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THE STEADY MHD-FLOW OF A LOW CONDUCTIVE FLUID IN THE NEIGHBOURHOOD OF AN INFINITE POROUS PLATE AT SIMULTANEOUS ROTATION OF A PLATE AND FLUID WITH STRONG MAGNETIC FIELD

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Abstract. There has been studied the steady MHD-flow of a conductive fluid at simultaneous rotation of a infinite porous plate and fluid near it with strong magnetic field for large values of injection velocity by means of the method of the consistent approximation, with the Green function and small parameter

The physical characteristics of fluid motion with respect to small parameter, there are represented by infinite series. There is given recurrent correlations with arbitrary precision. The first two approximations are found explicitly. There is calculated the resistance moment against rotation of plate.

Keywords and phrases: Flow, conductivity, magnetic field, injection velocity, porous.

AMS subject classification: 85W30.

In paper [1] it has been studied a stationary problem of the rotatory motion of conductive fluid on a fixed disk with magnetic field.

In paper [2] it has been investigated the motion of rotating porous plate in a low conductive fluid with weak magnetic field, taking into account heat-transfer.

In the present paper, by the method of succesive approximations, using the Green function and small parameter, it is studied the steady MHD-flow of a flow conductive fluid near an infinite porous plate at simultaneous rotation of a plate and fluid with different angular velocities, taking into account strong magnetic field and large values of injection velocity.

In this case the system of differential equations of the steady motion of conductive fluid with corresponding boundary conditions has the form

$$\begin{cases} v_r\dfrac{\partial v_r}{\partial r}+v_z\dfrac{\partial v_r}{\partial z}-\dfrac{v_\varphi^2}{r}=-\omega_2^2 r+\nu\Big(\Delta v_r-\dfrac{v_r}{r^2}\Big)-\dfrac{\sigma B_0^2}{\rho}v_r, \\ v_r\dfrac{\partial v_\varphi}{\partial r}+v_z\dfrac{\partial v_\varphi}{\partial z}+\dfrac{v_r v_\varphi}{r}=\nu\Big(\Delta v_\varphi-\dfrac{v_\varphi}{r^2}\Big)-\dfrac{\sigma B_0^2}{\rho}(v_\varphi-\omega_2 r), \\ v_r\dfrac{\partial v_z}{\partial r}+v_z\dfrac{\partial v_z}{\partial z}=-\dfrac{1}{\rho}\dfrac{\partial p}{\partial z}+\nu\Delta v_z, \\ \dfrac{\partial v_r}{\partial r}+\dfrac{v_r}{r}+\dfrac{\partial v_z}{\partial z}=0, \end{cases} \tag{1}$$

$$\begin{cases} z=0, \quad v_r=0, \quad v_\varphi=\omega_1 r, \quad v_z=-v_w, \\ z=\infty, \quad v_r=0, \quad v_\varphi=\omega_2 r, \end{cases} \tag{2}$$

where $\Delta = \dfrac{\partial^2}{\partial r^2} + \dfrac{1}{r}\dfrac{\partial}{\partial r} + \dfrac{\partial^2}{\partial z^2}$, while ω_1 and ω_2 are angular velocities of rotation of a plate and fluid, respectively.

Due to geometric and mechanic considerations we seek a solution of the problem in the form

$$\begin{cases} v_r = \omega_0 r f(\xi), \quad v_\varphi = \omega_0 r q(\xi), \quad v_z = \sqrt{\nu\omega_0}\,[g(\xi) - v_w], \\ z = \sqrt{\nu/\omega_0}\,\xi, \quad v_w = \sqrt{\nu\omega_0}\,v_w, \quad P = -\rho\nu\omega_0 P'(\xi). \end{cases} \tag{3}$$

If we introduce additionally variable $\eta = v_w \xi$, then by (3) from system (1)-(2) we receive the following system of equations and boundary conditions:

$$\begin{cases} f'' + f' - k^2 f = \varepsilon(gf') + \varepsilon^2(f^2 - q^2 + \omega_2^2), \\ q'' + q' - k^2 q = \varepsilon(gq') + \varepsilon^2(2fq), \\ \varepsilon P' = -g'' + \varepsilon(gg') - g', \\ g' = -\varepsilon(2f), \end{cases} \tag{4}$$

$$\begin{cases} \eta = 0, \quad f = 0, \quad q = \omega_1, \quad g = 0, \\ \eta = \infty, \quad f = 0, \quad q = \omega_2, \end{cases} \tag{5}$$

where $\varepsilon = \frac{1}{v_w}$, $m^2 = \frac{\sigma B_0^2}{\rho\omega_0}$, $k^2 = \frac{m^2}{v_w^2}$.

