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INVESTIGATION OF FEATURES OF AERO AND HYDRO-DYNAMIC FLOWS IN A NARROW CHANNEL IN VIEW OF THE RELIEF OF BOTTOM

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The question can be made so: to study the flow of aero-hydro-fluxes in a narrow channel of b width, inclined with an α angle in respect of the horizon, under the conditions of a week ($V<10m/s$) wind. There is discussed a stationary flow in xoz plane in view of gravity force and relief of the bottom. The point of origin is placed in the bottom of the river or the valley, ox axis is directed towards the flow of the flux and oz – vertically upwards. It is stipulated that the intensiveness of flow is constant in a small interval of time and the action of atmospheric pressure is constant. Thus we shall have [1,2]:

$$\frac{\partial p}{\partial x} = 0, \tag{1}$$

$$P_x = g\sin\alpha, P_z = g\cos\alpha, \tag{2}$$

where p is pressure, g is acceleration of gravity.

Under the indicated conditions the hydrodynamic equations set can be written in the following form [1, 2, 3]:

$$g\rho\sin\alpha + \mu\left(\frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}\right) = 0, \tag{3}$$

$$\frac{\partial P}{\partial y} = 0, \tag{4}$$

$$\frac{\partial p}{\partial z} + g\rho\cos\alpha = 0, \tag{5}$$

(3)-(5) are integrated in the following marginal terms:

$$\text{when} \quad z = 0, V = 0, \tag{6}$$

$$\text{when} \quad z = 0, \quad \text{then} \quad \frac{\partial V}{\partial z} = 0, \tag{7}$$

$$\text{when} \quad z = h, \quad \text{then} \quad p = p_0. \tag{8}$$

Here ρ is density of air (fluid), h is the height of the free surface, μ- the cinematic coefficient of viscosity. The problem formulated in such a form is solved [1, 2] and the corresponding parameters: velocity, pressure and the amount of inflowing flux Q is determined with such a relationship:

$$p = p_0 + g\rho(h - z)\cos\alpha, \tag{9}$$

$$V = \frac{g\rho(2h - z)}{2\mu}\sin\alpha, \tag{10}$$

$$Q = \frac{g\rho b h^2 \sin\alpha}{3\mu}, \tag{11}$$

where Q is the rate of flow in the case when the lower surface of the channel is covered of hilly cavities of a small height (of the range of several metres). In order to investigate the structure of movement instead of the z coordinate. Let's introduce a new coordinate $\acute{z_1}$ with the following relationship [3, 7]:

$$z_1 = \frac{z - \xi(x, y)}{H - \xi(x, y)} H, \tag{12}$$

where $\xi(x, y)$ is the form of bottom relief. In the new frame of reference the (3)-(5) equations set with the (6)-(8) marginal terms shall take the following form:

$$g\rho\sin\alpha + \mu\left(\frac{\partial^2 V}{\partial y^2} + \alpha^2\frac{\partial^2 V}{\partial z_1^2}\right) = 0, \tag{13}$$

$$\frac{\partial P}{\partial y} = 0, \tag{14}$$

$$\alpha\frac{\partial V}{\partial z_1} + g\cos\alpha = 0, \tag{15}$$

$$\text{When} \quad z_1 = \xi(x, y), \quad \text{then} \quad V = 0 \tag{16}$$

$$\text{When} \quad z_1 = h, \quad \text{then} \quad \frac{\partial V}{\partial z_1}, p = p_0. \tag{17}$$

Here $a = \frac{H}{H - \xi(x,y)}$ and is regarded to be constant. Taking into account the parameter a the formulae (9) - (11), can be rewritten in such a form:

$$p = p_0 + \frac{g\rho}{a}(h - z_1)\cos\alpha, \tag{18}$$

$$V = \frac{g\rho z_1(2h - z_1)}{2a^2\mu} + \sin\alpha, \tag{19}$$

$$Q = \frac{g\rho b h^3 \sin\alpha}{3a^2\mu}. \tag{20}$$

There is obvious from (18) - (20) that the velocity of air flux and its rate depend on the parameter of bottom relief inversely as its square value. Thus, taking into consideration the influence of the bottom relief in the channel has diminished both velocity of the flux and amount of intensiveness. Hence the source of local contaminations (spots of various admixtures) shift slowly and the period of self-cleaning shall be increased.

