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# NEW MATHEMATICAL MODEL OF MOVEMENT OF CONSTRAINT PARTICLE

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**Abstract**. It is advanced an opinion that the cause of the particle motion retardation, or dampening of vibrations, may not only be the factors being external with regard to the mass, but the nature of the mass itself. the article provides an explanation of the mechanism of realization of the delay effect.

**Keywords and phrases**: Newton's Law, motion of a particle, damping of vibrations, time lags argument.

**AMS subject classification**: 74K99.

Before beginning my report I like to make little introduction and explain, how the issue of classical mechanics, as 'constraint particle movement's, fall into sphere of my interests. I represent the Institute of Structural Mechanics and Earthquake Engineering, the main direction of which there is 'protection of population, cities and settlements from strong earthquakes'. It is noteworthy that the research of interaction between seismic waves and structure, i.e. the creation of mathematical simulation, began 150 years ego. In spite of this, the revision and accurate definition of this simulation continues up to present time. Moreover, the analysis of earthquakes those had place in XX century, has shown that the simulation, on the basis of which the earthquake engineering is implemented all over the world, is not reliable, and it must be changed right away! This is the world problem, and ways of quest of new mathematical simulation are outlined in different directions. As one of general directions must be considered the more deep study of material behaviour in conditions of strong dynamic effect, when at passage of seismic wave of high intensity the damage, cracking, destruction of material (rock, soil, concrete, stone, steel) occurs. The complexness of this phenomena forces investigators to apply to simulations those study the phenomena on the level of small particles, molecules or even atoms. This way was in antiquity, this way is at present. There is suspicion that even here something unknown exists that defines the unintelligible behaviour of material and structure sometimes at earthquakes.

We have applied to such simulation as well. We are examining the system of particles (masses) those are connected by bonds. Bonds can be elastic, sprig-like or one-sided bonds those break at strong dynamic effect and so on.

For study of such dynamical system we have elaborated the numerical algorithm that before was used by us in research of essentially nonlinear task [1]. The basic moment of this algorithm is the fact that masses periodically (on the discrete net of the time) are released from bonds and are replaced in small periods of time as free material particles, and then the restoration of bonds occurs.

We have checked this algorithm first on known tasks and noticed very interesting phenomena - there occurred the delay of movement and the damping of oscillator vibration.

The analysis of numerical algorithm has shown that this was caused by the circumstance that the mass (material particle) was reacted to external effect with delay, and then we realized, that this is the mass internal nature, that consists in following:

"Every body will no longer remain at rest or in uniform motion in a straight line if compelled to change its state by the action of an external force, but will do it with a delay. This text is a periphrasis of the Newton's famous Law, with a difference that classical mechanics usually asserts that the reaction to an external force takes place instantly, while we draw attention to the fact that the reaction occurs with a delay in time".

So, the research of present task let dare to formulate Newton's law little otherwise, and this, from it's part, forced us to make following step: to look for explanation, how the mass reaction delay causes the movement delay, or oscillation damping. This was conducted on the oscillator case study, the talk about which is given below.

The example has been taken from [2] and its summary follows the same source. Let point $M$ with the mass $m$ move in a straight line under the action of gravitational force $F$ to the stationary center $O$ (Fig 1). Suppose the force modulus $F$ is proportional to the distance of the particle $M$ from the center $O$, i.e.$F = cx\ OM$ where $c$ is the constant aspect ratio. The law of motion of the particle $M$ is to be found.

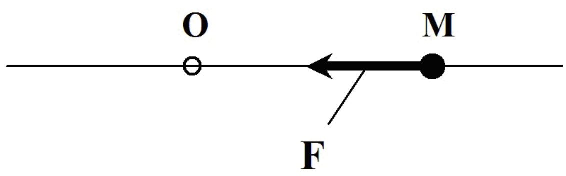


Fig.1.

A corresponding differential equation of motion of the particle $M$ will be recorded as follows:

$$\frac{d^2X(t)}{dt^2} + k^2X(t) = 0, \tag{1}$$

where $k^2 = \dfrac{c}{m}$.

The solution of the equation (1) is

$$X(t) = a\sin(kt + \alpha), \tag{2}$$

where $a$ and $\alpha$ have arbitrary constant values.

