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ON ONE CYLINDRICAL TYPE TRANSFORMATION OF A RANDOM MEASURE

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Abstract. In an infinite dimensional linear space the nonlinear transformations cylindrical type for random measures are considered. In some conditions the fact equivalence of the corresponding measure and the random measure is proved.

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Many problems connected with equations in measures [1] can be reduced to integral type transformations in infinite-dimensional spaces. In this work we study the problem of the equivalence of distributions of random values for a linear transformation that has an integral form and depends on fixed elements of the original σ-algebra.

Throughout the paper $\{\Omega, \mathcal{J}, P\}$ is assumed to be a fixed probability space. Let Q_m be an m-dimensional cube

$$Q_m = \mathop{X}_{k=1}^{m} [0,1] = [0,1]^m,$$

ν_m a measure on Borel subsets of Q_m. Let further $\{X, B\}$ be some measurable space.

We will consider the random measures $\mu(t, A) = \mu(t, A, \omega)$ defined on $Q_m \times B \times \Omega$ and with real values, which have the following properties:

M1) μ is a random value for fixed $t \in Q_m$ and $A \in B$; $E\mu^2(t, A) < \infty$ almost everywhere on the Lebesgue measure for all $t \in Q_m$ and $A \in B$.

M2) for fixed $t \in Q_m$, $\mu(t, A)$ is a measure with alternating signs almost for all ω.

M3) for fixed $A \in B$, with probability 1 μ is square-integrable with respect to a measure $\nu_m(dt)$.

Let $\widetilde{L}_2(Q_m)$ be a space of random measures $\mu(t, A)$ with properties M1), M2) and M3). It is understood that $\widetilde{L}_2(Q_m)$ is a Hilbert space with the scalar product

$$(\mu_1, \mu_2)_{\widetilde{L}_2(Q_m)} = E \int\limits_{Q_m} \mu_1(t, X)\mu_2(t, X)\nu_m(dt)$$

and, accordingly, with the norm

$$\|\mu\|^2_{\widetilde{L}_2(Q_m)} = E \int\limits_{Q_m} \mu^2(t, X)\nu_m(dt).$$

Denote the self-correlation measure by

$$\beta_{ts}^{\mu_1\mu_2}(A,B) = E\mu_1(t,A)\mu_2(s,B).$$

In terms of this measure we write

$$(\mu_1,\mu_2)_{\widetilde{L}_2(Q_m)} = \int\limits_{Q_m} \beta_{tt}^{\mu_1\mu_2}(X,X)\nu_m(dt)$$

and

$$\|\mu\|^2_{\widetilde{L}_2(Q_m)} = \int\limits_{Q_m} \beta_{tt}^{\mu\mu}(X,X)\nu_m(dt).$$

Let $\widetilde{L}_2^+(Q_m) \subset \widetilde{L}_2(Q_m) \subset \widetilde{L}_2^-(Q_m)$ be a quasikernel equipment of the basic space $\widetilde{L}_2(Q_m)$. Consider the transformation $\widetilde{L}_2(Q_m)$

$$\widetilde{\mu}(A,t) = \mu(A,t) + \\ + \int\limits_{Q_m} G\big(t,s,A,\mu(A_1,s),\mu(A_2,s),\ldots,\mu(A_n,s)\big)\nu_m(ds), \tag{1}$$

where $A_1,A_2,\ldots,A_n$ are fixed measurable sets from B and $G(t,s,A,x_1,x_2,\ldots,x_n)$ is a function on $Q_{2m}\times B\times R^n$. We call such a mapping a cylindrical type transformation (see [2], [3]).

Let us introduce the notation

$$\int\limits_{Q_m} G\big(t,s,A,\mu(A_1,s),\mu(A_2,s),\ldots,\mu(A_n,s)\big)\nu_m(ds) = \\ = g(t,A,\mu_1,\mu_2,\ldots,\mu_n);$$

$$\frac{\partial g(t,A_i,\mu_1,\ldots,\mu_{j-1},u_j,\mu_{j+1},\ldots,\mu_n)}{\partial u_j}\Big|_{u_j=\mu_j} = g'_{ij}(t,A_i,\mu_1,\ldots,\mu_n).$$

The distribution of a random measure μ is denoted by P_μ. A transformation of for (1) of the space $\widetilde{L}_2(Q_m)$ changes the measure μ to the measure $\widetilde{\mu}$ the distribution of which is denoted by $P_{\widetilde{\mu}}$. According to the Minlos-Sazonov theorem, in the above-mentioned conditions these distributions are concentrated in the space $\widetilde{L}_2^-(Q_m)$. We are interested in the conditions for which these distributions are equivalent.

