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THE MIXED BOUNDARY VALUE PROBLEM FOR THE NON-SHALLOW SPHERICAL SHELLS

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In the present paper the non-shallow spherical bodies of shell type are disscussed, when the dicplacement vector is independent from the thickness coordinate x_3:

$$\mathbf{u}(\mathrm{x}^1,\mathrm{x}^2,\mathrm{x}^3)=\mathbf{u}(\mathrm{x}^1,\mathrm{x}^2).$$

It is well known, that the equilibrium equations and stress-strain relations (Hook's Law) have the following complex form in the system of isometric coordinates [1].

$$\begin{cases} \dfrac{1}{\Lambda}\dfrac{\partial}{\partial z}(T_{11}-T_{22}+2iT_{12})+\dfrac{\partial}{\partial \bar z}T_\alpha^\alpha+\dfrac{2}{\rho}T_+ + F_+ = 0, \\ \\ \dfrac{1}{\Lambda}\left(\dfrac{\partial T_+}{\partial z}+\dfrac{\partial \overline{T}_+}{\partial \bar z}\right)+\dfrac{1}{\rho}\left(T^{33}-T_\alpha^\alpha\right)+F_3=0. \end{cases} \tag{1}$$

$$\begin{cases} T_{11}-T_{22}+2iT_{12}=4\mu\Lambda\dfrac{\partial u^+}{\partial \bar z}, \\ T_\alpha^\alpha = 2(\lambda+\mu)\left(\theta+\dfrac{2}{\rho}u_3\right), \\ T_+=T_{13}+iT_{23}=T_{31}+iT_{32}=\mu\left(2\dfrac{\partial u_3}{\partial \bar z}-\dfrac{1}{\rho}u_+\right), \\ T_{33}=\dfrac{\lambda}{2(\lambda+\mu)}T_\alpha^\alpha, \\ \theta=\dfrac{1}{\lambda}\left(\dfrac{\partial u_+}{\partial z}+\dfrac{\partial \overline{u_+}}{\partial \bar z}\right), \\ F_+=F_1+iF_2, \quad u_+=u_1+iu_2, \quad u^+=u^1+iu^2, \quad z=x^1+ix^2. \end{cases} \tag{2}$$

where $x^1=tg\dfrac{\theta}{2}cos\varphi$, $x^2=tg\dfrac{\theta}{2}sin\varphi$ are the isometric coordinats on the shell midsurface. Here we use the notations

$$\boldsymbol{\sigma}^\alpha(x^1,x^2,x^3)=\frac{1}{\left(1+\frac{x_3}{\rho}\right)^2}\mathbf{T}^\alpha(\mathrm{x}^1,\mathrm{x}^2),$$
$$\boldsymbol{\sigma}^3(x^1,x^2,x^3)=\frac{1}{1+\frac{x_3}{\rho}}\mathbf{T}^3(\mathrm{x}^1,\mathrm{x}^2),$$
$$\mathbf{\Phi}(x^1,x^2,x^3)=\frac{1}{\left(1+\frac{x_3}{\rho}\right)^2}\mathbf{F}(x^1,x^2).$$

$\boldsymbol{\sigma}^i$ are contravariant stress vectors, $\boldsymbol{\Phi}$ an external force, $\mathbf{u}$ the displacement vector, λ and μ are Lame's constants, ρ is a radius of sphere.

The general representations of this system are given in the forms

$$\begin{aligned} u_+ &= 2\rho\left[\frac{\partial\chi}{\partial\bar z} - \frac{z}{1+z\bar z}\overline{\varphi(z)} - \overline{\psi(z)}\right], \\ u_3 &= \chi(z,\bar z) + \frac{\lambda+3\mu}{4(\lambda+2\mu)}\left[\varphi(z) + \overline{\varphi(z)}\right], \end{aligned}$$

where $\varphi(z)$ and $\psi(z)$ are holomorphic functions of z and $\chi(z,\bar z)$ is solution of the equation [2]

$$\nabla^2\chi + \frac{2}{\rho^2}\chi = 0,$$

Let us consider the mixed boundary value problem for the non-shallow spherical shells. We have to find the elasticity balance, when the stresses are given on the some part of the boundary and the displacements are on the remainder.

Let us the boundary conitions have the following forms:

$$\begin{cases} u_{(l)} + iu_{(s)} = 2\rho i\left[\dfrac{\partial\chi}{\partial\bar z} - \dfrac{z}{1+z\bar z}\overline{\varphi(z)} - \overline{\psi(z)}\right]\dfrac{d\bar z}{ds} = \displaystyle\sum_{-\infty}^{+\infty}(A_n + iB_n)e^{in\varphi}, \quad r = r_0, \\ T_{(ln)} = -Im\left\{\mu\left[\dfrac{1}{2}\dfrac{\lambda+3\mu}{\lambda+2\mu}\overline{\varphi'(z)} + \dfrac{2z}{1+z\bar z}\overline{\varphi(z)} + 2\overline{\psi(z)}\right]\dfrac{d\bar z}{ds}\right\} = \displaystyle\sum_{-\infty}^{+\infty} M_n e^{in\varphi}, \quad r = r_0; \end{cases} \tag{3}$$

