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I.VEKUA APPROXIMATION $N = 1$ FOR THE NON-SHALLOW CYLINDRICAL SHELLS

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We shall consider the non-shallow cylindrical shells for I.N. Vekua approximation $N = 1$. The displacement vector $\mathbf{U}(x^1, x^2, x^3)$ are expressed by the following formula [1]

$$\mathbf{U}(x^1, x^2, x^3) = \mathbf{u}(x^1, x^2) + \frac{x^3}{h}\mathbf{v}(x^1, x^2).$$

Here $\mathbf{u}(x^1, x^2)$ and $\mathbf{v}(x^1, x^2)$ are the vector fields on the middle surface $x^3 = 0$, $2h$ is the thickness of the shell, x^3 is a thickness coordinate $(-h \leq x^3 \leq h)$, x^1 and x^2 are isometric coordinates on the cylindrical surface.

Let us construct the solutions of the form [2], [3]:

$$u_3 = \sum_{k=0}^{\infty} \overset{k}{u}_3 \varepsilon^k, \quad u_\alpha = \sum_{k=0}^{\infty} \overset{k}{u}_\alpha \varepsilon^k \ \ (\alpha = 1, 2),$$
$$v_3 = \sum_{k=0}^{\infty} \overset{k}{v}_3 \varepsilon^k, \quad v_\alpha = \sum_{k=0}^{\infty} \overset{k}{v}_\alpha \varepsilon^k \ \ (\alpha = 1, 2),$$

where u_1, u_2 and v_1, v_2 are the tangent components of the vectors $\mathbf{u}$ and $\mathbf{v}$ respectively, u_3 and v_3 denote the normal components of these vectors, $\varepsilon = \dfrac{h}{R_0}$ is a small parameter, R_0-the radius of the middle surface of the cylinder.

The system of equilibrium equations of the two-dimensional non-shallow cylindrical shells may be written in the following form [4], [5]:

$$\begin{aligned}
\mu\Delta \overset{k}{u}_1 &+ (\lambda+\mu)\partial_1 \overset{k}{\theta} + \lambda\partial_1 \overset{k}{v}_3 = \overset{k}{X}_1,\\
\mu\Delta \overset{k}{u}_2 &+ (\lambda+\mu)\partial_2 \overset{k}{\theta} + \lambda\partial_2 \overset{k}{v}_3 = \overset{k}{X}_2,\\
\mu\Delta \overset{k}{v}_3 &- 3\left[\lambda \overset{k}{\theta} + (\lambda+2\mu)\overset{k}{v}_3\right] = \overset{k}{X}_3,
\end{aligned} \tag{1}$$

$$\begin{aligned}
\mu\Delta \overset{k}{v}_1 &+ (\lambda+\mu)\partial_1 \overset{k}{\Theta} - 3\mu(\partial_1 \overset{k}{u}_3 + \overset{k}{v}_1) = \overset{k}{X}_4,\\
\mu\Delta \overset{k}{v}_2 &+ (\lambda+\mu)\partial_2 \overset{k}{\Theta} - 3\mu(\partial_2 \overset{k}{u}_3 + \overset{k}{v}_2) = \overset{k}{X}_5,\\
\mu\Delta \overset{k}{u}_3 &+ \mu \overset{k}{\Theta} = \overset{k}{X}_6,\\
&(k = 0, 1, 2, ...),
\end{aligned} \tag{2}$$

