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ON THE MATHEMATICAL MODEL DESCRIBING OF GEORGIAN TERRITORY POSSIBLE POLLUTION FROM THE HOT POINTS DISPERSED IN THE NORTHERN CAUCASUS

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Introduction. Environmental protection is one of the most urgent issues today. Anthropogenic sources of environmental pollution are more diverse, powerful and enduring as compared to natural. There are a number of artificial sources of environmental pollution (auto transport, air transport, enterprises, factories, etc.), presenting permanent sources of environmental contamination. One more source for anthropogenic pollution is harmful substances entered into the environment during military conflicts. Unfortunately local wars have been becoming frequent lately during which environment is polluted by military weapons as well as explosions at gas and oil terminals, environment is additionally polluted by gas and oil burning. Results of scientific research demonstrated that in the years of the Second World War pollination of Caucasian Glacier significantly increased (the process was caused by military operations under way in the Northern Caucasus). During Iraq-Kuwait, Iraq-USA, Chechnya-Russia conflicts up to million tons of oil was being daily burned on oil-mining sites. Huge amount of soot, carbonic acids, sulfur dioxide and other substances was being dispersed into atmosphere [1-4]. The above issue is a very urgent one as local conflicts are becoming more and more frequent in Trans Caucasian and Middle East regions.

Unfortunately the problem has touched Georgia (conflicts in Abkhazia and South Osetia) as well as its adjacent regions (Chechnya, Karabakh); even the military actions in Iraq and Kuwait (accompanied by environmental pollution with dangerous substances emanated from gas and oil terminals) have certain effect upon deterioration of environment in the neighbor countries (including Georgia) [5]. As seen above, confrontations between countries plays a very significant role in the process of environmental pollution. Not only population suffers from the polluted environment, additives transmitted through air and sea flows cause global pollution of the whole environment [5-9]. Therefore this issue needs to be examined in more detail. Here we decided to study the problem on the example of the Russian-Chechnya, conflicts.

Numerical Model With Account of Orography Influence. As is known, a substance transfer and diffusion through the atmosphere can be described by the following equation [5-8, 10-11]:

$$\frac{\partial C}{\partial t}+U\frac{\partial C}{\partial x}+V\frac{\partial C}{\partial y}+W\frac{\partial C}{\partial z}=\frac{\partial}{\partial x}\left(k_k\frac{\partial C}{\partial x}\right)+\frac{\partial}{\partial y}\left(k_p\frac{\partial C}{\partial y}\right)+\frac{\partial}{\partial z}\left(k_s\frac{\partial C}{\partial z}\right)-\alpha c+F, \quad (1)$$

where C is concentration; U, V, W are the axial components of wind velocity along axis ox, oy, and oz respectively; K_x, K_y and K_z are the coefficients of turbulent diffusion along axis ox, oy and oz; α is coefficient that determines the velocity of substance concentration changes during the process of substance decomposition and transformation; $F(x, y, z, t)$ are internal Sources.

As our purpose is to investigate harmful substances transfer and diffusion by the air currents from the North Caucasus (Grozny) toward the Georgian territory (Tbilisi) so let the axis ox be directed along the earth meridian (southward), the axis oy be directed along the parallel, and the axis oz be directed along the earth radius vertically upward. Let $C(x, y, z, t)$ be the concentration of aerosol substance that migrates through the atmosphere along with air Streams at the velocity $\bar{V}\ (x, y, z) = \bar{U}_i + \bar{V}_j + \bar{W}_k$and we are learning for its distribution in the area G with surfaces.

Analysis of synoptic processes in the Caucasus region shows that Georgian territory is dominated by western and eastern meteorological processes. This is stipulated by the geographical location of Georgia and features of its relief. The north Georgian territory is bordered by the Great Caucasus Range directed along parallel. It is a natural obstacle for meteorological processes coming from the north. But detailed analysis of the synoptic processes in Georgia shows that 10% of the cyclones coming from the north pass the Great Caucasus Range directed along parallel. And entered in the Georgian territory. Above mentioned gives us possibility to make some simplification for (1).

Now let us consider the following specific problem migration and diffusion of harmful substances ejected from a point or linear sources of power $\boldsymbol{m}$ located at altitude h_1 in the region of non homogeneous orography. Assume that meteorological situation stipulates transference of substance in the atmosphere in the direction of average wind velocity and axis ox has the same direction of an average velocity of atmospheric stream, oz be directed vertical upward and oy be perpendicular to the plane oxz (that is $V = 0$) and coefficient being constant. Under these conditions substance transference in the atmosphere is described by the following two-dimensional equation in area G:

$$\frac{\partial C}{\partial t} = k_x \frac{\partial^2 C}{\partial x^2} + \frac{\partial C}{\partial z}\left(k_z \frac{\partial C}{\partial z}\right) - U\frac{\partial C}{\partial x} - W\frac{\partial C}{\partial z} - ac + F(x, y, z, t), \qquad (2)$$

