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OPTIMIZATION PROBLEM OF THE CYLINDRICAL COVERING OF THE UNDERGROUND CONSTRUCTION

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Abstract. The work deals with the questions of optimal design of the cylindrical coverings of the underground constructions by using the half-moment theory. Based on the known transition of the yield conditions of brittle-ductile materials, the solution to the problem is reduced to a statically definable system. Provided the load on structure and mid-surface configuration are given, the optimal thicknesses received through calculations ensure its minimum volume.

Keywords and phrases: Cylindrical covering, semi-flexible, tensile, compression, optimal thickness, strength, force.

AMS subject classification: 74K25; 74K20.

The problem of optimal designing of the building structures, running at the margin of a deformable solid body theory and optimal control theory, considers the issue of a maximum planning efficiency of the study object. Selecting the optimality criterion is the main problem of optimal design. When solving practical problems, they mainly base themselves on such simple criteria, as the minimum weight (volume) of the structure by preserving its strength, rigidity and stability.

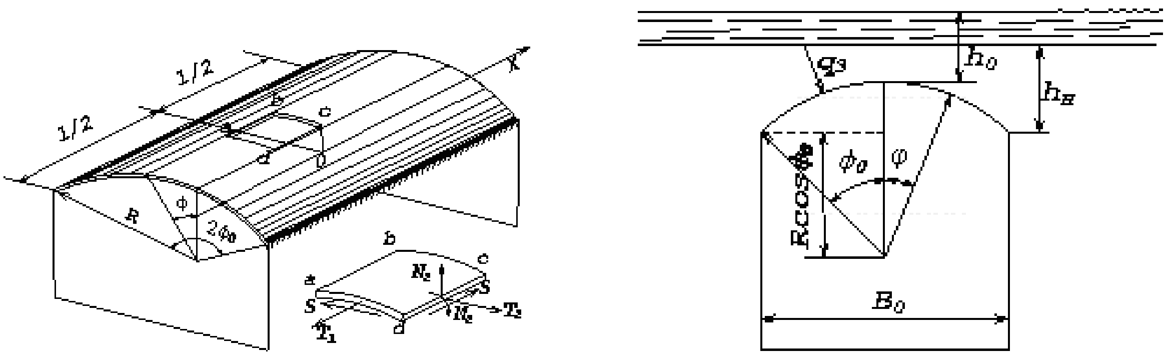

Fig.1.(a) Design model of a cylindrical covering of an underground construction
(b) Protodyakonov's scale to define vertical rock pressure.

The problem of minimization of the weight of the cylindrical covering of the underground construction may be solved by optimization of either its mid-surface, or its thickness, or both. The work gives the solution to the optimization problem of covering of the cylindrical constructions by selecting the thicknesses, which ensure full and instantaneous transition of the construction to the plastic state by considering different tensile and compressive strengths of the material (brittle-ductility).

The length of the cylindrical coverings of the underground constructions usually much exceeds its width in horizontal projection what along with the proper boundary conditions, enables us to estimate its mode of deformation in fact with an acceptable accuracy by using the theory of semi-flexible shells (Fig. 1, a) when the decisive system of differential equations is as follows [1]:

$$\begin{aligned}
&\frac{\partial T_1}{\partial x} + \frac{1}{R}\frac{\partial S}{\partial \varphi} + q_1 = 0,\\
&\frac{\partial S}{\partial x} + \frac{1}{R}\frac{\partial T_2}{\partial \varphi} + \frac{N_2}{R} + q_2 = 0,\\
&\frac{1}{R}\frac{\partial N_2}{\partial \varphi} - \frac{T_2}{R} + q_3 = 0,\\
&\frac{\partial M_2}{\partial \varphi} - RN_2 = 0.
\end{aligned} \tag{1}$$

In case of a large working when the width $B_0 > 6m$ and the coefficient of rock hardness $f_k > 4$, the vertical rock pressure is taken as evenly distributed with q_0 intensity, and its projections on the axes will be:

$$q_1 = 0; \quad q_2 = q_0 \sin\varphi, \quad q_3 = -q_0 \cos\varphi. \tag{2}$$

In more general cases when a vertical rock pressure acts on the covering, the hydrostatic pressure force of ground waters and the own gravity of the cupola (Fig. 1, b) may be calculated from [2]:

$$\begin{gathered}
q_1 = 0; \quad q_2 = 0;\\
q_3 = -\gamma_f(R\cos\varphi_0 + h_H - R\cos\varphi) + \gamma(h_0 + R - R\cos\varphi) + \gamma_b h,
\end{gathered} \tag{3}$$

where γ_f, γ and γ_b are volume weights of the moutain rock, water and concrete, respectively.