where

$$
\begin{aligned}
\overset{k}{X}_1 &= X_1-\mu\left(\overset{k-1}{v}_1+\frac{\partial_{22}\overset{k-1}{v}_1}{3}\right)+\sum_{s=0}^{\left[\frac{k-1}{2}\right]}\left(\frac{\lambda+2\mu}{2s+3}\partial_{11}\overset{k-2s-1}{v}_1-\frac{\lambda+3\mu}{2s+1}\partial_1\overset{k-2s-1}{u}_3\right)-\\
&- \mu\sum_{s=1}^{\left[\frac{k-1}{2}\right]}\frac{\overset{k-2s-1}{v}_1}{2s+1}-\sum_{s=1}^{\left[\frac{k}{2}\right]}\left(\frac{\lambda+2\mu}{2s+1}\partial_{11}\overset{k-2s}{u}_1-\frac{\lambda+3\mu}{2s+1}\partial_1\overset{k-2s}{v}_3-\frac{\mu}{2s-1}\overset{k-2s}{u}_1\right),\\
\overset{k}{X}_2 &= X_2-\lambda\partial_2\overset{k-1}{u}_3-\frac{\lambda+2\mu}{3}\partial_{22}\overset{k-1}{v}_2-\mu\left(\sum_{s=1}^{\left[\frac{k}{2}\right]}\frac{\partial_{11}\overset{k-2s}{u}_2}{2s+1}-\sum_{s=0}^{\left[\frac{k-1}{2}\right]}\frac{\partial_{11}\overset{k-2s-1}{v}_2}{2s+3}\right),\\
\overset{k}{X}_3 &= X_3+3\lambda\overset{k-1}{u}_3+(2\lambda-\mu)\partial_2\overset{k-1}{v}_2-\mu\partial_{22}\overset{k-1}{u}_3+3\sum_{s=0}^{\left[\frac{k-1}{2}\right]}\left(\frac{\mu}{2s+3}\partial_{11}\overset{k-2s-1}{u}_3+\right.\\
&+ \left.\frac{\lambda+3\mu}{2s+3}\partial_1\overset{k-2s-1}{v}_1\right)-3(\lambda+2\mu)\sum_{s=1}^{\left[\frac{k-1}{2}\right]}\frac{\overset{k-2s-1}{u}_3}{2s+1}-\\
&- 3\sum_{s=1}^{\left[\frac{k}{2}\right]}\left(\frac{\mu}{2s+1}\partial_{11}\overset{k-2s}{v}_3+\frac{\lambda+3\mu}{2s+1}\partial_1\overset{k-2s}{u}_1-\frac{\lambda+2\mu}{2s-1}\overset{k-2s}{v}_3\right),\\
\overset{k}{X}_4 &= X_4-\mu\partial_{22}\overset{k-1}{u}_1+3\sum_{s=0}^{\left[\frac{k-1}{2}\right]}\left(\frac{\lambda+2\mu}{2s+3}\partial_{11}\overset{k-2s-1}{u}_1-\frac{\lambda+3\mu}{2s+3}\partial_1\overset{k-2s-1}{v}_3+\right.\\
&+ \left.\frac{\mu}{2s+1}\overset{k-2s-1}{u}_1\right)-3\sum_{s=1}^{\left[\frac{k}{2}\right]}\left(\frac{\lambda+2\mu}{2s+3}\partial_{11}\overset{k-2s}{v}_1-\frac{\lambda+3\mu}{2s+1}\partial_1\overset{k-2s}{u}_3-\frac{\mu}{2s+1}\overset{k-2s}{v}_1\right),\\
\overset{k}{X}_5 &= X_5-(\lambda+2\mu)\partial_{22}\overset{k-1}{u}_2-(2\lambda-\mu)\partial_2\overset{k-1}{v}_3-\\
&- 3\mu\left(\sum_{s=1}^{\left[\frac{k}{2}\right]}\frac{\partial_{11}\overset{k-2s}{v}_2}{2s+3}-\sum_{s=0}^{\left[\frac{k-1}{2}\right]}\frac{\partial_{11}\overset{k-2s-1}{u}_2}{2s+3}\right),\\
\overset{k}{X}_6 &= X_6+\lambda\left(\overset{k-1}{v}_3+\partial_2\overset{k-1}{u}_2\right)-\frac{\mu}{3}\partial_{22}\overset{k-1}{v}_3+\\
&+ \sum_{s=0}^{\left[\frac{k-1}{2}\right]}\left(\frac{\mu}{2s+3}\partial_{11}\overset{k-2s-1}{v}_3+\frac{\lambda+3\mu}{2s+1}\partial_1\overset{k-2s-1}{u}_1\right)-(\lambda+2\mu)\sum_{s=1}^{\left[\frac{k-1}{2}\right]}\frac{\overset{k-2s-1}{v}_3}{2s+1}-\\
&- \sum_{s=1}^{\left[\frac{k}{2}\right]}\left(\frac{\mu}{2s+1}\partial_{11}\overset{k-2s}{u}_3+\frac{\lambda+3\mu}{2s+1}\partial_1\overset{k-2s}{v}_1-\frac{\lambda+2\mu}{2s-1}\overset{k-2s}{u}_3\right),
\end{aligned}
$$

