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ON THE PROBLEM OF CONVERGENCE OF COSTS

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1. The problem of convergence of costs in the Kalman–Bucy scheme of partially observable random processes has been studied in a lot of works (see, for example, [3], [4], [5]). In the present paper, this problem is being studied for the generalized Kalman–Bucy scheme ([2]).

Consider the generalized Kalman–Bucy scheme of partially observable random processes ([2])

$$Y_t = Y_0 + Y \circ A_1(t) + M_1(t), \quad t \geq 0, \tag{1}$$

$$X_t = X_0 + Y \circ A_2(t) + \varepsilon M_2(t), \quad t \geq 0, \tag{2}$$

where $M_1(t)$ and $M_2(t)$ are the local martingales, $A_1(t)$ and $A_2(t)$ are the deterministic functions, and $(Y_0, X_0, M_1(t), M_2(t))$ is the Gaussian system. $H \circ A$ denotes the Lebesgue–Stieltjes integral of the process H with respect to the process A.

Assume that the gain function is linear and has the form

$$g(t,x) = f_0(t) + f_1(t)x, \tag{3}$$

where $f_0(t)$ and $f_1(t)$ are the deterministic measurable functions.

Introducer the costs

$$s^0 = \sup_{\tau \in \mathfrak{M}^Y} Eg(\tau, Y_\tau), \tag{4}$$

$$s^\varepsilon = \sup_{\tau \in \mathfrak{M}^X} Eg(\tau, Y_\tau), \tag{5}$$

where $\mathfrak{M}^y$ and $\mathfrak{M}^X$ are the classes of stopping moments with respect to the families of σ-algebras $(\mathcal{F}_t^Y)$ and $(\mathcal{F}_t^X)$, $\mathcal{F}_t^Y = \sigma\{Y_s,\ 0 \leq s \leq t\}$, $\mathcal{F}_t^X = \sigma\{X_s,\ 0 \leq s \leq t\}$.

Suppose now that for the functions $g_0(t)$, $g_1(t)$, $g_2(t)$ and for the increasing process Z_t the following conditions are fulfilled:

I. $EY^2 g^2 \circ \langle M_2 \rangle$ is increasing;

II. $\langle M_1 \rangle = g_1 \circ Z$, $\langle M_2 \rangle = g_2 \circ Z$;

III. $\Delta A_2 = 0$, $\langle M_1, M_2 \rangle = 0$,

where $\langle M_1 \rangle$ and $\langle M_2 \rangle$ are quadratic characteristics of the local martingales M_1 and M_2, and $\langle M_1, M_2 \rangle$ is their mutual quadratic characteristic.

Further, let $\mathcal{E}_t(A_1)$ denote a stochastic exponent, or a solution of the linear stochastic Dolean equation

$$\mathcal{E}_t(A_1) = 1 + \mathcal{E}_t(A_1) \circ A_1, \tag{6}$$

and let $\rho(t)$ be a continuous increasing function, majorizing the function $(1+\Delta A_1)^2 g_2 | g_0^2$. Let, moreover,

$$m_t = E(Y | \mathcal{F}^X), \quad \gamma_t = E(Y_t - m_t)^2. \tag{7}$$

2. In the theorem below we will prove the convergence of the cost s^ε to the cost s^0, as $\varepsilon \to 0$. Assume that $0 \le f_1(t) \le F < \infty$.

Theorem. *Let a partially observable random process be given by the equations* (1), (2)*, the costs* (4), (5) *be defined, and the conditions* I–III *be fulfilled. Then the estimate*

$$s^0 - s^\varepsilon \le \varepsilon \cdot F \cdot \rho(t) \cdot \mathcal{E}_t^2(A_1) \tag{8}$$

holds.

proof. First of all, it should be noted that the difference of costs can be estimated by γ_t as follows ([4]):

$$s^0 - s^\varepsilon \le F \cdot \gamma_t, \tag{9}$$

where γ_t satisfies the equation

$$\gamma_t = \gamma_0 + \gamma_t(2 + \Delta A_1) \circ A_2 + \langle M_1 \rangle - q^2 \circ (\langle \varepsilon M_2 \rangle), ,$$

$$q = \frac{d[\gamma_t \cdot (1 - \Delta A_1) \circ A_2]}{d(\langle \varepsilon M_2 \rangle)} .$$

Introduce the transformation

$$\gamma_t = \varepsilon \cdot u_t \cdot \mathcal{E}_t^2(A_1) \tag{10}$$

and show that $u_t \le \rho(t)$ for every $t \ge 0$. Thus the theorem is complete. We have

$$\gamma_t = \int_0^t \gamma_s \cdot (2 + \Delta A_1) g_1(s)\, dZ_s +$$

$$+ \int_0^t g_1(s)\, dZ_s - \frac{1}{\varepsilon^2} \int_0^t \frac{\gamma_s^2 \cdot (1 + \Delta A_1)^2 g^2(s)}{g_2(s)}\, dZ_s,$$

whence for u_t we can write

$$\varepsilon \int_0^t \mathcal{E}_s^2(A_1)\, du_s = \int_0^t g_1(s)\, dZ_s -$$

$$- \int_0^t \frac{u_s^2 \cdot \mathcal{E}_s^4(A_1)(1 + \Delta A_1)^2 g^2(s)}{g_2(s)}\, dZ_s.$$

From the last relation it immediately follows that

$$u_t = \frac{1}{\varepsilon} \int_0^t \mathcal{E}_s^2(A_1) g_1(s)\, dZ_s - \frac{1}{\varepsilon} \int_0^t \frac{u_s^2 \cdot \mathcal{E}_s^2(A_1) g^2(s)}{g_2(s)}\, dZ_s =$$

$$= \frac{1}{\varepsilon} \int_0^t \frac{\mathcal{E}_s^2(A_1) g^2(s)}{g_2(s)} \Big[\frac{(1 + \Delta A_1)^2 g_2(s)}{g^2(s)} - u_s^2 \Big]\, dZ_s. \tag{11}$$

Just as in [4], from (11) we can conclude that $u_t \leq \rho(t)$, which provides us with the estimate (8).

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