The equation (2) is an equation of harmonic vibrations. Correspondingly, in the case of a straight-line motion under the action of the gravitational force proportional to the distance from the center of gravity, the particle will perform harmonic non-damped vibrations.

If the particles moves in a straight line along the axis $x$ in the resisting medium, say in air, and the resistance force $R$ is assumed to be proportional to the velocity of travel of the particle, then the differential equations of motion of the particle M could be recorded as follows (Fig. 2):

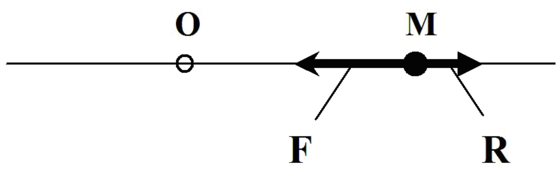


Fig.2.

$$\frac{d^2X(t)}{dt^2} + 2n\frac{dX(t)}{dt} + k^2X(t) = 0, \tag{3}$$

where $\mu$ is the constant aspect ratio characterizing the medium resistance, and $2n = \dfrac{\mu}{m}$
The solution of the equation (3) is

$$X(t) = ae^{-nt}x\sin(k_1t + \alpha), \tag{4}$$

where $k_1^2 = k^2 - n^2$.

This equation differs from the equation (1) of harmonious vibrations by the factor $e^{-nt}$; therefore the $M$ particle motions represent vibrations near the center $O$, but the peak-to-peak value of these vibrations no longer preserves its constant value; they tend to rapidly decrease with time. Therefore, these vibrations are called damped vibrations.

We have set forth the physical problem based on [2] where the resistance of medium, namely air, is referred to as the cause of the damping of vibrations. In classical mechanics, proposed are also other models explaining the fact of existence of damping of vibrations - the internal friction model, the energy dissipation model, etc., which does not alter the essence of the matter, for all of them consider that the cause of the damping of vibrations is beyond the particle.

In contrast to the above, we accentuate on the proposition that the cause of the damping of vibrations and the cause of the retardation of displacements is the mass of the particle itself which, as a result of the external action starts to move but with a delay; therefore, the reaction in a constraint also occurs with a delay [3]. In our specific case, this statement will sound as follows: if the external force $F(t)$ and the inertia force $m\frac{d^2X(t)}{dt^2}$ act on the mass $m$ at the instant $t$, then the motion of the mass $m$ originates with a certain delay, i.e. at the instant $(t + \tau)$.

If we accept this point of view, the equation (1) (without resistance of the medium) could be rewritten as follows:

$$\frac{d^2X(t)}{dt^2} + k^2X(t + \tau) = 0. \tag{5}$$

We obtain a differential equation with the time advanced argument [4], which enables to treat many phenomena from a new standpoint and, repeating the words of the author of the monograph, "opens a field full of attractive facts that so resembles and does not resemble the theory of ordinary differential equations".

While pursuing limited objectives, let us expand $X(t+\tau)$ in the Taylor series

$$X(t+\tau) = X(t) + \tau \frac{dX(t)}{dt} + \ldots \tag{6}$$

and, preserve in the expansion (6) the first two members. Inserting this in (5), we obtain:

$$\frac{d^2X(t)}{dt^2} + k^2\tau \frac{dX(t)}{dt} + k^2 X(t) = 0. \tag{7}$$

The equation (7) is identical to the equation (3), i.e. it produces the same damped vibrations of the particle $M$.

Thus, we have attempted to demonstrate that the retarding action of the inertia forces of the mass, i.e. the reaction delay effect in a constraint, results in damped vibrations and can be written down as a differential equation with the time advanced argument.

## REFERENCES

1. Gabrichidze G.K. Some nontraditional problems of interaction building structures with Geological Environment, Dissertation, Tbilisi, 1991 (in Russian).

2. Voronkov I.M. Theoretical mechanics, Moscow, 1961 (in Russian).

3. Gabrichidze G.K. Every body will no longer remain at rest or in uniform motion in straight line if compelled to change its state by the action of an external force, but will do it with a delay, Bulletin of the Georgian national Academy of Sciences, **2**, 1 (2008), 54-56.

4. Mishkis A.D. Linear differential equation with Delay arguments, Moscow, 1972 ( in Russian).

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