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ON HYDRAULIC CALCULUS OF THE MAIN PIPE-LINE OR A BASE OF THE NON-LINEAR MODEL

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The problems of hydrogasodynamics are described with the non-linear partial differential equations [1,2,3] and in every particular case they need different approach [4,5].

We consider the non-stationary gas flow in the main pipe-line, which is described by the system of non-linear partial differential equations [2,3]:

$$\frac{\partial P^2}{\partial x} = A\,Q^2, \qquad x \in (0,L), \quad t > 0, \tag{1}$$

$$\frac{\partial P}{\partial t} = B\,\frac{\partial Q}{\partial x}, \qquad x \in (0,L), \quad t > 0, \tag{2}$$

where $P = P(x,t)$ is the gas pressure, $Q = Q(x,t)$ is the outlay, A and B are the given constants. For this system according units of the parameters are known [2,3].

For the system (1), (2) we consider the initial and boundary conditions:

$$P(x,0) = f(x), \quad P(0,t) = P_1(t),\; Q(L,t) = Q_2(t), \tag{3}$$

$$x \in [0,L], \quad t \geq 0,$$

where L is the length of the pipe-line.

The existence of the solutions of the system (1), (2) is not known.

For the finding an approximate solutions we can use finite difference scheme or everageze the quantity Q/P by means of which we can linearized the system (1), (2).

The well-known everageze

$$\frac{\partial P}{\partial t} \approx \frac{1}{L}\int_0^L \frac{\partial P}{\partial t}\,dx = \varphi(t) \tag{4}$$

is also considered.

By this method we can obtain the approximate solution in the analytic form.

In [1] the approximate solutions are given in the analytic form, when P_1 and Q_1 are the constants.

We consider the non-stationary boundary conditions.

Taking into the account (4) in (2) and integrated along the segment $[x,L]$, we obtain

$$Q(x,t) = Q_2(t) - \frac{\varphi(t)}{B}\,(L-x). \tag{5}$$

Integrated the equation (1), using the formula (5) and the representation of square root by the series eliminated terms of the third and more power, we obtain

$$P(x,t) = P_1(t) +$$
$$+ \frac{A}{2P_1(t)} \Big\{ Q_2^2(t) \cdot x - \frac{Q_2(t) \cdot \varphi(t)}{B} \cdot [L^2 - (L-x)^2] \Big\}. \tag{6}$$

Hence, if we find the function $\varphi(t)$, the approximate solution of the system (1), (3) will be given by (5), (6).

Let us introduce the notations

$$\frac{A}{2P_1(t)} Q_2^2(t) = R_1(t),$$

$$\frac{A}{2P_1(t)} \cdot \frac{Q_2(t)}{B} = R_2(t),$$

then we obtain

$$P(x,t) = P_1(t) + R_1(t) \cdot x - R_2(t) \cdot [L^2 - (L-x)^2] \cdot \varphi(t),$$

$$\frac{\partial P}{\partial t} = P_1'(t) + R_1'(t) \cdot x - R_2'(t)[L^2 - (L-x)^2] \cdot \varphi(t) -$$
$$-R_2(t)[L^2 - (L-x)^2] \cdot \varphi'(t).$$

Integrated the previous formula along the segment $[0, L]$ and taking into the account (4) we obtain

$$\varphi'(t) + \frac{2L^2 R_2'(t) + 3}{2L^2 R_2(t)} \cdot \varphi(t) = \frac{6P_1'(t) + 3LR_1'(t)}{4L^2 R_2(t)}.$$

We have

$$\varphi(t) = e^{-\int\limits_0^t \alpha(\tau) d\tau} \Big[\int_0^t \beta(\tau) \cdot e^{\int\limits_0^\tau \alpha(\omega) d\omega} d\tau + \varphi(0) \Big],$$

where

$$\alpha(t) = \frac{2L^2 R_2'(t) + 3}{2L^2 R_2(t)}, \ \beta(t) = \frac{6P_1'(t) + 3LR_1'(t)}{4L^2 R_2(t)}.$$

Finally, putting $t = 0$ and $x = L$, we get

$$\varphi(0) = \frac{B \cdot Q_2^2(0)}{L} + \frac{B}{AL^2 \cdot Q_2(0)} \cdot (P_1^2(0) - f^2(L)).$$

R E F E R E N C E S

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