The solution of problem (4)-(5), by use the Green function, can be reduced to the solution of the system of integral-differential equations

$$\begin{cases} f = \int\limits_0^\infty [\varepsilon(gf') + \varepsilon^2(f^2 - q^2 + \omega_2^2)] G(\eta,\zeta) d\zeta, \\ q = \int\limits_0^\infty [\varepsilon(gq') + \varepsilon^2(2fq)] G(\eta,\zeta) d\zeta, \\ g = -\varepsilon \int\limits_0^\eta (2f) d\zeta, \end{cases} \tag{6}$$

where $G(\eta,\zeta)$ is the Green function of problem

$$G'' + G' - k^2 G = 0,$$
$$G|_{\eta=0} = 0, \quad G|_{\eta=\infty} = 0.$$

It has the form

$$G = \begin{cases} G_1 = \dfrac{e^{-n\eta} - e^{(n-1)\eta}}{2n-1}\, e^{-(n-1)\zeta}, & 0 \le \eta < \zeta, \\ G_2 = \dfrac{e^{-(n-1)\zeta} - e^{n\zeta}}{2n-1}\, e^{-n\eta}, & \zeta \le \eta < \infty, \end{cases}$$

where $n = \dfrac{\sqrt{1+4k^2}+1}{2}$.

Let us search a solution of system (6) in the form of series with respect to small parameter ε:

$$f=\sum_{i=0}^{\infty}\varepsilon^{4i+2}f_i,\quad q=\sum_{i=0}^{\infty}\varepsilon^{4i}q_i,\quad g=\sum_{i=0}^{\infty}\varepsilon^{4i+3}g_i \tag{7}$$

Introducing series (7) into system (6) and equating the coefficients of the same degrees of ε, we receive the following recurrence relations:

$$\begin{aligned}
f_0 &= \int_0^\infty (-q_0^2+\omega_2^2)G(\eta,\zeta)d\zeta,\\
f_j &= \int_0^\infty \Big[\sum_{\alpha=0}^{j-1}(g_\alpha f'_{j-\alpha-1}+f_\alpha f_{j-\alpha-1})-\sum_{\alpha=0}^{j}q_\alpha q_{j-\alpha}\Big]G(\eta,\zeta)d\zeta,\ (j\ge 1),\\
q_0 &= A(\eta),\\
q_j &= \int_0^\infty \Big[\sum_{\alpha=0}^{j-1}(2f_\alpha q_{j-\alpha-1}+g_\alpha q'_{j-\alpha-1})\Big]G(\eta,\zeta)d\zeta,\ (j\ge 1),\\
g_j &= -2\int_0^\eta f_j d\zeta,\quad (j\ge 0),
\end{aligned}$$

where $A(\eta)$ is the solution of the problem

$$\begin{aligned}
&A''(\eta)+A'(\eta)-k^2A(\eta)=0,\\
&A(0)=\omega_1-\omega_2,\quad A(\infty)=0.
\end{aligned}$$

The first two approximations f_0, f_1, q_0, q_1, g_0, g_1 have the form:

$$\begin{aligned}
f_0 =&\frac{2\omega_2(\omega_1-\omega_2)}{2n-1}\eta e^{-n\eta}+\frac{(\omega_1-\omega_2)^2}{n(3n-1)}(e^{-n\eta}-e^{-2n\eta}),\\
q_0 =&\omega_2+(\omega_1-\omega_2)e^{-n\eta},\\
g_0 =&\frac{4\omega_2(\omega_1-\omega_2)}{n^2(2n-1)}[(1+n\eta)e^{-n\eta}-1]-\frac{(\omega_1-\omega_2)^2}{n^2(3n-1)}(e^{-n\eta}-1)^2,\\
f_1 =&(\omega_1-\omega_2)^4\Big[\frac{(2n-1)^2A_1}{(4n-1)(5n-1)}(e^{-n\eta}-e^{-4n\eta})\\
&-\frac{(A_2+A_3)\omega_2-A_3\omega_1}{\omega_1-\omega_2}(e^{-n\eta}-e^{-3n\eta})+\frac{A_8}{(\omega_1-\omega_2)^2}(e^{-n\eta}-e^{-2n\eta})\Big]\\
&+(\omega_1-\omega_2)^3\Big[\frac{\omega_2}{n^2(2n-1)(4n-1)}e^{-2n\eta}+\frac{2A_1A_9}{\omega_1-\omega_2}e^{-n\eta}-\frac{A_{10}}{\omega_1-\omega_2}\Big]\\
&\times\eta e^{-n\eta}-2\omega_2^3(\omega_1-\omega_2)\\
&\times\Big[\frac{2}{3(2n-1)^3}\eta^3+\frac{(\omega_1-\omega_2)A_{11}}{\omega_2^2}\eta^2+\frac{4}{(2n-1)^5}\eta\Big]e^{-n\eta},\\
q_1 =&-\frac{2\omega_2^2(\omega_1-\omega_2)}{(2n-1)^3}\big[(2n-1)\eta+2\big]\eta e^{-n\eta}
\end{aligned}$$