$$
\left(k=1,2,...;\quad \overset{k}{u}_3=\overset{k}{v}_3=\overset{k}{u}_\alpha=\overset{k}{v}_\alpha=0,\ \text{ if } \ \mathrm{k}<0;\ \alpha=1,2,\right.
$$

$$
\left.\overset{k}{\theta}=\partial_1\overset{k}{u}_1+\partial_2\overset{k}{u}_2,\quad \overset{k}{\Theta}=\partial_1\overset{k}{v}_1+\partial_2\overset{k}{v}_2,\quad \partial_\alpha=\frac{\partial}{\partial x^\alpha}\right).
$$

Here λ and μ are Lame's constants, $X_i \quad (i=1,...,6)$ - the components of external force.

For each fixed k equations (1) and (2) coincide with equations of I. N. Vekua which is obtained for prismatic shells with constant thickness in case of approximation $N=1$. The right part of equations (1) and (2) is well-known quantity, defined by functions $\overset{0}{u}_i, ..., \overset{k-1}{u}_i, \overset{0}{v}_j, ..., \overset{k-1}{v}_j \quad (i,j=1,2,3)$.

The general solutions of systems (1) end (2) are written in the following form:

$$\begin{aligned}
2\mu \overset{k}{u}_+ &= æ\overset{k}{\varphi}(z) - z\overline{\overset{k}{\varphi}{}'(z)} - \overline{\overset{k}{\psi}(z)} - \frac{\lambda}{6(\lambda+\mu)}\frac{\partial \overset{k}{\chi}(z,\bar z)}{\partial \bar z} + \widehat{\overset{k}{u}}_+,\\
2\mu \overset{k}{v}_3 &= -\frac{2\lambda}{3\lambda+2\mu}\left(\overset{k}{\varphi}{}'(z) + \overline{\overset{k}{\varphi}{}'(z)}\right) + \overset{k}{\chi}(z,\bar z) + \widehat{\overset{k}{v}}_3,\\
2\mu \overset{k}{v}_+ &= \frac{4(\lambda+2\mu)}{3\mu}\overline{\overset{k}{f}{}''(z)} + z\overline{\overset{k}{f}{}'(z)} + \overset{k}{f}(z) - 2\overline{\overset{k}{g}{}'(z)} + i\frac{\partial \overset{k}{w}(z,\bar z)}{\partial \bar z} + \widehat{\overset{k}{v}}_+,\\
2\mu \overset{k}{u}_3 &= -\frac{1}{2}\left(\bar z\overset{k}{f}(z) + z\overline{\overset{k}{f}(z)}\right) + \overset{k}{g}(z) + \overline{\overset{k}{g}(z)} + \widehat{\overset{k}{u}}_3,
\end{aligned}$$

$$\left(\overset{k}{u}_+ = \overset{k}{u}_1 + i\overset{k}{u}_2, \quad \overset{k}{v}_+ = \overset{k}{v}_1 + i\overset{k}{v}_2, \quad z = x^1 + ix^2,\right.$$

$$\left.\frac{\partial}{\partial \bar z} = \frac{1}{2}\left(\frac{\partial}{\partial x^1} + i\frac{\partial}{\partial x^2}\right), \quad \frac{\partial}{\partial z} = \frac{1}{2}\left(\frac{\partial}{\partial x^1} - i\frac{\partial}{\partial x^2}\right)\right),$$

where $æ = \dfrac{5\lambda+6\mu}{3\lambda+2\mu}$, $\overset{k}{\varphi}(z)$, $\overset{k}{\psi}(z)$, $\overset{k}{f}(z)$ and $\overset{k}{g}(z)$ are any analytic functions of z, $\overset{k}{\chi}(z,\bar z)$ and $\overset{k}{w}(z,\bar z)$ are the general solutions of the following Helmholtz's equations, respectively:

$$\begin{aligned}
\Delta \overset{k}{\chi} &- \eta^2 \overset{k}{\chi} = 0 \quad \left(\eta^2 = \frac{12(\lambda+\mu)}{\lambda+2\mu}\right),\\
\Delta \overset{k}{w} &- \gamma^2 \overset{k}{w} = 0 \quad \left(\gamma^2 = 3\right).
\end{aligned}$$

Here $\widehat{\overset{k}{u}}_+$, $\widehat{\overset{k}{v}}_3$ and $\widehat{\overset{k}{v}}_+$, $\widehat{\overset{k}{u}}_3$ are particular solutions of the non-homogeneous equations (1) and (2), respectively.

