Reports of Enlarged Session of the
Seminar of I. Vekua Institute
of Applied Mathematics
Volume 21, 2006-2007

MATHEMATICAL MODELLING OF SOIL POLLUTION BY OIL FOR URBAN CONDITIONS

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It is very important to study the soil pollution by oil products on the urban territory, as it is directly connected with the possible pollution of water supply network and underground waters. In the territory of Georgia, there are many small and medium enterprizes of consumption and service of oil and gas production. Work of these enterprizes are directly or obliquely related with the pollution of surrounding places and underground waters, since oil products consumed by them get into deep layers of soil due to damages in the sewage system. Enterprizes of such type include: auto, railway and air transport repair services, petrol stations, oil and gas cisterns and tanks (in Batumi, Poti, Supsa), etc. Each of them is potential pollutant of the environment, therefore prediction of pollution of soil and underground waters for cities and regional centers represents a very actual problem. Our aim is to study oil filtration problem by means of mathematical modeling for soils typical for many Georgian cities (Tbilisi, Rustavi, Gardabani, Kaspi, Khashuri, Kutaisi, Batumi, Poti) to assess the risk of possible pollution of underground water basins. There were investigated oil filtration for some types of soils with flat surfaces [1-5]. Generally in nature soils seldom have flat surface and almost always they have surfaces with complex configuration. In the urban conditions, there are many cases when spilled oil cover as the flat surface, as the pit of rectangular type. In this case we conventionally divide the problem of oil and oil-products filtration in the soil into two stages. Namely, on the first stage oil filtration in the soil takes place (with account of oil evaporation), untill the flat surface will be cleared from the oil. On the second stage, oil infiltration into the soil from the pit proceeds intensively and then the oil infiltration process is continued by spilled on the surrounding places of the pit, and interaction of these two processes strengthen the saturation of the soil porosity by oil and the following process of oil infiltration. Hence, from the mathematical point of view, we have the problem of oil filtration into soil with non-stationary boundary conditions, which are varying both in space and in time. With purpose to simplify the problem and represent it in a better way, we restrict ourself to the two-dimensional model of oil filtration into soil, which show quite well all main properties of three-dimensional model.

Let us consider Cartesian coordinate system oriented on Oxz in such a way that Ox axis is directed along the ground surface, and Oz axis is directed vertically down. Suppose that oil with thickness h_0 was spilled on the surface and filled a pit with depth h_1. Our aim is to study the problem of filtration of spilled oil into soil (both from the surface and from the pit) with account of oil evaporation process and boundary conditions varying in time and space.

As it is known, filtration of oil and oil products into soil can be described by means of the following nonlinear parabolic equation [2,3]:

$$\frac{\partial S}{\partial t}+K_{11}\frac{\partial S^{11}}{\partial z}=D_{11}\frac{\partial^2 S^{n+m+1}}{\partial x^2}+D_{11}\frac{\partial^2 S^{n+m+1}}{\partial z^2}, \tag{1}$$

where $S=W-W_0$; W is the ratio of the unit oil mass existing in the soil to unit soil mass; W_0 is the ratio of the unit soil mass connected with water to the unit soil mass; K_11 and D_11 are filtration and diffusion coefficients of oil filtration into soil, respectively and they can be written as follows [2-4]:

$$\begin{aligned} K_{11} &= \frac{-K_1}{(\sigma-w_0)^n}\frac{Y_w}{Y_{oil}}, \\ D_{11} &= \frac{K_1\alpha_1}{(n+m+1)(\sigma-w_0)^m}, \end{aligned}$$

where K_1 is a water filtration coefficient at its full saturation; σ is a porosity of soil; Y_w and Y_{oil} are cinematic coefficients of water and oil viscosity, respectively; n and m are parameters, which describe the degree of nonlinearity of filtration and diffusion processes; α_1 is a parameter describing the pressure in pores and can be written as follows:

$$\alpha_1=\frac{P_0Y_w}{(\sigma-w_0)Y_{oil}},$$

where P_0 is a pressure of the liquid in case of its full saturation.