$$
\begin{aligned}
&+\frac{2\omega_2(\omega_1-\omega_2)^2(8n-3)}{n(2n-1)^2(3n-1)}\Big[\frac{2n-1}{3n-1}(e^{-n\eta}-e^{-2n\eta})-\eta e^{-n\eta}\Big]\\
&+\frac{(\omega_1-\omega_2)^3}{n(2n-1)(3n-1)}\Big[\frac{2n-1}{2n(4n-1)}(e^{-n\eta}-e^{-3n\eta})-\eta e^{-n\eta}\Big],\\
g_1=&-(\omega_1-\omega_2)^4\Big[\frac{(2n-1)^2A_1}{2n(4n-1)(5n-1)}(e^{-4n\eta}-4e^{-n\eta}+3)\\
&-\frac{2(A_2+A_3)\omega_2-2A_3\omega_1}{3n(\omega_1-\omega_2)}(e^{-3n\eta}-3e^{-n\eta}+2)\\
&+\frac{A_8}{n(\omega_1-\omega_2)^2}(e^{-n\eta}-1)^2\Big]+(\omega_1-\omega_2)^3\\
&\times\Big[\frac{2\omega_2}{9n^4(2n-1)(4n-1)}(3n\eta e^{-3n\eta}+e^{-3n\eta}-1)\\
&+\frac{A_1A_9}{n^2(\omega_1-\omega_2)}(2n\eta e^{-2n\eta}+e^{-2n\eta}-1)\\
&-\frac{2A_{10}}{n^2(\omega_1-\omega_2)}(n\eta e^{-n\eta}+e^{-n\eta}-1)\Big]-2\omega_2^3(\omega_1-\omega_2)\\
&\times\Big[\frac{4}{3n^4(3n-1)^3}(n^3\eta^3e^{-n\eta}+3n^2\eta^2e^{-n\eta}+6n\eta e^{-n\eta}-6)\\
&+\frac{2(\omega_1-\omega_2)A_{11}}{n^3\omega_2^2}(n^2\eta^2e^{-n\eta}+2n\eta e^{-n\eta}+2e^{-n\eta}-2)\\
&+\frac{8}{n(2n-1)^5}(e^{-n\eta}-1)\Big],
\end{aligned}
$$

where we use the following notations:

$$
\begin{aligned}
A_1=&\frac{1}{n^2(3n-1)^2(2n-1)^2},\\
A_2=&\frac{(16n^2+3n-3)(4n-1)+2(5n-1)(3n-1)}{n(4n-1)^2}(2n-1)A_1,\\
A_3=&\frac{3(2n-1)^2}{2n(4n-1)}A_1,\\
A_4=&-\frac{4n(8n-3)(2n-1)^2+8(3n-1)(2n^2-4n+1)}{n(2n-1)(3n-1)}A_1,\\
A_5=&\frac{2(16n^2+3n-3)(2n-1)-2(16n-3)(3n-1)}{n(3n-1)}A_1,\\
A_6=&\frac{(2n^2+7n-2)(2n-1)}{n(3n-1)(4n-1)}A_1,\\
A_7=&\frac{4(14n-5)}{n(3n-1)(2n-1)^3}-\frac{4n(8n-3)(2n-1)+8(3n-1)^2}{2n-1}A_1,\\
A_8=&\omega_2^2A_4+\omega_2(\omega_1-\omega_2)A_5-(\omega_1-\omega_2)^2A_6,\\
A_9=&\frac{4n(3n-1)\omega_2^2}{2n-1}+4(3n-1)\omega_2(\omega_1-\omega_2)+(2n-1)(\omega_1-\omega_2)^2,
\end{aligned}
$$

$$\begin{aligned}A_{10} =&\omega_2^2 A_7 + (\omega_1-\omega_2)^2(2n-1)A_1\\&+\frac{\omega_2(\omega_1-\omega_2)(28n^2-12n+1)}{(2n-1)(4n-1)}(3n-1)A_1,\\A_{11} =&\frac{2n(14n-5)(3n-1)A_1}{2n-1}\omega_2(\omega_1-\omega_2)\\&+\frac{4\omega_2^2}{(2n-1)^4}+n(3n-1)(\omega_1-\omega_2)^2A_1.\end{aligned}$$

The obtained solutions are valid for an infinite plate. However, for sufficiently large radius R, we can neglect the influence of an edge and calculate the value of a moment M of forces resistant to the rotation:

$$\begin{aligned}M =&\frac{\pi\mu\sqrt{\omega_0^3}v_w R^4(\omega_1-\omega_2)}{2\sqrt{\nu}}\Big\{n+\frac{2\varepsilon^4}{2n-1}\Big[\frac{2\omega_2^2}{(2n-1)^2}\\&-\frac{\omega_2(\omega_1-\omega_2)^2(8n-3)(2n^2-4n+1)}{n(2n-1)(3n-1)}+\frac{(\omega_1-\omega_2)^2}{(3n-1)(4n-1)}\Big]\Big\}.\end{aligned}$$

From solutions obtained above it is clear that the influence of injection velocity, magnetic interaction and angular velocities of the rotation of a plate and fluid on the physical characteristics of a flow.

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