Let's consider the following boundary value problems for system (1),(2).

Find the solutions of the homogeneous system of equations (1) and (2) compatible with the kinematic boundary conditions:

$$\overset{k}{u}_+ = \begin{cases} \overset{k}{F}{}'_+, & |z| = R_1,\\ \overset{k}{F}{}''_+, & |z| = R_2, \end{cases} \qquad \overset{k}{v}_3 = \begin{cases} \overset{k}{F}{}'_3, & |z| = R_1,\\ \overset{k}{F}{}''_3, & |z| = R_2, \end{cases} \tag{3}$$

$$\overset{k}{v}_+ = \begin{cases} \overset{k}{G}{}'_+, & |z| = R_1,\\ \overset{k}{G}{}''_+, & |z| = R_2, \end{cases} \qquad \overset{k}{u}_3 = \begin{cases} \overset{k}{G}{}'_3, & |z| = R_1,\\ \overset{k}{G}{}''_3, & |z| = R_2, \end{cases} \tag{4}$$

where $\overset{k}{F}_+,\ \overset{k}{F}_3,\ \overset{k}{G}_+,\ \overset{k}{G}_3$ are the known functions of $\overset{0}{u}_i, ..., \overset{k-1}{u_i},\ \overset{0}{v}_j, ..., \overset{k-1}{v_j}\ (i, j, = 1, 2, 3)$.

Let us introduce the functions $\overset{k}{\varphi}(z),\ \overset{k}{\psi}(z),\ \overset{k}{\chi}(z, \bar{z}),\ \overset{k}{F}_+$ and $\overset{k}{F}_3$ by the series

$$\begin{aligned}
&\overset{k}{\varphi}(z) = Azlnz + \alpha lnz + \sum_{n=1}^{\infty} \overset{k}{a}_n z^n,\ \ \overset{k}{\psi}(z) = \beta lnz + \sum_{n=0}^{\infty} \overset{k}{b}_n z^n,\\
&\overset{k}{\chi}(z, \bar{z}) = \sum_{-\infty}^{\infty} \left(\overset{k}{\alpha}_n I_n(\eta r) + \overset{k}{\alpha}_n{}' K_n(\eta r) \right) e^{in\theta},\\
&\overset{k}{F}_+ = \sum_{-\infty}^{\infty} A_n e^{in\theta},\ \ \overset{k}{F}_3 = \sum_{-\infty}^{\infty} B_n e^{in\theta},
\end{aligned} \tag{5}$$

where $I_n(\eta r)$ and $K_n(\eta r)$ are Bessel's modificed functions.

By substituting (5) into (3) we obtain the system of algebraic equations:

$$\begin{aligned}
&A = 0,\ \ æ\alpha + \bar{\beta} = 0,\\
&2æ lnR_1\alpha - 2R_1^2 \overline{\overset{k}{a}}_2 - \frac{\lambda\eta}{12(\lambda+\mu)} \left(I_0(\eta R_1)\, \overset{k}{\alpha}_{-1} - K_0(\eta R_1)\, \overset{k}{\alpha}{}'_{-1} \right) + æ\, \overset{k}{a}_0 - \overset{k}{b}_0 = A_0',\\
&2æ lnR_2\alpha - 2R_2^2 \overline{\overset{k}{a}}_2 - \frac{\lambda\eta}{12(\lambda+\mu)} \left(I_0(\eta R_2)\, \overset{k}{\alpha}_{-1} - K_0(\eta R_2)\, \overset{k}{\alpha}{}'_{-1} \right) + æ\, \overset{k}{a}_0 - \overset{k}{b}_0 = A_0'',\\
&-\overline{\alpha} + æ R_1^2\, \overset{k}{a}_2 - \frac{\lambda\eta}{12(\lambda+\mu)} \left(I_2(\eta R_1)\, \overset{k}{\alpha}_1 - K_2(\eta R_1)\, \overset{k}{\alpha}{}_1' \right) - R_1^{-2} \overline{\overset{k}{b}}_{-2} = A_2',\\
&-\overline{\alpha} + æ R_2^2\, \overset{k}{a}_2 - \frac{\lambda\eta}{12(\lambda+\mu)} \left(I_2(\eta R_2)\, \overset{k}{\alpha}_1 - K_2(\eta R_2)\, \overset{k}{\alpha}{}_1' \right) - R_2^{-2} \overline{\overset{k}{b}}_{-2} = A_2'',\\
&æ R_1^n\, \overset{k}{a}_n + (n-2) R_1^{-n+2} \overline{\overset{k}{a}}_{-n+2} - \frac{\lambda\eta}{12(\lambda+\mu)} \left(I_n(\eta R_1)\, \overset{k}{\alpha}_{n-1} - K_n(\eta R_1)\, \overset{k}{\alpha}{}'_{n-1} \right)\\
&\qquad\qquad\qquad\qquad\qquad\qquad - R_1^{-n} \overline{\overset{k}{b}}_{-n} = A_n',\\
&æ R_2^n\, \overset{k}{a}_n + (n-2) R_2^{-n+2} \overline{\overset{k}{a}}_{-n+2} - \frac{\lambda\eta}{12(\lambda+\mu)} \left(I_n(\eta R_2)\, \overset{k}{\alpha}_{n-1} - K_n(\eta R_2)\, \overset{k}{\alpha}{}'_{n-1} \right)\\
&\qquad\qquad\qquad\qquad\qquad\qquad - R_2^{-n} \overline{\overset{k}{b}}_{-n} = A_n'',\\
&I_1(\eta R_1)\, \overset{k}{\alpha}_1 + K_1(\eta R_1)\, \overset{k}{\alpha}{}_1' - \frac{2\lambda}{3\lambda+2\mu} \left(2R_1\, \overset{k}{a}_2 - \frac{\overline{\alpha}}{R_1} \right) = B_1',\\
&I_1(\eta R_2)\, \overset{k}{\alpha}_1 + K_1(\eta R_2)\, \overset{k}{\alpha}{}_1' - \frac{2\lambda}{3\lambda+2\mu} \left(2R_2\, \overset{k}{a}_2 - \frac{\overline{\alpha}}{R_2} \right) = B_1'',\\
&I_n(\eta R_1)\, \overset{k}{\alpha}_n + K_n(\eta R_1)\, \overset{k}{\alpha}{}_n' - \frac{2\lambda}{3\lambda+2\mu} \left((n+1) R_1^n\, \overset{k}{a}_{n+1} - (n-1) R_1^{-n} \overline{\overset{k}{a}}_{-n+1} \right) = B_n',\\
&I_n(\eta R_2)\, \overset{k}{\alpha}_n + K_n(\eta R_2)\, \overset{k}{\alpha}{}_n' - \frac{2\lambda}{3\lambda+2\mu} \left((n+1) R_2^n\, \overset{k}{a}_{n+1} - (n-1) R_2^{-n} \overline{\overset{k}{a}}_{-n+1} \right) = B_n'',\\
&(n = \pm 1,\ \ -2,\ \ \pm 3, ...).
\end{aligned} \tag{6}$$

For coefficients $\overset{k}{a}_n$ and α we have:

$$\begin{aligned}
\overset{k}{a}_2 &= \frac{E_1C_2 - E_2C_0}{E_1T_2 - E_2S_0},\ \ \overline{\alpha} = \frac{C_0T_2 - C_2S_0}{E_1T_2 - E_2S_0},\\
\overset{k}{a}_n &= \frac{T_{-n+2}C_n - S_nC_{-n+2}}{T_nT_{-n+2} - S_nS_{-n+2}},
\end{aligned}$$