Theorem. *Let conditions* M1), M2) *and* M3) *be fulfilled for transformation* (1), $A_1,A_2,\ldots,A_n$ *be fixed measurable sets from B and the following conditions be fulfilled for functions $G(t,s,A,x_1,x_2,\ldots,x_n)$ on $Q_{2m}\times B\times R^n$:*

- G1) *for and $A\in B$, $G(t,s,A,x_1,x_2,\ldots,x_n)$ is continuous with respect to $t,s\in Q_m$, differentiable with respect to $x_1,x_2,\ldots,x_n$ and square-integrable with respect to the measure $\nu_m\times\nu_m\times l_n$, where l_n is a Lebesgue measure in R^n.*
- G2) *for fixed $t,s\in Q_m$ and $x_1,x_2,\ldots,x_n\in R$, $G(t,s,A,x_1,x_2,\ldots,x_n)$ is a measure on B.*

G3) *there exists a nonzero determinant*

$$\Delta(x) = \begin{vmatrix} 1+g'_{11}(x) & g'_{12}(x) & \dots & g'_{1n}(x) \\ g'_{21}(x) & 1+g'_{22}(x) & \dots & g'_{2n}(x) \\ \dots & \dots & \dots & \dots \\ g'_{n1}(x) & g'_{n2}(x) & \dots & 1+g'_{nn}(x) \end{vmatrix}.$$

Then, if the self-covariational measure $\beta^{\mu\nu}_{ts}$ has a logarithmic derivative with respect to each argument along the constant directions $\widetilde{L}^{+}_{2}(Q_m)$, then the measures P_μ and $P_{\widetilde{\mu}}$ are equivalent and

$$\frac{dP_{\widetilde{\mu}}}{dP_\mu}(\mu) = \Delta(\mu)\exp\Big\{-\beta\big(t, g(t,A,\mu),\mu\big) - \frac{1}{2}\,\|g(t,A,\mu)\|^2_{\widetilde{L}_2(Q_m)}\Big\}, \tag{2}$$

where $\beta(t,g,A)$ denotes some measurable functional which is an abstract analogue of an extended stochastic integral ([1]).

Proof. To transformation (1) we apply a general theorem for the nonlinear transformation of smooth measures from [4]. The main requirement in this theorem consists in proving that transformation (1) is invertible. To establish this fact, we substitute step-by-step $A_1, A_2, \dots, A_n$ in (1) and form the system

$$\begin{aligned}
\widetilde{\mu}(t,A_1) &= \mu(t,A_1) + \int\limits_{Q_m} G\big(t,s,A_1,\mu(s,A_1),\dots,\mu(s,A_n)\big)\nu_m(ds),\\
\widetilde{\mu}(t,A_2) &= \mu(t,A_2) + \int\limits_{Q_m} G\big(t,s,A_2,\mu(s,A_2),\dots,\mu(s,A_n)\big)\nu_m(ds),\\
&\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\\
\widetilde{\mu}(t,A_n) &= \mu(t,A_n) + \int\limits_{Q_m} G\big(t,s,A_n,\mu(s,A_n),\dots,\mu(s,A_n)\big)\nu_m(ds).
\end{aligned}$$

According to condition G3) of the theorem, this system is solvable with respect to $\mu(t,A_1), \mu(t,A_2), \dots, \mu(t,A_n)$. Therefore there exist functions $H_1, H_2, \dots, H_n$ such that

$$\begin{aligned}
\mu(t,A_1) &= H_1\big(t,A_1,\dots,A_n,\widetilde{\mu}(t,A_1),\dots,\widetilde{\mu}(t,A_n)\big),\\
\mu(t,A_2) &= H_2\big(t,A_1,\dots,A_n,\widetilde{\mu}(t,A_1),\dots,\widetilde{\mu}(t,A_n)\big),\\
&\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\\
\mu(t,A_n) &= H_n\big(t,A_1,\dots,A_n,\widetilde{\mu}(t,A_1),\dots,\widetilde{\mu}(t,A_n)\big).
\end{aligned}$$

Then from (1) we obtain an inverse transformation of the form

$$\begin{aligned}
\mu(t,A) = \widetilde{\mu}(t,A) - &\\
- \int\limits_{Q_m} G\Big(t,s,A,H_1\big(s,A_1,\dots,A_n,\widetilde{\mu}(s,A_1),\dots,\widetilde{\mu}(s,A_n)\big),\dots,&\\
H_n\big(s,A_1,\dots,A_n,\widetilde{\mu}(s,A_1),\dots,\widetilde{\mu}(s,A_n)\big)\Big)\nu_m(ds).&
\end{aligned}$$

We observe now that the smoothness (in a sense of the existence of a logarithmic derivative along the constant directions of the subspace $\widetilde{L}_2^+(Q_m)$) of a self-correlation measure gives the same smoothness of the distribution P_μ. The other conditions of the theorem are fulfilled automatically, which shows that the assertion we wanted to prove and formula (2) are valid.

REFERENCES

1. DaletskiĭYu. L., Fomin S. V. Measures and differential equations in infinite-dimensional spaces, M.: 1983 (in Russian).

2. Koval'chik I.M. A nonlinear transformation of Gaussian measures, Mat. Metody i Fiz.-Mekh. Polya, **25** (1987), 13-16 (in Russian).

3. Sokhadze G.A. A note on the absolute continuity of measures for a cylindrical type transformation, Georgian Engineering News, **3** (2007), 13–15.

4. Sokhadze G.A., Tkeshelashvili A. S. On the transformation of a random measure, J. Intellect, **1**, 12 (2002), 39–42, Tbilisi.

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