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CALCULATION OF RECTANGULAR DOUBLE CURVATURE FLAT SHELL BY TRIGONOMETRICAL FUNCTIONS

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Abstract. In the work is considered the inclined rotational shell analysis. Integration the written down in correspondingly selected dimensionless quantities of system of the decision equilibrium equations gives the possibility to obtain from the general solution, as special case, the analytical solution s of the is mode of deformation of a plate, spherical, cylindrical and double curvature shells.

Keywords and phrases: Plate, shell, rise of arch, mode of deformation, inclined.

AMS subject classification: 74K25; 74K20.

The common technical theory of calculation of flat shells basically is developed by E. Reissner [1,2] and V.Vlasov [3]. Irrespective of the fact how the full system of the equations is written down, its integration demands knowledge of boundary conditions along borders. According to character of attaching the boundary conditions would be kinematic, static and mixed. There are five real boundary conditions on each border of shell but because of that in classical theory of shells the solving differential equation is 8 orders, on each edge (border) is necessary to satisfy four boundary conditions: this disparity, as many other, are resulted due to approximate nature of technical theory of. The "reconstruction" of real boundary conditions belongs to Kirchhoff and is proved by a St. Venant's principle, that emphasizes A. Love [4]. Therefore at application of classical theory of shells the exact satisfaction of all 5 real boundary conditions is impossible, and is clear so important is any new scientific research by means of which classical theory of shells will be released from this lack. Many well-known scientists paid to this problem extraordinary attention. Later the study of this problem was held by I. Vekua[5], who the system of differential equation of classical theory of shells instead 8 order reduced to system of differential equation 10 order, which is enough to satisfy all 5 real boundary conditions.

As to I. Gugushauri theory, [6] which is adequate to classical theory of shells and is distinctly differ by its mathematical simplicity, by it's practical application is easy to reach exact satisfaction of real boundary conditions due to accordingly selecting the appropriate interpolating function.

In technical theory instead stresses apply them equivalent equally effective values. Their amount is equal eight, five forces $(S_x, S_y, T^*, Q_x, Q_y)$ and three moments (M_x, M_y, H^*).

S_x and S_y are longitudinal forces, T^* is shear force, Q_x and Q_y are transverse or shear forces. M_x and M_y are bending moment, H^* is torsion torque.

Above-mentioned forces and moments enable to argue about mode of deformation the allocated from shell spatial element on corresponding flat element. The equilibrium condition of allocated flat element (Fig. 1) according I. Gudushauri theory is presented by T_xT_y - empty systems unity (Fig. 2) and (Fig. 3).

For T_xT_y - empty systems equilibrium equation are received by taking into account noted in (Fig. 2) and (Fig. 3) rectangular components of internal forces and interaction reactive force as well as rectangular components of external loads. At the same time above-mentioned interaction reactive forces and external loads also transferred to middle face of shell.

In paper is considered rectangular double curvature shell (Fig. 4). For accomplish a task are used the dimensionless quantities.

$$x = \frac{\overline{x}}{a}; \quad y = \frac{\overline{y}}{b}; \quad \eta = \frac{a}{b}; \quad \beta = \frac{h}{R_1}; \quad \gamma = \frac{a}{R_1}; \quad k = \frac{R_1}{R_2}, \tag{1}$$

where $\overline{x}$ and $\overline{y}$ are orthogonal coordinates, $2a$ and $2b$ dimensions of covered by shell rectangle, $2h$ - shell thickness, R_1 and R_2 radius of convexity, f - rise (Fig. 4).