where

$$
\begin{aligned}
E_1 &= 2æln\frac{R_2}{R_1}-\frac{\lambda^2\eta(I_0(\eta R_2)-I_0(\eta R_1))(R_1K_1(\eta R_1)-R_2K_1(\eta R_2))}{6(\lambda+\mu)(3\lambda+2\mu)R_1R_2(I_1(\eta R_2)K_1(\eta R_1)-I_1(\eta R_1)K_1(\eta R_2))}\\
&+\frac{\lambda^2\eta(K_0(\eta R_2)-K_0(\eta R_1))(R_1I_1(\eta R_1)-R_2I_1(R_2))}{6(\lambda+\mu)(3\lambda+2\mu)R_1R_2(I_1(\eta R_2)K_1(\eta R_1)-I_1(\eta R_1)K_1(\eta R_2))},\\
E_2 &= R_1^2-R_2^2-\frac{\lambda^2\eta(R_2^2I_2(\eta R_2)-R_1^2I_2(\eta R_1))(R_1K_1(\eta R_1)-R_2K_1(\eta R_2))}{6(\lambda+\mu)(3\lambda+2\mu)R_1R_2(I_1(\eta R_2)K_1(\eta R_1)-I_1(\eta R_1)K_2(\eta R_2))}\\
&+\frac{\lambda^2\eta(R_2^2K_2(\eta R_2)-R_1^2K_2(\eta R_1))(R_1I_1(\eta R_1)-R_2I_1(R_2))}{6(\lambda+\mu)(3\lambda+2\mu)R_1R_2(I_1(\eta R_2)K_1(\eta R_1)-I_1(\eta R_1)K_1(\eta R_2))},\\
C_n &= A_n^{''}R_2^n-A_n^{'}R_1^n\\
&+\frac{\lambda\eta(R_2^nI_n(\eta R_2)-R_1^nI_n(\eta R_1))(B_{n-1}^{''}K_{n-1}(\eta R_1)-B_{n-1}^{'}K_{n-1}(\eta R_2))}{12(\lambda+\mu)(I_{n-1}(\eta R_2)K_{n-1}(\eta R_1)-I_{n-1}(\eta R_1)K_{n-1}(\eta R_2))}\\
&-\frac{\lambda\eta(R_2^nK_n(\eta R_2)-R_1^nK_n(\eta R_1))(B_{n-1}^{''}I_{n-1}(\eta R_1)-B_{n-1}^{'}I_{n-1}(\eta R_2))}{12(\lambda+\mu)(I_{n-1}(\eta R_2)K_{n-1}(\eta R_1)-I_{n-1}(\eta R_1)K_{n-1}(\eta R_2))},\\
T_n &= æ(R_2^{2n}-R_1^{2n})\\
&-\frac{\lambda^2\eta n(R_2^nI_n(\eta R_2)-R_1^nI_n(\eta R_1))(R_2^{n-1}K_{n-1}(\eta R_1)-R_1^{n-1}K_{n-1}(\eta R_2))}{6(\lambda+\mu)(3\lambda+2\mu)(I_{n-1}(\eta R_2)K_{n-1}(\eta R_1)-I_{n-1}(\eta R_1)K_{n-1}(\eta R_2))}\\
&+\frac{\lambda^2\eta n(R_2^nK_n(\eta R_2)-R_1^nK_n(\eta R_1))(R_2^{n-1}I_{n-1}(\eta R_1)-R_1^{n-1}I_{n-1}(\eta R_2))}{6(\lambda+\mu)(3\lambda+2\mu)(I_{n-1}(\eta R_2)K_{n-1}(\eta R_1)-I_{n-1}(\eta R_1)K_{n-1}(\eta R_2))},\\
S_n &= (n-2)\left[R_2^2-R_1^2\right.\\
&+\frac{\lambda^2\eta(R_2^nI_n(\eta R_2)-R_1^nI_n(\eta R_1))(R_2^{-n+1}K_{n-1}(\eta R_1)-R_1^{-n+1}K_{n-1}(\eta R_2))}{6(\lambda+\mu)(3\lambda+2\mu)(I_{n-1}(\eta R_2)K_{n-1}(\eta R_1)-I_{n-1}(\eta R_1)K_{n-1}(\eta R_2))}\\
&-\left.\frac{\lambda^2\eta(R_2^nK_n(\eta R_2)-R_1^nK_n(\eta R_1))(R_2^{-n+1}I_{n-1}(\eta R_1)-R_1^{-n+1}I_{n-1}(\eta R_2))}{6(\lambda+\mu)(3\lambda+2\mu)(I_{n-1}(\eta R_2)K_{n-1}(\eta R_1)-I_{n-1}(\eta R_1)K_{n-1}(\eta R_2))}\right].
\end{aligned}
$$