Now let's discuss such a aero- hydro- flux which contains punctual or linear sources of contamination and during the flow in the horizontal plane form hurricanes of intensiveness, circulatory flows [1]. Let's use the hydrodynamic equations set in the form of Lamba-Gromoc which admits potentiality of velocity and force field, i.e. [3]:

$$U = \frac{\partial \Phi}{\partial x}, V = \frac{\partial \Phi}{\partial y}, W = \frac{\partial \Phi}{\partial z}. \tag{21}$$

That's why the equations set shall be written in such a way:

$$grad(\phi + \Pi + E) = grad\left(\frac{\partial \Phi}{\partial t}\right). \tag{22}$$

Here ϕ is a potential of massive forces, $\Phi-$ the potential of the field of velocities, $E-$ kinetic energy, $\Pi = \frac{P}{\rho}$ for the non-squeezable environment, $\Pi = \frac{\kappa-1}{\kappa}\frac{P}{\rho}$ and for squeezable-adiabatic conditions,$\chi = \frac{C_p}{C_v}$, C_p-specific heat capacity under the constant pressure, C_{v-}specific heat capacity under the constant volume. (22) yields Lagrange integral [1, 3]:

$$-(\phi + \Phi + E) = \frac{\partial \Phi}{\partial t} + c(t), \tag{23}$$

where c(t) is constant and when it equals to 0, (23) takes the form of Euler formula, which for the barotropic environment has the appearance of

$$\rho\phi + P + \frac{\rho V^2}{2} + \rho\frac{\partial \Phi}{\partial t} = 0. \tag{24}$$

Assuming that during the flow in stationary small time interval ($\Phi = const$) the direction of ox axis coincides with the axis of the "hurricane" (ϕ =0), then (24) shall be rewritten in such a form:

$$P - P_\infty = \frac{\rho V^2}{2}, \tag{25}$$

where p_∞corresponds to the value of the pressure in a long distance from the source, where $V = 0$. As it is known from the theory, velocities of the flow [1, 8] decrease inversely as the r value of the distance from the source, i.e.,

$$V = \frac{G}{2\pi r}. \tag{26}$$

Taking into account (26), (25) shall be rewritten in such a way:

$$P - P_\infty = -\frac{\rho G^2}{6\pi^2 r^2}. \tag{27}$$

I.e. in case of hurricane flows the pressure decreases inversely as the square of the distance from the centre. At the same time, as there $p < p_\infty$ occurs the suction of the air (water) influx into the centre. This fact also contributes to the diminution of the transportation - scatter of the contaminating admixtures existing in the flowing flux.

It is interesting to mention that this is exactly this feature of the "hurricane" velocity of the wind that explains the formation of "tornado". When tornado moves, it suck in the air masses and takes various objects with itself. It dries up water in river basins, breaks down houses and trees. $p - p_\infty$Rapid increase of the value subtraction conditions the fact that tornado spreads onto a narrow stripe and doesn't touch the object at the border, whereas it destroys similar objects in the centre.

When processes are adiabatic and the environment is squeezable, then like (27) we shall have for the temperature

$$T - T_\infty = -\frac{x-1}{x}\frac{G^2}{8\pi^2 r^2}\frac{1}{R}, \tag{28}$$

where R is the universal gas constant. (28) shows well that in hurricane fluxes temperature diminishes towards the centre. This provides for the fact there along the flow there is discovered ,,cold" and "hot" areas. Such areas are indeed observed in large-scale (atomic, volcanic) explosions and their creation is expected along the oil and gas pipelines in the case of catastrophic situations, too.

One of the main actual problems is of interest, namely, the value of excess pressure during an explosion (during a relatively large interval of time). For the generalization let's allow that the flow of explosion spread spherically (it is obvious that spread of the flows can occur only in certain sectors, especially in view of the direction of dominating winds). In such case velocity of the flow in connected with its source rate of flow Q with the following formula [1, 8]:

$$V = \frac{Q}{4\pi r^2}. \tag{29}$$

It must be remarked that the movement is non-stationary at this time, in the initial moment velocity is equal to zero everywhere, and after the interval of time Δt velocity becomes equal to the value given by the expression (29). It is obvious that,

$$\phi = -\frac{Q}{4\pi r}. \tag{30}$$

Lagrange integral shall yield

$$\frac{\partial \phi}{\partial t} + \Pi + \frac{V^2}{2} = \Pi_\infty. \tag{31}$$