In case of a complex state of stress, in order to write down the equation of plastic fluidity (strength), M. Mikeladze [1] when presenting the pressure components to the mid-surface as paired and unpaired functions in the Stas strength condition for brittle-ductile materials and after proper transitions, gains [1]:

$$T_1 = 2T_2 + (\rho - 1)\sigma_S h, \tag{4}$$

$$(1-\rho)^2\sigma_S^2 h^4 + 6(1-\rho)T_1\sigma_S h^3 - 3(T_1^2 + 4S^2)h^2 - 64M_2^2 = 0, \tag{5}$$

where σ_S and $\rho\,\sigma_S$ are tensile strength and compression strength. A set of equations (1) and (4) together with proper boundary conditions, as for the statically definable system, enables us to define all unknown forces and torques (T_1, T_2, S, N_2, M_2). Unknown thicknesses of covering, which ensure the equal danger of the different points of its body to destruction, are determined by the equation of strength (fluidity) (5). The section sizes of the covering defined by such an approach allow for designing the construction of a practically minimum volume.

By considering the relationship (4) in the first three equations of the system of equilibrium equations (1) as a result of elimination of T_1, S and N_2, we gain the decisive differential equation to T_2 longitudinal force.

$$\frac{\partial^2 T_2}{\partial \varphi^2} - 2R^2 \frac{\partial^2 T_2}{\partial x^2} - (\rho - 1)\sigma_S R^2 \frac{\partial^2 h}{\partial x^2} + T_2 + 2q_0 R \cos\varphi = 0.$$

If considering that the thickness of covering along axis x changes smoothly, we can ignore the influence of $(\rho - 1)\sigma_S R^2 \frac{\partial^2 h}{\partial x^2}$ members what is equal to the assumption $\rho = 1$and which we will use when specifying the boundary conditions. By considering the above-mentioned, the decisive equation will be presented as follows:

$$\frac{\partial^2 T_2}{\partial \varphi^2} - 2R^2 \frac{\partial^2 T_2}{\partial x^2} + T_2 + 2qR \cos\varphi = 0. \tag{6}$$

For circular edges of the covering with an articulated support, when $x = \pm\frac{\ell}{2}$, $T_1 = 0$, that is by considering (4) $T_2 = \frac{1}{2}(1 - \rho)\sigma_S h$. If assuming that $\rho = 1$, then $T_2 = 0$. In order to meet these boundary conditions, let us present the solution to the decisive equation and load on it with the aim of separation of the variables as follows:

$$T_2(x, \varphi) = T_2(\varphi) \cos\frac{\pi}{\ell}x, \quad q(\varphi) = q_0(\varphi) \cos\frac{\pi}{\ell}x. \tag{7}$$

By considering (7), the decisive equation (6) will be presented as follows:

$$\frac{\partial^2 T_2(\varphi)}{\partial \varphi^2} + \omega^2 T_2(\varphi) = -2q_0 R \cos\varphi, \tag{8}$$

where $\omega^2 = \frac{2R^2\pi^2}{\ell^2} + 1$.

The reference data of the function $T_2(\varphi)$ when $\varphi = 0$ will be:

$$T_2(\varphi) = 0, \quad \frac{\partial T_2(\varphi)}{\partial \varphi} = 0. \tag{9}$$

The first condition is the result of the study implying that the yaw rate along the edge ($\varphi = 0$) due to the boundary state approaches infinity, and the appropriate normal stress approaches zero [1]. The second condition follows from the second equation of system (1) by considering that due to the symmetry of the covering and load, the intersecting force $N_2(x, 0)$ along the edge and the external load component $q_2 = 0$. The shear force S equal to zero and its derivative $\left.\frac{\partial S}{\partial x}\right|_{\varphi=0} = 0$.