Let

$$
\begin{aligned}
&\overset{k}{f}(z)=Czlnz+\gamma lnz+\sum_{n=1}^{\infty}\overset{k}{c}_nz^n,\quad \overset{k}{g}(z)=\delta lnz+\sum_{n=0}^{\infty}\overset{k}{d}_nz^n,\\
&\overset{k}{w}(z,\bar z)=\sum_{-\infty}^{\infty}\left(\overset{k}{\beta}_nI_n(\gamma r)+\overset{k}{\beta}{}'_nK_n(\gamma r)\right)e^{in\theta},\\
&\overset{k}{G}_+=\sum_{-\infty}^{\infty}M_ne^{in\theta},\quad \overset{k}{G}_3=\sum_{-\infty}^{\infty}N_ne^{in\theta}.
\end{aligned}
$$

We now find the coefficients $\overset{k}{c}_n$, $\overset{k}{d}_n$, $\overset{k}{\beta}_n$ and $\overset{k}{\beta}{}_n'$ from following system of algebraic equations

$$
\begin{aligned}
&\frac{i\gamma}{2}\left(I_1(\gamma R_1)\overset{k}{\beta}_0-K_1(\gamma R_1)\overset{k}{\beta}{}_0'\right)+R_1\left(\overset{k}{c}_1+\overline{\overset{k}{c}_1}\right)-\frac{2\overline{\delta}}{R_1}=M_1^{'},\\
&\frac{i\gamma}{2}\left(I_1(\gamma R_2)\overset{k}{\beta}_0-K_1(\gamma R_2)\overset{k}{\beta}{}_0'\right)+R_2\left(\overset{k}{c}_1+\overline{\overset{k}{c}_1}\right)-\frac{2\overline{\delta}}{R_2}=M_1^{''},
\end{aligned}
$$

$$\frac{i\gamma}{2}\left(I_n(\gamma R_1)\overset{k}{\beta}_{n-1} - K_1(\gamma R_1)\overset{k}{\beta}{}'_{n-1}\right) + R_1^n\overset{k}{c}_n + \frac{4(\lambda+2\mu)}{3\mu}(n-1)(n-2)R_1^{-n}\overline{\overset{k}{c}}_{-n+2}$$
$$-(n-2)R_1^{-n+2}\overline{\overset{k}{c}}_{-n+2} + 2(n-1)R_1^{-n}\overline{d}_{-n+1} = M'_n,$$
$$\frac{i\gamma}{2}\left(I_n(\gamma R_2)\overset{k}{\beta}_{n-1} - K_1(\gamma R_2)\overset{k}{\beta}{}'_{n-1}\right) + R_2^n\overset{k}{c}_n + \frac{4(\lambda+2\mu)}{3\mu}(n-1)(n-2)R_2^{-n}\overline{\overset{k}{c}}_{-n+2}$$
$$-(n-2)R_2^{-n+2}\overline{\overset{k}{c}}_{-n+2} + 2(n-1)R_2^{-n}\overline{d}_{-n+1} = M''_n,$$
$$\overset{k}{d}_0 + \overline{\overset{k}{d}}_0 - \frac{R_1^2}{2}\left(\overset{k}{c}_1 + \overline{\overset{k}{c}}_1\right) + 2\delta ln R_1 = N'_0,$$
$$\overset{k}{d}_0 + \overline{\overset{k}{d}}_0 - \frac{R_2^2}{2}\left(\overset{k}{c}_1 + \overline{\overset{k}{c}}_1\right) + 2\delta ln R_2 = N''_0,$$
$$R_1^n\overset{k}{d}_n + R_1^{-n}\overline{\overset{k}{d}}_{-n} - \frac{1}{2}\left(R_1^{n+2}\overset{k}{c}_{n+1} + R_1^{-n+2}\overset{k}{c}_{-n+1}\right) = N'_n,$$
$$R_2^n\overset{k}{d}_n + R_2^{-n}\overline{\overset{k}{d}}_{-n} - \frac{1}{2}\left(R_2^{n+2}\overset{k}{c}_{n+1} + R_2^{-n+2}\overset{k}{c}_{-n+1}\right) = N''_n.$$

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