We consider the problem of transfer and diffusion of adverse substances in an atmosphere for the Caucasus regions, where one of the determining factors for circulation process is particularities of orography. In order to reflect correctly the impact of complex relief in the mathematical model, let us rewrite (2) in a coordinate system

$$t^1 = t_1,\ x^1 = x_1,\ z^1 = \frac{z - h(x)}{h_0}$$

(where function $h(x)$ describes non homogeneity of the earth surface, ho is altitude of border layer of the atmosphere), then we will receive[5,10, 12-14]:

$$\frac{\partial C}{\partial t} = k_x \frac{\partial^2 C}{\partial x^2} + \frac{\partial C}{\partial z}\left(k_z \frac{\partial C}{\partial z}\right) - U\frac{\partial C}{\partial x} - \overline{W}\frac{\partial C}{\partial z} - ac + F(x^1, z^1, t^1), \qquad (3)$$

where $\bar{W} = \frac{U\partial h}{\partial x} - W$ is analogue of vertical wind velocity in the new coordinate system (for simplicity x_1, y_1, z_1 in (3) are changed by x, y, z).

Horizontal and vertical wind velocity components U and $\bar{W}$ in (3) are satisfied the following equation of continuity:

$$\frac{\partial U}{\partial x}+\frac{\partial W}{\partial z}=0. \tag{4}$$

Let integrate (4) from z_1 to z, then we will obtain:

$$\bar{W}=W_1+\frac{1}{h_0}\frac{\partial h}{\partial x}(U-U_1)+\int\limits_{z_1}^{z}\frac{\partial U}{\partial x}dz, \tag{5}$$

where U_1 and W_1 are horizontal and vertical components of the wind velocity at the z_1 altitude; $z_1 = 1\ m$.

While making prognosis of atmosphere pollution it is very important to have U(x,z) and $K_z(z)$ functions behavior in the near-earth and border-layer atmosphere of the atmosphere.

As it is known in the near-earth layer atmosphere height distribution of wind velocity component U has the following form [11]:

$$U(x,z)=U_1(x)\frac{\ln\frac{(z-h(x))}{h_z-z_0}U}{\ln\frac{z_1}{z_0}}, \tag{6}$$

where z_0 is roughness of the earth surface, $z_1 =$ const;

It we assume that roughness along ox is constant z_0 =0.01 and z_1=1, then we will have:

$$U(x,z)=U_1(x)\left(1+\frac{1}{2}\lg(z)\right). \tag{7}$$

In order to give horizontal character to the distribution of the $U_1(x)$ along ox we use the values of well-known for meteorologists distribution of $U_1(x)$ when air masses pass a natural obstacles (mountains). On the basis [10] we use the following formula:

$$U(x,z)=U_0(x)(1+\gamma(x)), \tag{8}$$

Where U_0 is average value of the wind velocity at the level z_1;$\gamma(x)$ is a weight function, its value depends from the form h(x). $\gamma(x)\in[-0.3, 0.25]$

So we will use the following formula:

$$U(x,z)=\begin{cases} U_0(1+\gamma(x))\left(1+\frac{1}{2}\lg(z)\right) & \text{when} \quad z\leqslant z_1,\\ U_0(1+\gamma(x))\left(1+\frac{1}{2}\lg(z_1)\right) & \text{when} \quad z>z_1,\end{cases} \tag{9}$$

where z_1 s altitude of near-earth layer.

As it is known in the near-earth layer of the atmosphere the vertical thermal currents and momentum are maintained according to altitude and vertical component $k_z(z)$ of turbulent stream variation coefficient increases in proportion of altitude [9-13]:

$K_z(z)=Y-K_1\frac{z}{z_1}$, where Y is coefficient of molecular diffusion; $k_1=k_z(z_1)$;

As a result of theoretical studies I. Kibel has received analogous formula for $K_z(z)$, [10,15].

$$K_z(z) = \begin{cases} K_1 \left(\frac{z_1}{r_1}\right)_0^{1-z} & \text{when} \quad z \leqslant r_1, \\ K_1 & \text{when} \quad z > r_1, \end{cases} \tag{10}$$

where $-1 \leqslant \varepsilon < 1$ and by selecting its values it is possible to select thermal stratification of the environment.

Initial values for function $h(x)$ describes non homogeneity of the earth surface is obtained from the topographical maps with the horizontal step 10 km. After that for each mountain-pass areas we used the following interpolation formula:

$$h(x) = h_i^{\max}(1 - \frac{x - x_{\max}}{d})^2)^2, \tag{11}$$

where $h_i^{\max}$ is a maximum height for each i mountain-pass area; d is horizontal size of each i mountain-pass area.