For a short interval of time from (31) we shall have

$$p - p_\infty = \rho\frac{Q}{4\pi r \Delta t} - \frac{\rho Q^2}{32\pi^2 r^4}. \tag{32}$$

In interval Δt of time r is about zero and that's why in (32) the second member in the right hand-side can be negligible because of its smallness. Thus there remains

$$p - p_\infty = \rho \frac{Q}{4\pi r \Delta t} \frac{1}{2}. \tag{33}$$

We have assumed that as we approach the centre the pressure increases inversely to distance, this is a conclusion different from the previous case which is in view of the non-stationary process.

Let's take advantage of the (9) and (10), in (19) and (20) formulae and calculate the flow velocity (rate) as the dependence on the angle of slope of relief in reference with the horizon. Characterizing parameters of the valley are listed in Table 1.

Table 1

Param.Envir.	ρ(kg/m^2)	h(m)	z_1(m)	μ(kg/msec)	a	b
Air	1,3	5·10^2	10-500	1,7·10^5; $2 \cdot 10^4$	1.0-10.10	10-50
Water	10^3	0-5	0-2	1,1·10^6; $1,2 \cdot 10^5$	10-3.0	10-50

And the values of velocity with and without consideration of the relief of the valley, V and V(a) for four values of the angle , for two values of are cited in Table 2.

Table 2- From these date it is obviously seen the proportional growth of velocity as the angle of the horizontal slope increases. The calculated values of the wind velocity are closer to the real values if we consider influence of the bottom relief and we take the cinematic coefficient of turbulence almost even for the upper layer of water and in its adjacent layer of air (cm in the water marginal level).

The cited mathematical theory is used to determine velocity of the air (water) flow in valleys of various rivers or in lowlands amongst the mountains, and, correspondingly, to study spread of contamination. Evaluation of excess pressure and temperature, formed because of contamination, is possible on the basis of the formulae (28) - (33). According to the formula (33) for various values of the rate Q dependence of the excess pressure on r.

The values of Q increase in connection with the angleα, but not strictly proportionally. For instance,$\rho h^3 = 8 \cdot 10^3$ km,$\alpha = 150$,$b = 20m, Q = 2.08 \cdot 10^4\ kg$ and $Q(a) = 1.4 \cdot 10^4 kg$, and in the case of the angle $\alpha = 60^0$, respectively,$Q = 6,4 \cdot 10^4\ kg$ and $Q(a) = 1.4 \cdot 10^4 kg$.

The obtained results were compared to the calculations based on the determination of admixtures in a turbulent environment, solution of the classical equation of diffusion under real marginal and initial conditions [2,8].

Table 2.

	for Water				for Air			
Vα^o	V(m/s)		V(a)(m/s)		V(m/s)		V(a)(m/s)	
15^0	4.69	2.35	3.35	1.64	2.08. 10^{-2}	0.195	1.44. 10^{-2}	0.14
30^0	8.82	4.53	6.3	3.15	4.0. 10^{-2}	0.375	2.78. 10^{-2}	0.27
45^0	12.35	6.35	8.8	4.41	5.6. 10^{-2}	0.525	3.89. 10^{-2}	0.38
60^0	14.11	7.21	10.1	5.04	6.4.10^{-2}	0.60	4.44.10^{-2}	0.43

There was taken Rioni Valley, anticipating that the place of pollution of oil products is at least 5 km distant from the sea and the time of flow is 2 hours (in such a small period of time evaporation and sedimentation on the bank can be neglected). There was found that maximum of the concentration of oil products reaches the Black Sea in 15-20 minutes, when the velocity of water flow amounts to 2-3 m/s (which is consistent with our theoretical calculations). In the case water shallowness in the rivers, then the speed of movement of the oil products spot is small and amounts only to 0.3 - 0.4 m/s [5] (this magnitude is obtained in view of the influence of theoretical parameter). and the risk of its sedimentation onto the banks increases during 2 hours since the moment of accident. Results of the calculation have shown that if the accident happens at the distance of 5 km from the sea, then almost 80% of the products reach the sea in about 25 minutes.

At the same time, if the accident happened at a relatively long distance from the sea, then the basic products do not reach the sea and remain on the bottom of the valley and on its banks for a long period of time.

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