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NON-STATIONERY PROBLEM OF TRANSVERSELY DISPLACED CRACK PROPAGATION IN AN INFINITE ELASTIC COMPOUND ZONE

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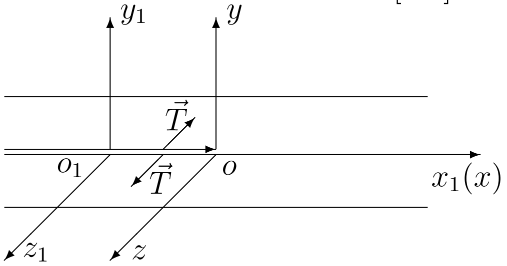
In this article the solution of the problem of a half-infinite crack propagation in a compound elastic zone is considered. The crack extends with constant speed $\vec{v}$ along the x-axis, which separates two phases with different elastic characteristics. An antisymmetric tangent forces, oriented along the z_1-axis, are applied to the edges of the crack. The edges of the zones are free from stresses. The components of displacement along the x_1 and y_1 axises equal to zero, while the component of displacement along the z_1-axis is the function of x_1 and y_1; $u = 0,\ v = 0,\ w = w(x_1, y_1, t)$.

The deformation of this type is anti-plane. The solutions of the problems for infinite area with finite or half-infinite eracks, when the crack extends with constant or variable speed are considered in the articles [1-3].



The equation of motion for the zones 1 and 2 will be

$$\begin{aligned} \frac{\partial^2 \widetilde{w_1}}{\partial t^2} &= C_{21}^2 \left(\frac{\partial^2 \widetilde{w_1}}{\partial x_1^2} + \frac{\partial^2 \widetilde{w_1}}{\partial y_1^2} \right) \\ \frac{\partial^2 \widetilde{w_2}}{\partial t^2} &= C_{22}^2 \left(\frac{\partial^2 \widetilde{w_2}}{\partial x_1^2} + \frac{\partial^2 \widetilde{w_2}}{\partial y_1^2} \right) \end{aligned} \tag{1}$$

Where $\widetilde{w}$ is the component of displacement along the z_1-axis and the C_{21} and C_{22} are the wave extension speeds accordingly in the zones 1 and 2.

The boundary conditions will be

$$\begin{array}{ll} \widetilde{\tau}_{yz}^{(1)} = -T,\ \widetilde{\tau}_{yz}^{(2)} = T, & y_1 = 0,\ x_1 < \nu t, \\ \widetilde{w}_1 = \widetilde{w}_2, & y_1 = 0,\ x_1 > \nu t, \\ \tau_{zx}^{(1)} = \tau_{zx}^{(2)}, & y_1 = 0,\ -\infty < x_1 < \infty, \\ \tau_{yz}^{(1)} = 0, & y_1 = h,\ -\infty < x_1 < \infty, \\ \tau_{yz}^{(2)} = 0, & y_1 = -h,\ -\infty < x_1 < \infty, \end{array} \tag{2}$$

Let us assume that the $\vec{T}$ forces are acting along the edges of the crack. Now we consider equations (1) and (2) in the moving coordinate system

$$y = y_1,\ x = x_1 - \nu t \tag{3}$$

and introduce the new functions $w_1(x,y,t)=\widetilde{w}_1(x_1,y_1,t_1);\; w_2(x,y,t)=\widetilde{w}_2(x_1,y_1,t_1)$.

Then the equation of motion (1) in the moving coordinate system will be [4]

$$\begin{aligned}
&\left(1-\frac{v^2}{C_{21}^2}\right)\frac{\partial^2 w_1}{\partial x^2}+\frac{\partial^2 w_1}{\partial y^2}=\frac{1}{C_{21}^2}\left(-2\nu\frac{\partial^2 w_1}{\partial x\partial t}+\frac{\partial^2 w_1}{\partial t^2}\right),\\
&\left(1-\frac{v^2}{C_{22}^2}\right)\frac{\partial^2 w_2}{\partial x^2}+\frac{\partial^2 w_2}{\partial y^2}=\frac{1}{C_{22}^2}\left(-2\nu\frac{\partial^2 w_2}{\partial x\partial t}+\frac{\partial^2 w_2}{\partial t^2}\right).
\end{aligned} \tag{4}$$

The boundary conditions (2) are

$$\begin{array}{ll}
\mu_1\dfrac{\partial w_1}{\partial y}-\mu_2\dfrac{\partial w_2}{\partial y}=-2T, & y=0,\; x<0,\\
w_1=w, & y=0,\; x>0,\\
\mu_1\dfrac{\partial w_1}{\partial x}-\mu_2\dfrac{\partial w_2}{\partial x}=0, & y=0,\; -\infty<x<\infty,\\
\dfrac{\partial w_1}{\partial y}=0, & y=h,\; -\infty<x<\infty,\\
\dfrac{\partial w_2}{\partial y}, & y=-h,\; -\infty<x<\infty,
\end{array} \tag{5}$$

we apply the Laplace transform method $\overline{w}=\int_0^\infty w\,e^{-pt}dt$ and the Fourier transform method $\overline{\overline{w}}=\dfrac{1}{\sqrt{2\pi}}\displaystyle\int_{-\infty}^{\infty}\overline{w}\,e^{isx}dx$ to the differential equations (4) and to the boundary conditions (5).