Generally problem (1)-(11) is integrated in $G = \{0 \leqslant x \leqslant L, 0 \leqslant z \leqslant 1\}$ with the following initial and boundary conditions:

$$\begin{aligned} &C_{t=0} = C_0, \\ &C_{t=0} = M\delta(z - H), \\ &K_z \left.\frac{\partial C}{\partial z}\right|_{x=l} = 0, \\ &\left.\frac{\partial C}{\partial z}\right|_{x=1} = 0. \end{aligned} \tag{12}$$

Let us covering area $\bar{G}$with grid: , $x_i = ix\Delta l ... Y_k = k + \Delta l ... i = 1..N, k = 1..M$ where is a step in the directions OX and OY.

Primary values of the grid function r = r(x,y) are taken from the topographic maps (the step on the topographic maps as twice less than). Further these obtained values of the z(x,y) are smoothed with aid of second order derivative and linear interpolation's finite-deference schemes. Specifically for the points i+1/2, i-1/2 of the grid we have:

$$\left(\frac{\partial^2 r}{\partial x^2}\right)_{i+\frac{1}{2},k} = \frac{1}{\Delta L^2}\left(r_{i+1,k} + r_{i,k} - 2r_{i+\frac{1}{2},k}\right), \tag{13}$$

$$\left(\frac{\partial^2 r}{\partial x^2}\right)_{i-\frac{1}{2},k} = \frac{1}{\Delta L^2}\left(r_{i,k} + r_{i-1,k} - 2r_{i+\frac{1}{2},k}\right), \tag{14}$$

$$\left(\frac{\partial^2 r}{\partial x^2}\right)_{i,k} = \frac{1}{\Delta L^2}\left(r_{i+1,k} + 2r_{i,k} + r_{i-1,k} - 2r_{i+\frac{1}{2},k} - 2r_{i-\frac{1}{2},k}\right), \tag{15}$$

from other hand we have:

$$\left(\frac{\partial^2 r}{\partial x^2}\right)_{i,k} = \frac{1}{\Delta L^2}\left(r_{i+\frac{1}{2},k} + r_{i-\frac{1}{2},k} - 2r_{i,k}\right) \tag{16}$$

consequently we have:

$$r_{i,k}=\left(\frac{2}{3}r_{i+\frac{1}{2},k}+\frac{2}{3}r_{i+\frac{1}{2},k}-\frac{1}{6}r_{i+1,k}-\frac{1}{6}r_{i-1,k}\right). \qquad (17)$$

Similar formulas we can obtain for the points $\left(i,k-\frac{1}{2}\right)$ and $\left(i,k+\frac{1}{2}\right)$of the grid .

Further, for the purpose to remove peaks in the field of the relief, we expand values of the r(x,y) on the row of the Furie and third's of the last terms of the Furie's row are taken out.

The numerical modeling is realized with the help of the well known scheme of F. Shuman [16].

In order to remove orographycal "noise" from the numerical sheme during calculations, influence of the orographycaly effects on atmosphere processes is eliminated step by step.

Values of orographic elevations used in our numerical model are applied as orographic discrete functions.

We have investigated influence of the military actions (conflicts) of small and regional scales on background value of pollution of environment. Both local and regional distribution of harmful substances dispersed in the atmosphere from the conflict zone as a result of using various weapons are also studied. In particular, for the solution of the task we were necessary to find and process the following materials, which were directly connected to pollution of environment during military operations.

1. Study of arms used at each military action and evaluation of intensity of their use and mass of harmful substances emitted in an environment.

2. Quantity of explosions of oil pipelines, terminals and boreholes, their power, essence of harmful substances emitted in an environment, their masses and duration of their action.

3. Study of meteorological processes of region both their seasonal and monthly classification.

4. Study of climate, geographical and orographic features of the region.

On the basis of above mentioned real materials we have studied migration of the harmful substances, which have been ejected in an atmosphere, in space and in time. For the solution of this task it became necessary to execute the following research:

To study influence of orography on transfer and diffusion of harmful substances we have created regional two-dimensional numerical model of the equation of diffusion of a non-stationary general view, which correctly took into account orography, real parameters of speed of a wind, conditions of stratification and turbulence of an atmosphere and multiple polluting sources located on a surface of ground. With the help of the model we have carried out numerical simulations accounts.

By the numerical model we have studied migration of harmful substances sprayed in an atmosphere for two cases.

1. Taking into account influence of a relief.

2. Without taking into account influence of a relief.

As it was shown by obtained results in the case when influence of relief is taken into account values of concentrations of harmful substances are much lower than in the second case, and that is natural, because relief has essential influence on migration of harmful substances, on their gravitational fallout. Obtained results were verified with the real data and were found high accuracy of our calculations.

In both cases the numerical experiments were different from each other by: 1) speed of a wind; 2) power of a source.

As it was shown by numerical simulations, actually, there is no migration of harmful substances in Georgia due to dominated in the region meteorological processes and orographic, and geographic features. Comparison of our results with data of Hydrometeorological Department of Georgia on concentrations of harmful substances during considered period gave a good coincidence.

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