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MAGNETIC BOUNDARY LAYER OF SECOND KIND WITH STRONG SUCTION

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Abstract. In this paper an asymptotic approximate solution of quations of magnetic boundary layer of second kind with strong suction using the Watson method is given.

Keywords and phrases: Steady magnetic boundary layer, the Watson method, inductive magnetic field, asymptotic approximate solution, strong suction.

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Two dimensional magnetic boundary layer, where magnetic field is perpendicular to the plane of flow, we called boundary layer of second kind [1,2,3].

In this paper an asymptotic approximate solution of equations of magnetic boundary layer of second kind with strong suction using Watson method [4,5,6], is given. Inductive magnetic field and distribution of temperature with dissipation of energy in a viscous liquid as well as Joule heat is founded.

Equations of two dimensional steady magnetic boundary layer of second kind on the plane plate, when magnetic field is directed to the oz axis in non-dimensional quantities are [2,3]:

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{\partial^2 u}{\partial y^2}-ALB_z\frac{\partial B_z}{\partial x}, \tag{1}$$

$$u\frac{\partial B_z}{\partial x}+v\frac{\partial B_z}{\partial y}=\frac{\partial^2 B_z}{\partial y^2}, \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{1}{pr}\frac{\partial^2 T}{\partial y^2}+Ec\left(\frac{\partial u}{\partial y}\right)^2+ALEc\left(\frac{\partial B_z}{\partial y}\right)^2, \tag{3}$$

$$\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=0, \tag{4}$$

where $Al=\dfrac{B_0^2}{\rho\mu_0u_0^2}$ - Alfven number, $P_r=\dfrac{c_\tau\eta}{\lambda}$ - Prandtl number, $Ec=\dfrac{u_0^2}{c_\tau T_0}$ - Eckert number.

Functions $u(x,y), v(x,y), B_z(x,y), T(x,y)$ must be obtained by following boundary conditions:

$$\left.\begin{aligned}&u(x,0)=0,\ v(x,0)=-v_w=\text{const},\ \mathrm{B_z(x,0)=B_w(x)},\ \mathrm{T(x,0)=T_w}=\text{const},\\&u(x,\infty)=u_\infty,\ \ B_z(x,\infty)=0,\ \ T(x,\infty)=T_\infty=\text{const}.\end{aligned}\right\} \tag{5}$$

If velocity $v(x, y)$ from equation of continuity (4) and boundary condition (5) is defined, we will have

$$v = -v_w - \int_0^y \frac{\partial u}{\partial x} dy \tag{6}$$

if new changes $\xi = x$, $z = v_w y$ and also new unknown functions

$$u(x, y) = f(\xi, z),\ B_z(x, y) = \varphi(\xi, z),\ T(x, y) = \theta(\xi, z) \tag{7}$$

are introduced from equations (1)-(3) we well receive:

$$\left.\begin{aligned}
&\frac{\partial^2 f}{\partial z^2} + \frac{\partial f}{\partial z} = \frac{1}{v_w^2}\left(f\frac{\partial f}{\partial \xi} - \frac{\partial f}{\partial z}\int_0^z \frac{\partial f}{\partial \xi} dz + AL\varphi\frac{\partial \varphi}{\partial \xi}\right),\\
&\frac{\partial^2 \varphi}{\partial z^2} + \frac{\partial \varphi}{\partial z} = \frac{1}{v_w^2}\left(f\frac{\partial \varphi}{\partial \xi} - \frac{\partial \varphi}{\partial z}\int_0^z \frac{\partial f}{\partial \xi} dz\right),\\
&\frac{1}{P_r}\frac{\partial^2 \theta}{\partial z^2} + \frac{\partial \theta}{\partial z} + Ec\left(\frac{\partial f}{\partial z}\right)^2 + ALEc\left(\frac{\partial \varphi}{\partial z}\right)^2 = \frac{1}{v_w^2}\left(f\frac{\partial \theta}{\partial \xi} - \frac{\partial \theta}{\partial z}\int_0^z \frac{\partial f}{\partial \xi} dz\right)
\end{aligned}\right\} \tag{8}$$

Let velocity of suctions be large, then $\dfrac{1}{v_w^2} = \varepsilon$ small. We will find an asymptotic series that satisfy equations (8) that are formally assumed as follows:

$$f(\xi, z) = \sum_{k=0}^{\infty} \varepsilon^k f_k(\xi, z),\quad \varphi(\xi, z) = \sum_{k=0}^{\infty} \varepsilon^k \varphi_k(\xi, z),\quad \theta(\xi, z) = \sum_{k=0}^{\infty} \varepsilon^k \theta_k(\xi, z), \tag{9}$$