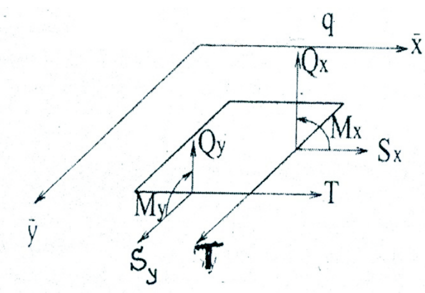


Fig. 1

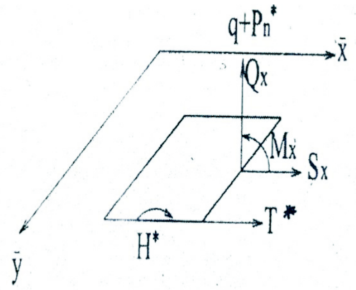


Fig. 2

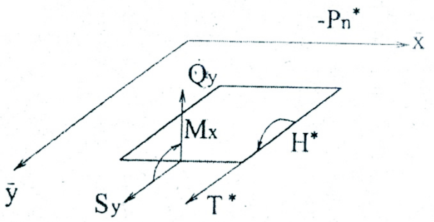
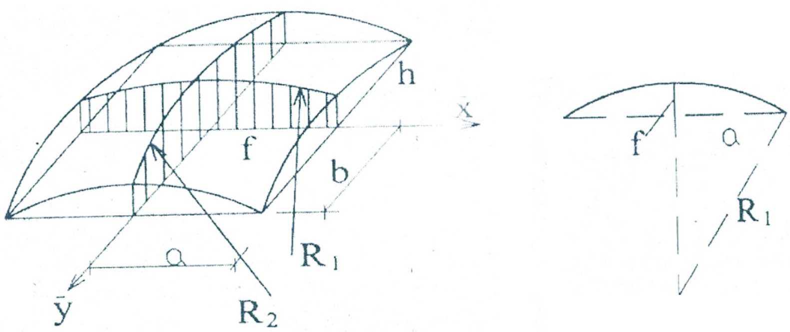


Fig. 3



$$\frac{a}{R_1} = \sin\alpha,$$
$$\frac{f}{a} = \mathrm{tg}\frac{\alpha}{2}.$$

Fig. 4

The equilibrium equation T_x and T_y for empty systems (1) with taking into account notes will becomes.

For T_x - empty systems

$$\begin{cases} \dfrac{\partial S_x}{\partial x} + \eta\,\dfrac{\partial T^*}{\partial y} = 0, \\ \dfrac{\partial Q_x}{\partial x} + yS_x + \alpha P_n^* + \alpha q = 0, \\ \dfrac{\partial M_x}{\partial x} + \eta\,\dfrac{\partial H^*}{\partial y} + \alpha Q_x = 0. \end{cases} \tag{2}$$

For T_y - empty systems:

$$\begin{cases} \dfrac{\partial S_x}{\partial y} + \dfrac{1}{\eta}\,\dfrac{\partial T^*}{\partial x} = 0, \\ \eta\,\dfrac{\partial Q_y}{\partial y} + K_y S_y - \alpha P_n^* = 0, \\ \eta\,\dfrac{\partial M_y}{\partial y} - \dfrac{\partial H^*}{\partial x} + \alpha Q_y = 0, \end{cases} \tag{3}$$

where P_n^* represents the reactive force of interaction of empty systems. If from (2) and (3) equilibrium equation excludes P_n^* reactive force received well-known in technical theory of shells system of equations.

For deformation equation we have:

$$\begin{cases} \dfrac{\partial^2 W_x}{\partial x^2} = -\dfrac{\alpha^2}{D(1-v^2)}\,(M_x - vM_y), \\ \dfrac{\partial^2 W_y}{\partial y^2} = -\dfrac{\alpha^2}{D(1-v^2)\eta^2}\,(M_y - vM_x), \\ \dfrac{\partial U}{\partial x} = -\dfrac{\alpha}{B(1-v)^2}\,(S_x - vS_y) - \gamma W_x, \\ \dfrac{\partial V}{\partial y} = -\dfrac{\alpha}{B(1-v)^2}\,(S_y - vS_x) - \dfrac{\gamma}{\eta}\,W, \\ \dfrac{\partial V}{\partial x} + \eta\dfrac{\partial U}{\partial y} = \dfrac{2\alpha T^*}{B(1-v)}\,, \end{cases} \tag{4}$$

where $D = \frac{Eh^3}{12(1-v^2)}$, $B = \frac{Eh}{1-v^2}$, however W_x and W_y empty T_x and T_y are the deflection of system.