If the functions $\varphi(z)$, $\psi(z)$, $f(z)$ are introduced by series [3]

$$\varphi(z) = \sum_{n=0}^{\infty} a_n z^n, \quad \psi(z) = \sum_{n=0}^{\infty} b_n z^n, \quad f''(z) = \sum_{n=0}^{\infty} c_n z^n,$$

then the solutions of this system (3) have the following forms:

$$\begin{aligned} a_0 &= -\frac{1}{r_0}(\bar A_0 - i\bar B_0); \\ a_1 &= \frac{4(\lambda+2\mu)}{\lambda+3\mu}\frac{1}{1+r_0^2}[A_1 - iB_1 + \frac{2\rho}{\mu}M_1]; \\ a_n &= \frac{2(\lambda+2\mu)}{\lambda+3\mu}\frac{1}{n}\left\{\left[\frac{(n+1)(n+(n-2)r_0^2)}{r_0^{n+1}} + \frac{2r_0^{3-n}}{1+r_0^2}\right](A_n + iB_n)\right. \\ &+ \left.\frac{2r_0^{1-n}}{1+r_0^2}(A_n - iB_n) + \frac{4\rho}{\mu}\frac{r_0^{1-n}}{1+r_0^2}M_n\right\}, \quad n \ge 2; \\ b_0 &= -\frac{1}{(1+r_0^2)^2}\left\{\left[1 + \frac{r_0^2(5\lambda+11\mu)}{\lambda+3\mu}\right](A_1 - iB_1) + \frac{8\rho}{\mu}\frac{\lambda+2\mu}{\lambda+3\mu}r_0^2 M_1\right\}; \\ b_n &= -\left\{\left[\frac{(n+2)(n+1+(n-1)r_0^2)}{2r_0^{n+2}} + \frac{r_0^{2-n}}{1+r_0^2}\right](A_{n+1} + iB_{n+1})\right. \\ &+ \left.\frac{r_0^{-n}}{1+r_0^2}(A_{n+1} - iB_{n+1})\right\}\left[1 + \frac{4}{n+1}\frac{\lambda+2\mu}{\lambda+3\mu}\frac{r_0^2}{1+r_0^2}\right] \end{aligned}$$

$$\begin{aligned} &- \frac{8\rho}{\mu(n+1)}\frac{\lambda+2\mu}{\lambda+3\mu}\frac{r_0^{2-n}}{(1+r_0^2)^2}M_{n+1}, \quad n \geq 1; \\ c_0 &= -\frac{3(1+r_0^2)}{r_0^3}(A_2+iB_2); \\ c_n &= -\frac{n+2}{2}\frac{1+r_0^2}{r_0^{n+3}}[(n+2)^2-1](A_{n+2}+iB_{n+2}); \quad n \geq 1. \end{aligned}$$

Let us the boundary conditions are equal to constans on the boundary points

$$\begin{cases} u_{(l)} + iu_{(s)} = p_1 + ip_2, \quad r = r_0, \\ T_{(ln)} = q, \quad r = r_0. \end{cases} \tag{4}$$

The solutions of the system (4) have the following forms:

$$\begin{aligned} a_0 &= -\frac{1}{r_0}(\bar{p}_1 - i\bar{p}_2), \\ a_n &= 0, \quad n \geq 1, \\ b_n &= 0, \quad n \geq 0, \\ c_n &= 0, \quad n \geq 0. \end{aligned}$$

In this case for the holomorpic functions $\varphi(z)$, $\psi(z)$, $f(z)$ we have

$$\varphi(z) = -\frac{1}{r_0}(\bar{p}_1 - i\bar{p}_2), \quad \psi(z) = 0, \quad f''(z) = 0.$$

For the components of the stresses and displacements we obtain:

$$\begin{aligned} T_{(ll)} &= \frac{\lambda+\mu}{\lambda+2\mu}\frac{1}{r_0}q + \frac{\mu}{\rho}\frac{r^2}{r_0}p_1, \\ T_{(ls)} &= \frac{\mu}{\rho}\frac{r^2}{r_0}p_2, \\ T_{(ln)} &= \frac{r}{r_0}q, \\ u_{(l)} &= \frac{r}{r_0}p_1, \\ u_{(s)} &= \frac{r}{r_0}p_2, \\ u_3 &= \frac{\rho}{2\mu r_0}\frac{\lambda+3\mu}{\lambda+2\mu}q, \end{aligned}$$

where

$$\sigma_{(ll)} = \frac{1}{1+\frac{x^3}{\rho}}T_{(ll)}, \quad \sigma_{(ls)} = \frac{1}{1+\frac{x^3}{\rho}}T_{(ls)}, \quad \sigma_{(ln)} = \frac{1}{1+\frac{x^3}{\rho}}T_{(ln)}.$$

Therefore, three-dimensional mixed boundary value problem for the spherical segment has been solved.

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