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TO THE QUESTION OF THE REPRESENTATION OF IMAGE ELEMENTS ON A CODE NET

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Abstract

In this paper the presentation of the image elements on the code net related to the questions of information compressing and coding is discussed. It is obtained the characteristics of the code fragments, which allows arranging them in definite classes for their following automatic recognition.

In this paper, along with questions of image coding, we consider the problem of local invariance and compression, assuming that the following conditions are fulfilled:

1. Information is coded and compressed without any loss (reduction) of initial data.
2. The specific characteristics of the image must organically include the respective criteria and properties of code fragments, which allow us to classify them into separate classes and recognize them automatically by computers.

Similarly to how the image is constructed from combination of totalities of elementary squares (pixels) on visual systems, in the present work four invariant (code) states of segments are discussed, from which elements of image are realized.

For the reason of obviousness, only one k-th layer of multilayer code net is considered which consists of the most elementary (by area) pixels [1-4].

Let us introduce the following notations and definitions: .

Let $\operatorname{Im} F$ be the image of a code fragment F on the code network $\Omega(n^k)$. Then

$$\operatorname{Im} F = \begin{cases} \operatorname{Im} L(g) & \textit{if } \operatorname{Im} F \textit{ is a fragment of the horizontal straight line}(-), \\ \operatorname{Im} L(v) & \textit{if } \operatorname{Im} F \textit{ is a fragment of the vertical straight line}(|), \\ \operatorname{Im} L(d_1) & \textit{if } \operatorname{Im} F \textit{ is a fragment of the diagonal straight line on the right}(/), \\ \operatorname{Im} L(d_2) & \textit{if } \operatorname{Im} F \textit{ is a fragment of the diagonal straight line on the left}(\backslash). \end{cases} \quad (1)$$

Definition 1: The part of the vector *S*, which is bounded by two extreme units on the right and on the left, is called the kernel (ker*S*).

By Definition 1 we have $\dim(\ker S) \le \dim S$, where $\dim(V_i)$ is the dimension of the vector V_i.

Definition 2: The decimal equivalent of the resultant (compressed) vector is called the scalar, Sc(*S*).

It is obvious that $Sc(S) \ge Sc(\ker S)$, and also $Sc(S^{/}) \ge Sc(\ker S^{/})$, where $S^{/}$ is the normalized resultant (compressed) vector.

Definition 3: A distance equal to the number of zeros between the nearest units in the kernel is called an internal step St (ker *S*').

Definition 4: An internal step of the kernel is called uniformly symmetrical if $Sc(St(\ker S^{/})) = const$ and is the same for all pairs of the nearest units in the kernel.

Remark: In the sequel, without any significant loss of generality, we will consider square blocks (packages, cells) of $n = m \times m$ dimension, where *n* is the number of pixels.

The following theorem holds true.

Theorem 1: *If* $\operatorname{Im} F$ *is the straight line on* $\Omega(n^k)$, *then*

$$St(\ker S^{/}) = \begin{cases} 0, & \operatorname{Im} F = \operatorname{Im} L(g) \\ m-2, & \operatorname{Im} F = \operatorname{Im} L(d_1) \\ m-1, & \operatorname{Im} F = \operatorname{Im} L(v) \\ m, & \operatorname{Im} F = \operatorname{Im} L(d_2) \end{cases} \qquad (2)$$

The proof of this theorem follows from the specific characteristics of the structure of a coding net $[1-4]$.

Corollary 1: The kernel of the compressed normalized vector on $\Omega(n^k)$ is uniformly symmetrical. $St(\ker S^{/})$ and $St(\ker S)$ completely describe the kind (parallel, intersecting, perpendicular and so on) of straight lines or their segments.

Corollary 2: The images of the straight lines $L(g)$, $L(d_1)$, $L(v)$, $L(d_2)$ are the scalars of these straight lines to within an isomorphism, i.e.,

$$\begin{aligned} &\operatorname{Im} L(g) \to Sc(\ker L(g)) = \bar{S}_g, \\ &\operatorname{Im} L(d_1) \to Sc(\ker L(d_1)) = \bar{S}_{d1}, \\ &\operatorname{Im} L(v) \to Sc(\ker L(v)) = \bar{S}_v. \\ &\operatorname{Im} L(d_2) \to Sc(\ker L(d_2)) = \bar{S}_{d2}. \end{aligned} \qquad (3)$$

Respectively, we have

$$\begin{aligned} &Sc(St(\ker S_g^{/})) = \bar{S}_g^{/} \le \bar{S}_g, \\ &Sc(St(\ker S_{d1}^{/})) = \bar{S}_{d1}^{/} \le \bar{S}_{d1}, \\ &Sc(St(\ker S_v^{/})) = \bar{S}_v^{/} \le \bar{S}_v. \\ &Sc(St(\ker S_{d2}^{/})) = \bar{S}_{d2}^{/} \le \bar{S}_{d2}. \end{aligned} \qquad (4)$$

Definition 5: A sequence of sets of kernels S_g, S_{d1}, S_v, S_{d2} or $S^{/}{}_g$, $S^{/}{}_{d1}$, $S^{/}{}_v$, $S^{/}{}_{d2}$, which maps a given two-dimensional image to within an isomorphism, is called a proper spectrum of this image.

