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ABOUT THE ANALYTIC REPRESENTATION OF A CLASS OF GEOMETRICAL FIGURES, SURFACES AND LINES

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Abstract

Aim of this article is analitical represantation of one class of geometrical figures, surfaces and lines. This class of surfaces appear, when we study the problems of spreading of smoke- rings, also this class of lines describe the complicated orbit of some celestial objects. In particulary cases of this analytic represantation give as classical objects (torus, helicoid, helix and ...).

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In the papers [1-4] there is defined the Generalized Möbius Listing's body - GML_n^m. In this paper we consider particular case of this class. In other hands we give analytic represantation of one large class of geometrical figures, surfaces, lines and traectory.

We have analitic represantation with following formula

$$X(\tau,\psi,\theta,t) = [R(\theta,t) + p(\tau,\theta,t)\cos\psi\cos n\theta - p(\tau,\theta,t)\sin\psi\sin n\theta]\cos\theta$$

$$Y(\tau,\psi,\theta,t) = [R(\theta,t) + p(\tau,\theta,t)\cos\psi\cos n\theta - p(\tau,\theta,t)\sin\psi\sin n\theta]\sin\theta$$

$$Z(\tau,\psi,\theta,t) = [K(\theta) + T(t) + p(\tau,\theta,t)\cos\psi\sin n\theta + p(\tau,\theta,t)\sin\psi\cos n\theta$$

or

$$X(\tau,\psi,\theta,t) = [R(\theta,t) + p(\tau,\theta,t)\cos(\psi+n\theta)]\cos\theta$$

$$Y(\tau,\psi,\theta,t) = [R(\theta,t) + p(\tau,\theta,t)\cos(\psi+n\theta)]\sin\theta$$

$$Z(\tau,\psi,\theta,t) = [K(\theta) + T(t) + p(\tau,\theta,t)\sin(\psi+n\theta)$$

where we use following notations:
τ,ψ,θ - space values $\tau \in [\tau_*,\tau^*],\ \tau^* \geq \tau_* \geq 0,\ \psi \in [0,2\pi],\ \theta \in [0,2q\pi]\ q \in \mathbb{Z}$;
t - time value - $t \in [0,\infty)$;
$R(\theta,t)$ - parameter or function (usualy $R(\theta,t) > R_* = \text{const.} \geq \tau^*$;
n, q - integer parametres;
$K(\theta), \tau(t)$- some functions;

In this article we assume that every parameters and functions in formula don't depend on time argument t. Also:

- $T(t) = 0$

Case I.
- $\boldsymbol{K(\theta) = 0}$;
- $\boldsymbol{R(\theta, t) = R_* =}$ **const.** $\boldsymbol{> 0}$;
- $\boldsymbol{p(\tau, \theta, t) = \tau}$ - independent value.

In this case we have analitic representation of following geometrical objects:

Case Ia. - **n=0, q=1**

• If $\tau_* = 0,\ R_* > \tau^*$, then we have a **"Ring torus"** (with two radius R_* and τ^*), (body - 3 values (τ, ψ, θ)) (see e.g.[5], p.1816)

• If $\tau_* = 0,\ R_* = \tau^*$, then we have a **"Horn torus"** (with two radius R_* and τ^*), (body - 3 values (τ, ψ, θ)) (see e.g.[5], p.1816)

• If $\tau_* = 0,\ R_* < \tau^*$, then we have a **"Spindle torus"** (with two radius R_* and τ^*), (body - 3 values (τ, ψ, θ)) (see e.g.[5], p.1816)

• If $\tau_* > 0,\ R_* \geq \tau^*$, then we have a **"Toroidal shell"** (with thickness $\tau^* - \tau_*$), (body - 3 values (τ, ψ, θ));

• If value $\tau = \tau_0 =$ const., $R_* \geq \tau^*$, then we have **"Surface of torus"** (with two radius R_* and τ_0), (surface -2 values (ψ, θ));

• If value $\theta = \theta_0 =$ const., and $R_* \geq \tau^*$, then we have a **"Plane disk"** if $\tau_* = 0$ or **"Plane ring"** if $\tau_* > 0$;(surface - 2 values (τ, ψ));

Case Ib. **n - integer number and q=1,** then:

• If $\psi = \{\psi_0$ **and** $\psi_0 + \pi\}$, $\tau \in [0, \tau^*]$, or $\psi = \psi_0$, **and** $\tau \in [-\tau^*, \tau^*]$, then we have

"Two-sided surfaces of Mobius-Lising's type", n - number of rotation (if $n > 0$, then rotation is counter-clockwise, if $n < 0$, then rotation is clockwise), (surface - 2 values (τ, θ)); (see [1-3]);

