} = \frac{d}{ds}\left(c_p\left(T_m\right) \cdot T_m \cdot \rho\left(T_m\right) \cdot u_m \cdot \delta\right) + \lambda'\left(T_n\right)\frac{2\left(T_m - T_n\right)}{\delta}, \tag{10'}$$

$$\lambda'\left(T_n\right)\frac{2\left(T_m - T_n\right)}{\delta} = r \cdot \rho^m\left(T_n\right) \cdot \nu_p\frac{dz}{ds} - \lambda''\left(T_n\right)\left.\frac{\partial T}{\partial \xi}\right|_{\zeta=\delta^+}. \tag{11'}$$

Let us define the heat current of radiation (5):

$$dQ_l = B \cdot D \cdot ds \cdot d\sigma \cdot \frac{x(s) \cdot \left(x(s)\frac{dz}{ds} - \left(z(s) - \sigma\right)\frac{dx}{ds}\right)}{\left(x^2(s) + \left(z(s) - \sigma\right)^2\right)^2}.$$

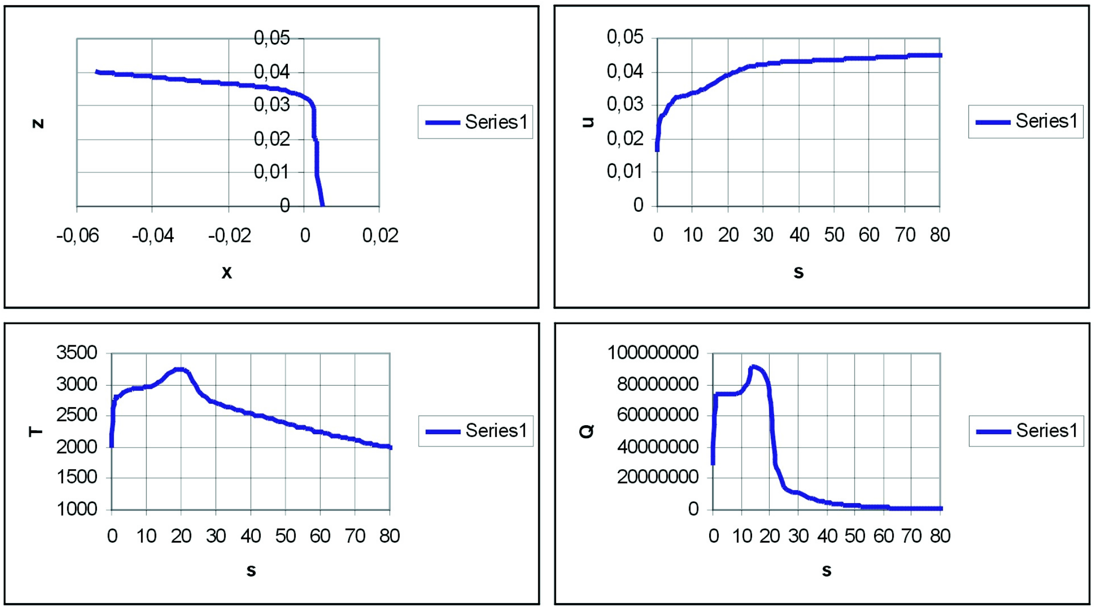
By integration of the relation by the arc length we will receive the expression for $\frac{dQ}{ds}$.

The system of the ordinary differential equations relative to the six unknown functions $x(s)$, $z(s)$, $Q(s)$, $u_m(s)$, $T_m(s)$, $\delta(s)$ is received. Provided that the intensity of radiation B, diameter of the arc - D and tangent stress - $\tau(s)$ on the membrane surface are known, then (12) and geometrical correlation

$$\left(\frac{dx}{ds}\right)^2 + \left(\frac{dz}{ds}\right)^2 = 1$$

close the system.

The received system of equations was solved by approximate numerical procedures. Some results are shown on the pic. 5 - the battle front of melting $z(x)$ and distribution of length of the front of average speed $u_m(s)$, average temperature $T_m(s)$ of the liquid metal and thermal current from the arc $\frac{dQ}{ds}$. The approximate numerical solutions of the model problem qualitatively well reflect the experimental data about the influence of speed of cut and power of the arc on the geometry of cutting.



Pic.5. Some results of the numerical solution of the model problem

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