Let us search solution of the problem in the area $\Omega_2=\Omega\backslash\Omega_1$, where $\Omega = \{0\le x\le x_5, 0\le z\le z_2\}$ and $\Omega_1=\{x_2<x<x_3, 0\le z\le z_1\}$. Let us denote border of Ω_2 by S_2, border of Ω_1 by $S_1$and $\{x_1\le x<x_2\cup x_3<x\le x_4, z=0\}$ by S_1^*. Let us search solution of problem (1) in Ω_2 with the following initial and boundary conditions:

$$S(0,x,z)=\begin{cases}\sigma-w_0 \ \text{ if } \ x,z\in S_1\cup S_1^*, \ \text{ when } \ t=0,\\ -w_00 \ \text{ if } \ x,z\in\Omega_2 \ \text{ and } \ x,z\notin S_1\cup S_1^*,\end{cases} \tag{2}$$

$$\frac{\partial S}{\partial x}=0 \ \text{ when } \ x=0 \ \text{ and } \ x=x_5, \tag{3}$$

$$\frac{\partial S}{\partial t}=0 \ \text{ when } \ z=z_2, \tag{4}$$

$$\frac{\partial S}{\partial z}=0 \ \text{ when } \ z=0 \ \text{ and } \ x\in]0,x_1]\cup]x_4,x_5], \tag{5}$$

$$\frac{\partial S}{\partial z}=\begin{cases}\sigma-w_0 \ \text{ if } \ h(t)=h_0-\int\limits_t\int\int\limits_{S_2}SdS-h^*(t)>0,\\ 0 \ \text{ if } \ h(t)=h_0-\int\limits_t\int\int\limits_{S_2}SdS-h^*(t)=0,\end{cases} \tag{6}$$

$$\text{and} \quad \text{if } \ x\in S_1^* \ \text{ and } \ z=0, \tag{7}$$

$$\frac{\partial S}{\partial z} = \begin{cases} \sigma - w_0 \ \text{ if } \ h\left(t\right) = h_1 - \int\limits_t \int\int\limits_{S_1} SdS - h^{**}\left(t\right) > 0, \\ 0 \ \text{ if } \ h\left(t\right) = h_1 - \int\limits_t \int\int\limits_{S_1} SdS - h^{**}\left(t\right) = 0, \end{cases}$$

$$h_2 = 0 \quad \text{ and } \ x \in [x_2, x_3] \quad \text{ and } \ z = z_1, \tag{8}$$

$$\frac{\partial S}{\partial z} = \begin{cases} \sigma - w_0 \ \text{ if } \ h_1\left(t\right) = h_1 - \int\limits_t \int\int\limits_{S_1} SdS - h^{**}\left(t\right) \geq z_1 - z, \\ -w_0 \quad if \quad h_1\left(t\right) = h_1 - \int\limits_t \int\int\limits_{S_1} SdS - h^{**}\left(t\right) < z_1 - z, \end{cases}$$

$$h_0 = 0 \quad \text{ and } \ z \in [0, z_1] \ \text{ and } \ x = x_2 \quad \text{ or } \ x = x_3, \tag{9}$$

where function $h(t)$ describes the change of the height of the spilled oil in time, caused by oil infiltration and evaporation in the equation (6) and function $h(t)$ describes the same processes, but in the pit with boundary conditions (7) and (8); functions $h_*(t)$ and $h_* * (t)$ describe change of the oil height on the surface and in the pit due to oil evaporation.

We have investigated oil filtration process into soil by three and two dimensional models. Results of calculations have shown that oil penetration process into soil from the pit occurs almost qualitatively equally. That is why in present work for simplicity we present results of numerical calculations by two-dimensional model of oil filtration into soils from the pit area.

The sizes of the spilled oil on the surface of ground were 2m., 0.03m. along the axes $0x$ and $0z$, respectively. The pit was located in the central part of the spilled oil and had the following sizes: 0.6m, 0.5m along the axes 0x and 0z, respectively.

For numerical solving of problem (1)-(9) let us introduce denotations: $x_i = ih_1,\ y_i = jh_2,\ z_i = kh_3,\ i = 0, \pm1, \pm2, ...\ j = 0, \pm1, \pm2, ...\ k = 0, \pm1, \pm2,\ ...\ h_1 > 0,\ h_2 > 0,\ h_3 > 0,\ t_l = t_{l-1} + t,\ l = 1, 2, ..., N,\ t_0 = 0,\ \tau = \frac{T}{N},\ S^l_{i,j,k}$–is a net function.