In particularly when **n=0** (surfaces - 2 values (τ, θ)):

• If $\psi_0 = 0$, then we have **"Plane ring"** when $\mathbf{R}_* > \tau^*$, or **"Plane disk"** when $\mathbf{R}_* = \tau^*$;

• If $\psi_0 = \pi/2$, then we have **"Surface of cylinder"**;

• If $\psi_0 \neq 0$, or $\pi/2$, then we have **"Surface of a cone"** or **"Surface of frustrum of a cone"**, $\{(\pi - 2\psi_0)$ - angle of a vertex of this cone);

Case Ib1. **n - integer number and q=1, then:**

• If $p(\tau, \theta, t) = (\tau^* + \tau_1)\cos\psi_0 + \tau\cos\psi,\ \tau \in [\tau^*, \tau_1]$, where $R_* >> \tau^* > \tau_1 > \tau_*$ and are some relations between this parameters and number n, then we have **"Cylindrical pipe (if** $\tau_* > 0$) or **bar** ($if \tau_* = 0$)**, which radius is** τ_1 **winding around the torus** (R_*, τ^*) **and make n-coils",** (body - 3 values $(\tau, \psi, \theta),$);

in this case if $\tau = \tau_0$ =const. and $\psi = \psi_0$ =const., then we have **"Helix line",** which winding around the torus (R_*, τ^*) (value (θ));

Case Ic. $\boldsymbol{n = i + 1/2,\ i \in \Im, q = 1}$

• If $\psi = \{\psi_0$ **and** $\psi_0 + \pi\}, \tau \in [0, \tau^*]$,, or $\psi = \psi_0$ and $\tau \in [-\tau^*, \tau^*]$, then we have a

"One-sided surfaces of Mobius-Lising's type", (2 values (τ, θ));

particularly

• if $i = 0$ or - 1, then we have **"Classical Mobius Strip"**; (see [1]);

Case Id. Lines, when $\boldsymbol{\tau = \tau_0}$ and $\boldsymbol{\psi = \psi_0}$:

• If $\boldsymbol{n \in \Im}$ **(integer) and** $\boldsymbol{q = 1}$, then we have a closed curve, which is winding around the "little parts" (cross section is a disk, with radius τ^*) of torus and **n** - is a number of coils, (value $\theta \in [\mathbf{0}, \mathbf{2}\boldsymbol{\pi}]$);

• If $\boldsymbol{n \in Q}$ **(Rational**, $\boldsymbol{n = i/j}$**) and** $\boldsymbol{q = j}$, then we have closed curve, after j-**circuits** around of "big parts" of the torus appear **i - coils** around of "little parts" of the torus (value $\theta \in [\mathbf{0}, \mathbf{2j}\pi]$);

• If $\boldsymbol{n \in \Re}$ **(Irrational) and** $\boldsymbol{q = \infty}$, then we have nonclosed curve, which make **infinite coils** around of "little parts" of the torus after **infinite circuits** around of "big parts" of the torus, but this curve is not self crossing $(0 \in [0\infty))$; (see [2]);

Case II.

$\boldsymbol{R = R_*,\ K(\theta) \neq 0,\ p(\tau, \theta, t) = \tau, n = 0, q \in \Im :}$

• If $K(\theta) = K \cdot \theta, |K| \geq \tau^*/2\pi, \tau_* = 0$, then we have **"Helix body"**, with "big radius" R_* and "little radius" τ^* (body - 3 values (τ, ψ, θ)) (This body physically similar a vertical spring);

Remark 1. If $K > 0$, then "Helix body" disposed on the plane XOY and if $K < 0$, then "Helix body" disposed under the plane XOY;

Remark 2. K - is a coefficient of expansion of the "Helix body";

Remark 3. If $K = \tau^*/2\pi$, then coils of "Helix body" are tangential;

Remark 4. q - number of coils of the "Helix body", $\theta \in [0, 2q\pi]$.

• If in this case $\tau_* > 0$, then we have **"Pipes Helix body"** ($(\tau^* - \tau_*)$ - thickness of the wool's of the pipes) (body - 3 values (τ, ψ, θ));

• If in this case $\tau = \tau_0 =$ const. and $\psi = \psi_0 =$ const., then we have classic **"Helix"** (line - value θ), (see [5], pp- 811);

Case IIe. $\boldsymbol{K(\theta) = K \cdot \theta,\ \theta \in [0, 2q\pi],\ q \in \Im,\ R_* > \tau^*,\ n \neq 0 :}$

• If $R_* = \tau^*, \psi = 0, \tau_* = 0$, then we have **"Surface of straight Helicoid"** (2 values (τ, θ)), (see [5] pp. 810) ;