Then by substituting (9) in equations as (8) and equating to zero the coefficients of the various powers of ε we will obtain the following set of differential equations, that may be solved in two approximation:

$$\frac{\partial^2 f_0}{\partial z^2} + \frac{\partial f_0}{\partial z} = 0,\quad f_0(\xi, 0) = 0,\quad f_0(\xi, \infty) = u_\infty,$$

$$\frac{\partial^2 f_1}{\partial z^2} + \frac{\partial f_1}{\partial z} = f_0\frac{\partial f_0}{\partial \xi} - \frac{\partial f_0}{\partial z}\int_0^z \frac{\partial f_0}{\partial z} dz + AL\varphi_0\frac{\partial \varphi_0}{\partial \xi},\quad f_1(\xi, 0) = 0,\quad f_1(\xi, \infty) = 0,$$

....

$$\frac{\partial^2 \varphi_0}{\partial z^2} + \frac{\partial \varphi_0}{\partial z} = 0,\quad \varphi_0(\xi, 0) = B_w(\xi),\quad \varphi_0(\xi, \infty) = 0,$$

$$\frac{\partial^2 \varphi_1}{\partial z^2} + \frac{\partial \varphi_1}{\partial z} = f_0\frac{\partial \varphi_0}{\partial \xi} - \frac{\partial \varphi_0}{\partial z}\int_0^z \frac{\partial f_0}{\partial z} dz,\quad \varphi_1(\xi, 0) = 0,\quad \varphi_1(\xi, \infty) = 0,$$

....

$$\frac{1}{P_r}\frac{\partial^2 \theta_0}{\partial z^2} + \frac{\partial \theta_0}{\partial z} = -Ec\left(\frac{\partial f_0}{\partial z}\right)^2 - ALEc\left(\frac{\partial \varphi_0}{\partial z}\right)^2,\ \theta_0(\xi, 0) = T_w,\ \theta_0(\xi, \infty) = T_\infty,$$

$$\frac{1}{P_r}\frac{\partial^2 \theta_1}{\partial z^2} + \frac{\partial \theta_1}{\partial z} = f_0\frac{\partial \theta_0}{\partial \xi} - \frac{\partial \theta_0}{\partial z}\int_0^z \frac{\partial f_0}{\partial z} dz - 2Ec\left[\frac{\partial f_0}{\partial z}\frac{\partial f_1}{\partial z} + AL\frac{\partial \varphi_0}{\partial z}\frac{\partial \varphi_1}{\partial z}\right],$$

$$\theta_1(\xi, 0) = 0,\ \theta_1(\xi, \infty) = 0.$$

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The solutions of this system in case $P_r = 1$ are:

$$f_0(\xi, z) = u_\infty(1 - e^{-z}),$$
$$f_1(\xi, z) = \frac{1}{2}ALB_w \frac{dB_w}{d\xi}(e^{-z} - e^{-2z}),$$
$$\varphi_0(\xi, z) = B_w(\xi)e^{-z}, \;\; \varphi_1(\xi, z) = \frac{u_\infty}{2}\frac{B_w}{d\xi}(e^{-z} - 2ze^{-z} - e^{-2z}),$$
$$\theta_0(\xi, z) = T_\infty + (T_w + T_\infty)e^{-z} + \frac{Ec}{2}(u_\infty^2 + AlB_w^2)(e^{-z} - e^{-2z}),$$
$$\theta_1(\xi, z) = ALEcU_\infty B_w \frac{dB_w}{dz}\left(\frac{13}{12}e^{-z} - ze^{-z} - \frac{5}{4}e^{-2z} + \frac{1}{6}e^{-3z} - ze^{-2z}\right).$$

We are now in a position to obtain series for the skin friction, the displacement and momentum thickness of the boundary layer and other physical parameters of boundary layer.

REFERENCES

1. Vatazhin A.B., Lubimov G.A., Regirer S.A. Magnetohydrodynamic flow in channels, Nauka, M., 1970 (in Russian).

2. Dzhaugashtin K.E. Some problems of the theory boundary layer of conducting fluid, Journ. thech. phisics, **33**, 7 (1963), 843-850 (in Russian).

3. Watson E.J. The asymptotic theory of boundary layer flow with suction, Reports and Memoranda, **2610** (1947), 1-26.

4. Sharikadze J.V. Solution of boundary layer equations of conducting fluid with strong suction, Reports of Enlarged Session of the Seminar of I. vekua Institute of Applied mathematics, **16**, 1 (2001), 30-34.

5. Sharikadze J. Falkner-Skan problems for conducting fluid with strong magnetic field, Intern. conf. " The problems of continium mechanics", Transactions, Tbilisi (2007), 83-86.

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