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ALGORITHMS FOR CALCULATION OF OPTIMAL DYNAMICAL PROCESSES AND RATIONAL DESIGNING OF MACHINE SYSTEMS

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Abstract. Algorithms of optimization synthesis of linear multimass mechanical systems oscillation processes and rational design of structural schemes is offered, allowing to choose rational relationship of machine transmission elastic-mass (inertial and stiffness) parameter ensuring their steady functioning with minimum dynamic coefficient at transition regimes.

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Basic scientific-technical task of modern mechanical engineering is connected with the creation of high productive and durable machines and constructions with low metal-intensivity.

The productivity of the machines and mechanisms depends upon the speed of their working parts and solving actual task of minimizing duration of transitional regimes and similar characteristics of technological cycles.

Unfortunately calculating and designing principal link parameters are mostly carried out on the basis of energy-force parameters and safety margin value without sound theoretical grounding and without taking into account dynamic characteristics of machines and peculiarity of transitional regimes.

If safety margin is taken more than is necessary it causes manufacturing of too heavy, massive machines and hence unjustified expenditure of metals. If safety margin is less than necessary, it leads to machine failure.

Modern industry (especially heavy engineering industry) is subjected to great loss nowadays of fatigues failure of important parts of machines. It is the same about mobile means of civil and defence purpose (Units of transmission and lories, tracks, tanks, vessel and aircraft engines of rotor type etc.).

Main reasons of the failure are mistakes made on initial stage of projecting machines and incorrect selection of constructive parameters (mass and rigidity of links and quantity of their relationship), neglecting dynamic vibrating processes.

According to statistics heavy engineering industry (particularly metallurgy) is subjected to considerable damage because of machine failure.

According to experimental data 70% of failure is caused by internal resonance regimes in the mechanical and electromechanical systems which has a character of oscillation beatings of elastic force amplitudes, and even at the small technological loading fast fatigues breakage of important parts (especially transmission) is caused.

It is known that oscillation beating arise at the approach of lowest values of natural oscillation frequencies, and the nearer these frequencies, the more is the negative effect. Coincidence of these natural frequencies causes the fastest breakage of construction.

Unfortunately this negative effect of oscillation beating is unknown for majority of engineer-designers and they neglect this effect during designing.

According to the result of investigations, oscillation frequency is functionally connecting with the values of the system constructive parameters, especially with ratio of elastic and masses parameters.

To determine natural oscillation frequencies, it is necessary to solve a characteristic equation of differential equations.

At high order equation (at design of real machine-aggregates), it is necessary to use computing technique. In this way a designer can determine the value of natural oscillations of designed machines, but he cannot solve the problem of optimization synthesis.

Difficulties of these problems can easily be solved by engineer method of optimizing synthesis of dynamic processes suggested by us. The method need not to be solved with high order characteristic equation, and using easily attainable mathematics body, designers are able to determine systems generalized dimensionless parameters, proportional to the mechanical system and to constructive parameters of the examined system.

For solving the problem it is necessary to consider high order different equation

$$\left.\begin{aligned} &X^{2n} + a_0 X^{[2(n-1)]} + \cdots + a_{n-2}\ddot{X} + a_{n-1}X = 0 \\ &(n = 1, 2, 3, \dots), \end{aligned}\right\} \tag{1}$$

where $a_1, a_2, \dots a_{n-1}$ are determined by the value of system parameters.

If argument t is substituted by value $t = \tau/a_0$, following equation will be obtained:

$$X^{(2n)} + X^{[2(n-1)]} + C_1 X^{[2(n-2)]} + \cdots + C_{n-1}X = 0, \tag{2}$$

where C_k are generalized dimensionless parameters:

$$\left.\begin{aligned} &C_1 = \frac{a_1}{a_0^2},\ \ C_2 = \frac{a_2}{a_0^3}, \dots C_n = \frac{a_{n-1}}{a_0^n}, \\ &\left(n = 1, 2, 3, \dots, \frac{k-2}{2}\right). \end{aligned}\right\} \tag{3}$$

It is determined that C_k parameter can alter in the limits:

$$0 \le C_k \le \frac{n-1}{2n} \qquad (k = 1, 2 \dots n-1). \tag{4}$$

Passing on from equation (1) to equation (2) an important practical significance for simplified the problem of optimizing synthesis of mechanical systems.

In equation (1), the range of alteration of a_1, a_2, a_3 determined by constructive parameters, is very wide and indefinite. That is why, using this equation to solve the problem of optimizing synthesis, designer would have to calculate infinite variants by using computers, which is uneconomic and unjustified.

Passing to generalized dimensionless parameters C_k significantly narrows the range of alteration of differential equation coefficients and contracts the volume of computing.

For example, in the case of differential equation of the eight order, generalized parameters have the following maximum values:

$$C_1 = 3/8,\ C_2 = 1/16,\ C_3 = 1/256.$$

The values of coefficients a_i are determined by elastic and inertia parameters of the system.