Theorem 1 and Definition 5 give rise to

Corollary: On $\Omega\left(n^k\right)$, the image contour has a unique proper spectrum.

Definition 6: A spectrum, possessing among other possible invariant spectra, a minimal sum of scalar kernels, is called canonical.

Definition 7: A kernel, possessing a minimal scalar value, is called an essential kernel of the spectrum.

It is obvious that for such kernels

$$St\left(\ker S^{/}\right)=0. \tag{5}$$

Let us introduce the notion of a cyclic permutation function σ, and let $P=\{P_1,P_2,P_3,P_4\}$, where $P_1=m,\quad P_2=m-1,\quad P_3=m-2,\quad P_4=0.$

By Corollary 1 of Theorem 1, between the elements of the set P we can establish a regular transformation law

$$\sigma: \quad P\Rightarrow P \tag{6}$$

which is a particular case of Peano's successor function [5]

$$\sigma(i)=m-i \quad i=0,1,2,\cdots,m\,, \tag{7}$$

$$\sigma: P_1\Rightarrow P_2,\quad P_2\Rightarrow P_3,\quad P_3\Rightarrow P_4,\quad P_4\Rightarrow P_1. \tag{8}$$

The following theorem holds true.

Theorem 2: *The spectrum of an image, which corresponds to some of its invariant states, can always be reduced to the canonical form by means of the function* σ.

This theorem is proved using Theorem 1 and its corollaries.

Corollary: The image is recognized, i.e., reduced to the symmetrical canonical form as a result of a cyclic permutation number $t\le 4$.

Example: Let $n=9$ and $U_0=000001100$ be the base-vector for the package (cell) on which the respective matrix is stretched:

$$\begin{pmatrix}000001101\\000001110\\000001000\\000000100\\000011100\\000101100\\001001100\\010001100\\100001100\end{pmatrix}\quad\begin{array}{c}V_1\\V_2\\V_3\\V_4\\V_5\\V_6\\V_7\\V_8\\V_9\end{array}$$

A fragment of the coding network for this package is shown in Fig. 1.

Fig. 1 Fig. 2

We have the following four cases:

1. The resultant vector mapping the horizontal straight line (denoted by $L(g)$ in Fig. 2) is equal to $S_g = 000001011.$ After normalization, it becomes $S_g^{/} = 000000111.$
2. Therefore $Sc\left(\ker S_g^{/}\right) = 7, \quad St\left(\ker S_g^{/}\right) = 0.$

Analogously, we write:

2. For the right diagonal straight line (denoted by $L(d_1)$ in Fig. 2)

$S_{d1} = 001011000, \quad S_{d1}^{/} = 001010100, \quad Sc\left(\ker S_{d1}^{/}\right) = 21, \quad St\left(\ker S_{d1}^{/}\right) = 1.$

3. For the vertical straight line (denoted by $L(v)$ in Fig. 2)

$S_v = 100101000, \; S_v^{/} = 100100100, \; Sc\left(\ker S_v^{/}\right) = 73, \; St\left(\ker S_v^{/}\right) = 2.$

4. For the left diagonal (denoted by $L(d_2)$ in Fig. 2)

$S_{d2}^{/} = 100010001, \; Sc\left(\ker S_{d2}^{/}\right) = 273, \; St\left(\ker S_{d2}^{/}\right) = 3.$

Therefore

$$\bar{\sigma}: \quad St\left(\ker S_g^{/}\right) \Rightarrow St\left(\ker S_{d1}^{/}\right),$$
$$\bar{\sigma}: \quad St\left(\ker S_{d1}^{/}\right) \Rightarrow St\left(\ker S_v^{/}\right),$$
$$\bar{\sigma}: \quad St\left(\ker S_v^{/}\right) \Rightarrow St\left(\ker S_{d2}^{/}\right),$$
$$\bar{\sigma}: \quad St\left(\ker S_{d2}^{/}\right) \Rightarrow St\left(\ker S_g^{/}\right).$$

By Definition 5, the compressed spectrum of straight lines with horizontal trajectory is canonical. In our example $Sc\left(\ker S_g^{/}\right) = 7,$ in one package (cell).

The considered invariant state and transformation are general for a block, a package, a cell, i.e., they do not depend on a scale of the coding net geometry [3,4].

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