Let us present the solution to the heterogeneous differential equation (8) as a sum of a general solution to the homogenous equation and particular solution of the heterogeneous equation relevant of (8):

$$T_2(\varphi) = C\cos\omega\varphi + D\sin\omega\varphi + \frac{2q_0 T\cos\varphi}{1 - \omega^2}.$$

By considering the boundary condition (9), we will gain:

$$C = \frac{2q_0 R}{\omega^2 - 1} \quad \text{and} \quad D = 0.$$

Then

$$T_2 = \frac{2q_0R}{\omega^2-1}\,(\cos\omega\varphi - \cos\varphi)\cos\frac{\pi}{\ell}\,x. \tag{10}$$

The intersecting force N_2 is defined from equation (3) of system (1). Let us present this force as follows:

$$N_2(x,\varphi) = N_2(\varphi)\cos\frac{\pi}{\ell}\,x. \tag{11}$$

The reference condition for function $N_2(\varphi)$ will be $N_2(0) = 0$ determined by the symmetry of load and construction.

According to the reference condition, for integration function $c(\varphi)$ we assume that $c(0) = 0$ and then, according to (11), we will gain:

$$N_2 = \left[\frac{2q_0R}{\omega^2-1}\left(\frac{1}{\omega}\,\sin\omega\varphi - \sin\varphi\right) - q_0R\sin\varphi\right]\cos\frac{\pi}{\ell}\,x. \tag{12}$$

We define the tangential force S from the first equation of system (1), where if equaling $\frac{\partial h}{\partial x}$ to 0 and integration function, when $\varphi = 0$, we will gain:

$$S = \frac{4q_0R^2\pi}{(\omega^2-1)\ell}\left(\frac{1}{\omega}\,\sin\omega\varphi - \sin\varphi\right)\sin\frac{\pi}{\ell}\,x. \tag{13}$$

For defining the bending moment, we use the equation four of system (1), which if presented by a single trigonometric series and after integration, will be presented as follows:

$$M_2(\varphi) = \frac{2q_0R^2}{\omega^2-1}\left(\cos\varphi - \frac{1}{\omega^2}\,\cos\omega\varphi\right) - q_0R^2\cos\varphi + c(\varphi).$$

For determining the function of integration $c(\varphi)$, we must assume that the value of the bending moment M_0 along the edge $\varphi = 0$ is known, i.e.

$$c(\varphi) = M_0 - \frac{2q_0R^2}{\omega^2-1}\left(\cos\varphi - \frac{1}{\omega^2}\,\cos\omega\varphi\right) + q_0R^2\cos\varphi.$$

When the longitudinal edges are rigidly fastened, then $M_2(0) = M_0$ and $c(\varphi) = M_0 - \frac{2q_0R}{\omega} + q_0R^2$, and when the longitudinal edges are fastened in an articulated manner, then

$$M_2(\varphi_0) = 0 \quad\text{and}\quad c(\varphi_0) = -\frac{2q_0R^2}{\omega^2-1}\left(\cos\varphi_0 - \frac{1}{\omega^2}\,\cos\omega\varphi_0\right) + q_0R^2\cos\varphi_0,$$

and

$$M_2 = M_2(\varphi)\cos\frac{\pi}{\ell}\,x. \tag{14}$$

Aiming at simplifying the numerical implementation of the problem the influence of the shear force in expression (5) to define the optimal values of thickness is better to ignore and by considering dependence (4), to present it as follows:

$$\left(\frac{4M_2}{\sigma_S h^2}\right)^2 + 6(\rho-1)\,\frac{T_2}{\sigma_S h} + 3\left(\frac{T_2}{\sigma_S h}\right)^2 = (2\rho-1)(2-\rho). \tag{15}$$

The problem is better to solve by the method of gradual approximation. In the first approximation, let us assume that $\rho = 0$ and let us ignore the weightiness of the construction in the zero approximation. Let us determine forces and moments, etc. in the first approximation by using the thicknesses gained as a result of such an assumption. The calculation will be continued unless desirable accuracy is gained.

The methods of calculation is the same when force q acting on the covering is defined by equation (3).

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