We approximate (5) with respect to time according to difference scheme of Adams-Beshfort

$$S^{(l+1)*} = S^{(l)} + \frac{\Delta t}{2} f^{(l)},$$

$$S^{(l+1)} = S^{(l)} + \Delta t \left[\frac{3}{2} f^{(l+1)*} - \frac{1}{2} f^{(l)}\right].$$

This is a scheme of the second order accuracy with respect to time. With respect to space (1) is approximated according to Schumann scheme, which also is of the second order accuracy.

Also we have used implicit finite difference schemes for multidimensional parabolic type equation (1) with a simple united algorithm for construction of absolutely stable economical schemes to solve multidimensional parabolic type equations, where each difference equation completely approximates the given differential equation [3]. It is worthy to note that for the first time the constructed schemes are dependent on the dimension only and they are not dependent on the weight [3]. Steps of the grid along the axes $0x$ and $0z$ are 5cm and time step is equal 1 min.

The calculations were made for the meadow-alluvial, sandy and sub-sandy soil. The values of the parameter we have used were the following: $Q_0 = 0.00001$; $\nu_{wat} = 0.0106$ cm/s; $\nu_{oil} = 0.28$ cm/s; $K_w = 0.25$ 10 cm/s; $D_w = 22.4cm/h$.

Numerical calculations were carried out during $t = 180$ days. Numerical calculation have shown that the process of oil filtration into soils was various for different period of the numerical integration of the (1). Particularly it was possible to distinguish three stages during the numerical calculations. The first stage continued until inequality $0 < -z < h$ was true. For the second and third stages we have analogous inequalities $0 < z, < H$; and $z. > h$, respectively.

The first stage contained intensive processes of oil evaporation and oil filtration in the soil. This process continued about 1.8 days until ground's surface was not cleared from the spilled oil (of course with the exception of the pit area). During the second stage there existed both processes: oil filtration and evaporation in soil, and oil infiltration into the soil from neighboring areas of the pit, until the pit was not cleared from the spilled oil. Numerical calculations have shown that this stage continued about 31 days. The third stage of oil infiltration in soil was less intensive than the first and second stages. We have kept the process of oil infiltration in soil under our observation until a velocity of front of oil penetration to soil was not infinitesimal i.e. until the process became almost stable. Numerical calculations have shown that the third stage continued about 147 days.

As we have mentioned above (numerical calculations have shown that) the process of oil filtration in soil during the first stage was much more intensive than during other two stages. These phenomena that were presented for the first stage was stipulated by the existence of the mutual process of oil filtration in soil from the surface and pit boundaries simultaneously. For the first stage distribution of oil in soil was almost similar both in the vertical and in the horizontal directions. This picture of oil distribution in soil was especially evident in neighboring areas of the lateral boundaries of the pit. Although we can denote that oil penetration in soil in the vertical direction was more intensive than in the horizontal direction in neighboring areas of the lower border of the pit. That is why a line of front of the oil penetration in soil has sine character at the end of the first stage process.

The second stage mainly was characterized by oil propagation in soil from the pit borders but we note that during the second stage we have used non-stationary boundary conditions at the lateral borders of the pit, taking into account oil filtration and evaporation processes simultaneously, until the pit clearing process has not accomplished. For the second stage, we have studied oil propagation in soil from the pit borders, on the one hand with account of evaporation from the pit and on the other hand without it. Numerical calculations have shown that the results of calculations were rather different. For the first case the pit clearing process accomplished in 31 days and maximum depth of oil penetration in soil reached 3.1m at the end of the second process, whereas without of evaporation the pit clearing process accomplished in 47 days and maximum depth of oil penetration in soil reached 4.05m at the end of the second stage.

On the third stage of oil infiltration to the soil there oil penetration in soil took place as in vertical as well in the horizontal directions. We note that this process in the vertical direction was more intensive than in the horizontal direction and in 180 days of the numerical calculations the line of the front of oil distribution in soil was not very well expressed, but had sine character. The maximum depth of the front of oil distribution in soil was observed on the depth 5.7m almost along the symmetric axes of the pit, and the maximum distance of oil penetration in the horizontal direction was observed about 4.1m from the pit symmetric axes.

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Received 7.09.2006; revised 20.03.2007; accepted 25.12.2007.