Then the differential equations (4) will be transformed into the equations

$$\frac{d^2\overline{\overline{w}}_1}{dy^2}-\gamma_1^2(s,p)\overline{\overline{w}}=0,\frac{d^2\overline{\overline{w}}_2}{dy^2}-\gamma_2^2(s,p)\overline{\overline{w}}=0, \tag{6}$$

where $\gamma_1^2(s,p)=\dfrac{(p+i\nu s)^2}{C_{21}^2}+s^2;\;\;\gamma_2^2(s,p)=\dfrac{(p+i\nu s)^2}{C_{22}^2}+s^2.$

The boundary conditions will be

$$\begin{array}{ll}
\mu_1\dfrac{d\overline{\overline{w}}_1}{dy}-\mu_2\dfrac{d\overline{\overline{w}}_2}{dy}=-\dfrac{2T}{ips}+\Phi_+(s), & y=0,\\
\overline{\overline{w}}_1-\overline{\overline{w}}_2=\Phi_-(s) & y=0,\\
\mu_1\overline{\overline{w}}_1-\mu_2\overline{\overline{w}}_2=0, & y=0,\\
\dfrac{d\overline{\overline{w}}_1}{dy}=0, & y=h,\\
\dfrac{d\overline{\overline{w}}_2}{dy}=0, & y=-h
\end{array} \tag{7}$$

$\Phi_+(s)$ and Φ_- are the analytic functions accordingly in the upper and lower half-planes.

Substituting general solutions of the equations (6) into the boundary conditions (7) and eliminating the constants, we obtain

$$\Phi_+(s)-\frac{\mu_1\mu_2}{\mu_1-\mu_2}(\gamma_1 th\gamma_1 h+\gamma_2 th\gamma_2 h)\Phi_-(s)=\frac{2T}{ips} \tag{8}$$

We carry out the factorization of the functions [5]

$$E(s)=\frac{\gamma_1 th\gamma_1 h+\gamma_2 th\gamma_2 h}{(k_1+k_2)\sqrt{s^2+p^2}} \tag{9}$$

where $k_1^2=1-\dfrac{\nu^2}{C_{21}^2},\ k_2^2=1-\dfrac{\nu^2}{C_{22}^2}$.

The factorization of the function $E(s)$ comes to the problem of linear conjugation [6]

$$E(s)=\frac{\chi+(s)}{\chi-(s)},\ \ln E(s)=\ln\chi_+(s)-\ln\chi_-(s),\ \ln\chi(s)=\frac{1}{2\pi i}\int_{-\infty}^{\infty}\frac{\ln E(t_1)dt_1}{t_1-s} \tag{10}$$

From the equations (8) we obtain

$$\frac{\mu_1-\mu_2}{\mu_1\mu_2}\frac{\Phi_+(s)}{\chi_+(s)\sqrt{s+pi}}-\frac{\mu_1-\mu_2}{\mu_1\mu_2}\frac{2T}{ips\chi_+(s)\sqrt{s+pi}}=\frac{(k_1+k_2)\sqrt{s-pi}}{\chi_-(s)}\Phi_-(s) \tag{11}$$

In accordance with the theorem of Liouville [5] we obtain

$$\frac{\mu_1-\mu_2}{\mu_1\mu_2}\left(\frac{\Phi_+(s)}{\chi_+(s)\sqrt{s+pi}}-\frac{2T}{ips\chi_+(s)\sqrt{s+pi}}\right)=\frac{c}{s} \tag{12}$$

$$\frac{(k_1+k_2)\sqrt{s-pi}}{\chi_-(s)}\Phi_-(s)=\frac{c}{s} \tag{13}$$

Multiplying the equation (12) by s and moving to the limit as $s\to 0$, we obtain the meaning of C. Finally we obtain

$$\Phi_+(s)=-\frac{2T\chi_+(s)\sqrt{s+pi}}{ips\chi_+(0)\sqrt{s+pi}}+\frac{2T}{ips}$$

$$\Phi_-(s)=-\frac{\mu_1-\mu_2}{\mu_1\mu_2}\frac{2T\chi_-(s)}{ips(k_1+k_2)\chi_+(0)\sqrt{pi}\sqrt{s-pi}}$$

In order to determine stress components in the environment of the point $x=0$, we need first to determine the sort of the mode of function in the environment of the point $s\to\infty$.

We obtain

$$\overline{\overline{\tau}}_{yz}=\frac{T}{ip\sqrt{p}\chi_+(0)}\frac{1}{\sqrt{1+is}}$$

Then the stress component in the environment of the crack tip is

$$\overline{\tau}_{yz}=\frac{T}{ip\sqrt{p}\chi_+(0)}\frac{1}{\sqrt{x}}$$

Where $\chi_+(s)=\dfrac{1}{2\pi i}\displaystyle\int_{-\infty}^{+\infty}\frac{\ln E(t)dt_1}{t_1-s}$, $\mathcal{I}_m(s)>0$.

The determination of $\chi_+(s)$ comes to the calculation of the integral on the finite segment between the branch points and the calculation of residue in the point C_R, which corresponds to the speed of the Rayleigh wave.

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