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THE LINEAR SYSTEM OF DIFFERENTIAL EQUATIONS WITH DEVIATING ARGUMENTS

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Abstract. A certain sufficient condition for the oscillation of proper solutions of the system of third order linear system of differential equations with deviating arguments is established in the present paper.

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Consider the linear system

$$\begin{cases} x_i'(t) = x_{i+1}(\beta_{i+1}t) \qquad (i = 1, 2, \dots n-1), \\ x_n'(t) = p(t)x_1(\beta_1 t), \end{cases} \tag{1}$$

where $p \in L_{loc}(R_+; R)$, $\beta_i \in]0; +\infty[\quad (i = 1, 2, \dots n)$.

Definition 1. A continuous vector function

$$X = (X_i)_{i=1}^n : [t_0; +\infty[\to R^n,$$

with $t_0 \in R_+$ is said to be a proper solution of the system (1) if it is locally absolutely continuous on $[t_0; +\infty[$, almost everywhere on this interval the equality (1) is fulfilled, and

$$sup\{\|x(s)\| : s \in [t; +\infty[\} > 0, \qquad for \quad t \in [t_0; +\infty[.$$

Definition 2. A proper solution of the system (1) is said to be oscillatory if every component of this solution has a sequence of zeroes tending to $+\infty$. Otherwise the solution is said to be non-oscillatory.

Definition 3. We say that the system (1) has the property A provided its every proper solution is oscillatory if n is even, and lither is oscillatory or satisfies

$$|x_i(t)| \downarrow 0, \text{ for } t \uparrow +\infty, \ (i = 1, 2, \dots n), \tag{2}$$

if n is odd.

Definition 4. We say that the system (1) has the property B provided its every proper solution either is oscillatory or satisfies either (2) or

$$|x_i(t)| \uparrow +\infty, \text{ for } t \uparrow +\infty, \ (i = 1, 2, \dots n), \tag{3}$$

if n is even, and either is oscillatory or satisfies (3) if n is odd.

Theorem 1. *Let* $p \in L_{loc}(R_+; R)$,

$$\prod_{i=1}^{n} \beta_i \geq 1 \qquad (\prod_{i=1}^{n} \beta_i \leq 1)$$

and

$$\overline{lim}_{t\to+\infty} t \int_t^{+\infty} s^{n-2}|p(s)|ds > (n-1)! \prod_{i=2}^{n} \beta_i^{i-1} \qquad ((n-1)! \prod_{i=1}^{n-1} \beta_i^{i-n}).$$

Then the system (1) has the property A.

Theorem 2. *Let* $p \in L_{loc}(R_+; R)$,

$$\prod_{i=1}^{n} \beta_i \geq 1 \qquad (\prod_{i=1}^{n} \beta_i \leq 1)$$

and

$$\overline{lim}_{t\to+\infty} \frac{1}{t} \int_0^{+\infty} s^{n}|p(s)|ds > (n-1)! \prod_{i=2}^{n} \beta_i^{i-1} \qquad ((n-1)! \prod_{i=1}^{n-1} \beta_i^{i-n}).$$

Then the system (1) has the property A.

Theorem 3. *Let* $p \in L_{loc}(R_+; R_+)$,

$$\prod_{i=1}^{n} \beta_i \geq 1 \qquad (\prod_{i=1}^{n} \beta_i \leq 1).$$

Moreover, let

$$\overline{lim}_{t\to+\infty} t \int_t^{+\infty} s^{n-2}|p(s)|ds > 2(n-1)! \prod_{i=1}^{n} \beta_i^{i-2} \qquad (2(n-2)! \prod_{i=1}^{n} \beta_i^{i+1-n}),$$

if n *is even and*

$$\overline{lim}_{t\to+\infty} t \int_t^{+\infty} s^{n-2}|p(s)|ds > (n-1)! \prod_{i=2}^{n} \beta_i^{i-1} \qquad ((n-1)! \prod_{i=1}^{n} \beta_i^{i+1-n}),$$

if n *is odd. Then the system (1) has the property* B.

Theorem 4. *Let* $p \in L_{loc}(R_+; R_+)$,

$$\prod_{i=1}^{n} \beta_i \geq 1 \qquad (\prod_{i=1}^{n} \beta_i \leq 1).$$

Moreover, let

$$\overline{lim}_{t\to+\infty} \frac{1}{t} \int_0^{t} s^{n}p(s)ds > 2(n-2)! \prod_{i=1}^{n} \beta_i^{i-2} \qquad (2(n-2)! \prod_{i=1}^{n} \beta_i^{i+1-n}),$$

if n is even, and

$$\overline{lim}_{t\to+\infty}\frac{1}{t}\int_0^t s^n p(s)ds > (n-1)!\prod_{i=2}^{n}\beta_i^{i-1} \qquad ((n-1)!\prod_{i=1}^{n}\beta_i^{i+1-n}),$$

if n is odd. Then the system (1) has the property B.

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