— **"Helicoid with a Hole" or "Surfaces of Helix body with plane coils":**

• If $R_* > \tau^*, \psi = 0$, then we have "Surface of Helix body with plane coils" (coil's surface is perpendicular of OZ axis);

• If $\psi = \pi/2$, then we have "Surface of Helix body with plane coils" (coil's surface is parallel of OZ axis);

• If $\psi \neq 0$ and $\pi/2$, then we have **"Surface of Helix body with plane coils"** ($(\pi - \psi)$ - angle between coil's surface and OZ axis);

Remark. q- number of coils, (if q- arbitrary real number, then $[q]$ - is a number of coils, and (q, K) - define the length of this figures);

If $\tau = \tau_0 =$ const., $\psi = \psi_0 =$ const., then we have **"Line winding around Helix body"**;

If $p(\tau, \theta, t) = (\tau^* + \tau_1)\cos\psi_0 + \tau\cos\psi$, where $R_* >> \tau_* > \tau^* > \tau_1$ are some relation between this parameters and number n - then we have **"Cylindrical pipe** (if $\tau_* > 0$) or **bar** (if $\tau_* = 0$), which radius is τ_1 **winding around Helix body** (R_*, τ^*) with q-coils and make n-coils, (body - 3 values (τ, ψ, θ)).

case III.

$\mathbf{K}(\boldsymbol{\theta}) = \mathbf{0}, \boldsymbol{\theta} \in [\mathbf{0}, \mathbf{2q}\boldsymbol{\pi}],\ \mathbf{q} \in \Im,\ \mathbf{n} = \mathbf{0},\ \mathbf{R}(\boldsymbol{\theta}) = (\mathbf{1} + \boldsymbol{\alpha\theta})\mathbf{R}_*, |\boldsymbol{\alpha}| \geq \boldsymbol{\tau^*/\pi}\mathbf{R}_* :$

• If $\tau_* = 0$, then we have a body -**"Spooling bar"** with radius τ, situated between two flats $\{Z = \tau^*\}$ and $\{Z = -\tau^*\}$, (3-values);

• If $\tau_* > 0$, then we have a body - **"Spooling pipe"** with radius $\tau^*, (\tau^* - \tau_*)$ - thikcness of pipe situated between two flats $\{Z = \tau^*\}$ and $\{Z = -\tau^*\}$, (3-values (τ, ψ, θ));

• If $\psi = 0$, then we have **"Spooling stripe"** situated on the plane **XOY**, width of $(\tau^* - \tau_*)$ - stripe, (2 - values $(\tau\theta)$);

• If $\psi = \pi/2$, then we have a surface of the **"Rolling stripe"**, width of $(\tau^* - \tau_*)$ - stripe, (2 - values (τ, θ));

Remark. If $\alpha > 0$, then surface is extended and rolling on the cylinder with radius R_* , if $\alpha < 0$, then surface is narrowed and rolling into the cylinder with radius R_* and q is a number of coils.

Case IIIf. $\boldsymbol{K(\theta) = 0, \quad \theta \in [0, 2q\pi], \quad R(\theta) = (1 + \alpha\theta)R_*,}$
$\boldsymbol{p(\tau, \theta, t) = (1 + \gamma\theta)\tau, \quad |\alpha| \geq (1 + \gamma\pi)\tau^*/\pi R_*, q \in \Im, n = 0 :}$

• If $\tau_* > 0, \gamma > 0, \alpha > 0$, then we have a body - **"Extended Spooling pipe with extended** radius" - simillar smails shell, but generator of this figur is a plane spirall on the XOY (3-values (τ, ψ, θ));

• If $\gamma < 0$, then "Extended Spooling pipe with narrowed radius"

• If $p(\tau, \theta, t) = (1 + \gamma \sin\theta)\tau, \alpha > 0$, then we have a body - "Extended Spooling pipe with changed radius" -, but generator of this figur is a plane spirall on the XOY (3 - values (τ, ψ, θ));

• If $\tau_* > 0,\ \gamma > 0,\ \alpha > 0,\ n = 0$, then we have a body - **"Extended Spooling pipe with extended radius"** - simillar smails shell, but generator of this figur is a space spirall (3- values (τ, ψ, θ));

• If $\gamma > 0,\ \alpha > 0,\ n = 0,\ \psi =$ **const**., then we have a surface - **"Extended Heliciod with a hole and with changed width of coils"** (q - number of the coils of helicoid);

• If $\gamma > 0,\ \alpha > 0,\ n \neq 0,\ \psi =$ const., then we have a surface - **"Extended Heliciod with a hole and with changed width of coils and surface of this coil also rotate"** (q - number of the coils of helicoid and n- number of rotation of the surface;

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