Let's accept that shell by the contour is handedly motionlessly fixed on rigid diaphragms and is subjected to uniformly distributed q normal load action. In this conditions the boundary and initial conditions becomes

$$\begin{cases} x = \pm 1, \quad M_x = 0, \quad U = 0, \quad V = 0, \quad W_x = 0, \quad \dfrac{\partial W_x}{\partial y} = 0, \\ y = \pm 1, \quad M_y = 0, \quad U = 0, \quad V = 0, \quad W_y = 0, \quad \dfrac{\partial W_y}{\partial x} = 0, \end{cases} \tag{5}$$

$$\begin{cases} x=0, \;\; Q_x=0, \;\; U=0, \;\; \dfrac{\partial W_x}{\partial y}=0, \;\; H^*=T^*=0, \\ y=0, \;\; Q_y=0, \;\; V=0, \;\; \dfrac{\partial W_y}{\partial y}=0, \;\; H^*=T^*=0. \end{cases} \tag{6}$$

Hence for common case of calculation of rectangular flat shell for any kind of boundary conditions are produced three interaction forces (H^*, T^*, P_n^*) which are in general presented by interpolating of series which with taking into account the boundary conditions of given task becomes.

$$\begin{cases} P_n^* = \sum_m \sum_n A_{mn} \cos mx \, \cos ny, \\ H^* = \dfrac{\alpha^2}{\eta} \sum_m \sum_n B_{mn} \dfrac{1}{mn} \sin mx \, \sin ny, \\ T^* = \dfrac{\alpha}{\gamma\eta} \sum_m \sum_n C_{mn} \dfrac{m}{n} \sin mx \, \sin ny \end{cases} \tag{7}$$

unknown A_{mn}, B_{mn}, C_{mn} factors are defined from three similar equality. Taking into account the equilibrium and deformation equations (7) enables us to define the internal forces and displacements A_{mn}, B_{mn}, C_{mn} by unknown factors as well as these lasts by three equal equalities which gives us system of the algebraic equations concerning unknown factors.

$$\frac{\partial V}{\partial x} + \eta \frac{\partial U}{\partial y} = \frac{2\alpha(1+n)}{B(1-n^2)} T^*, \tag{8}$$

$$W_x(x,y) = W(x,y), \quad W_y(x,y) = W(x,y).$$

System of algebraic equation (8) is solved by application of collocation method.

The mathematical algorithm and the program which enable by preservation of corresponding amount of members of series to define unknown A_{mn}, B_{mn}, C_{mn} factors and then to define internal forces and moments according to following equality

1. $S_x = \alpha q \left[C_{mn} \dfrac{1}{\gamma} \cos mx \, \cos ny + \dfrac{1}{\gamma} C_1(y) \right],$
2. $S_y = \alpha q \left[C_{mn} \dfrac{m^2}{n^2\eta^2\gamma} \cos mx \, \cos ny + \dfrac{1}{\gamma} D_1(x) \right],$
3. $Q_x = -\alpha q \left[x + xC_1(y) + (A_{mn} + C_{mn}) \dfrac{1}{m} \sin mx \, \cos ny \right],$
4. $Q_y = -\alpha q \left[\dfrac{K}{\eta} y \, D(x) - \left(A_{mn} - \dfrac{Km^2}{n^2\eta^2} C_{mn} \right) \dfrac{1}{n\eta} \cos mx \, \sin ny \right],$
5. $M_x = \alpha^2 g \left[\dfrac{1}{2} (x^2-1)(1+C_1(y)) - (A_{mn} + B_{mn}) \dfrac{1}{n\eta} \cos mx \, \sin ny \right].$

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