For example, in the case of three mass torsional system, values of a_0 and a_1 are calculating according to the formulas:

$$\left.\begin{aligned} a_0 &= C_{12}\frac{\Theta_1+\Theta_2}{\Theta_1\Theta_2} + C_{23}\frac{\Theta_2+\Theta_3}{\Theta_2\Theta_3}, \\ a_1 &= C_{12}\cdot C_{23}\frac{\Theta_1+\Theta_2+\Theta_3}{\Theta_1\Theta_2\Theta_3}, \end{aligned}\right\} \tag{5}$$

where C_{12} and C_{23} are rigidities of corresponding elastic shafts and - Θ_i $(i = 1, 2, 3)$ inertia moment of corresponding masses.

Substituting (5) values in equation (3) generalized dimensionless parameters can be determined, which, in turn, define the degree of closeness of natural oscillations frequencies, and hence, actual possibilities of origination of oscillation beatings.

It follows from equation (4) that value of generalized parameters for any degree of freedom not exceed - 0.5.

Thus, if degree of freedom $n = 100$, that means, that we deal with differential equation of 200 degree, then:

$$\max C = \frac{n-1}{2n} = \frac{100-1}{200} \approx 0.5.$$

It follows from equation (4) that in the case of differential equation of the fourth order $(n = 2)$, variation of generalized parameters takes place in the range

$$0 \le C_1 \le 0.25. \tag{6}$$

In the case of differential equation of the sixth order $(n = 3)$

$$0 \le C_1 \le 0.33,\ 0 \le C_2 \le 1/27. \tag{7}$$

For equation of the eighth order

$$0 \le C_1 \le 3/8,\ 0 \le C_2 \le 1/16,\ 0 \le C_3 \le 1/256. \tag{8}$$

As a result of numerous experiments and a lot of computing, it has been stated that the more the value of C_i (in its turn it testifies high degree of nearness of law frequencies of natural frequency of oscillations of the system) depending on the ratio of constructive parameters of mechanical systems of any degree of freedom, the sharper the oscillation beating in mechanical systems.

Maximum value of $C_{1\max} = 0.25$ - for differential equation of the fourth order, $C_{1\max} = 1/3$ - for differential equation of sixth order etc stipulates the pure oscillation beating with the highest value of elastic force amplitudes in mechanical systems. In this case the smallest natural frequencies of the system become equal to each other ($\beta_1 \approx \beta_2$), that is the internal resonance is originated.

To avoid that phenomenon, designer should try to remove the smallest frequencies of natural oscillation maximal from each other, that is, to withdraw the system from the zone of maximum values of generalized parameters.

The investigation has determined that the upper nonoptimal range for C_1 value exists for the mechanical systems with the degree of freedom two (n=2) under the range characterized by

$$C_1 = (0.18 \div 0.25). \tag{9}$$

For the system with the degree of freedom three ($n = 3$)

$$C_1 = (0.25 \div 1/3). \tag{10}$$

For the system with the degree of freedom four ($n = 4$)

$$C_1 = (0.33 \div 0.375). \tag{11}$$

In general, maximum permissible value of the generalized parameter C_1, should be 20% less than its maximum value, determined according to expression (4). Otherwise, at initial stage of designing a designer must change the constructive parameters ratio, so that new selected parameter values would ensure the withdrawing of C_1 from the range of nonoptimal values ($9 \div 11$).

We have determined that there exist the second (lower) range of nonoptimal values of parameters C_1 and for the systems with the degree of freedom two it is defined with the range

$$0 \leq C_1 \leq= 0.04.$$

For the systems with the degree of freedom three ($n = 3$)

$$0 \leq C_1 \leq= 0.07.$$

For the systems with the degree of freedom four ($n = 4$)

$$0 \leq C_1 \leq 0.09.$$

It is determined that mechanical systems whose ratio of constructive parameters stipulates the lower nonoptimal value range of the generalized parameter C_1, are characterized by high sensibility (reaction) towards external, particularly impact forces. Minimum value of C_1 ensures the maximum removal of natural oscillation frequencies, caused by the increased characteristics of transmission elastic link stiffness, but such mechanical systems, at the same time, are characterized by the high clearances which represent impact type.

The range of lower nonoptimal values of generalized parameters is particularly dangerous for object with reverse working regimes and are characterized by periodical opening and closing of clearances.

From the above mentioned it is clear that there is optimal alteration of a range of generalized parameter, practical realization of which ensure a minimal reaction of mechanical systems to the influence of any type external forces.

For mechanical systems with the degree of freedom two this range

$$0.05 \leq C_1 \leq 0.0.18.$$

In the case of differential equation of the sixth order ($n = 3$)

$$0.08 \leq C_1 \leq 0.25.$$

For equation of the eighth order

$$0.12 \leq C_1 \leq 0.3.$$

Thus, the simplest method of engineer solving of the problem of synthesis of mechanical systems for dynamic processes optimization, ensuring minimal dynamic loading of machine aggregates, reliability and decreasing of metal-intensivity, is enclosed by the algorithm based on the simplest arithmetic operations, which demands the definition of elastic-mass parameters of the designed objects and the calculation of generalized dimensionless parameters.

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