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ON APPLICATION OF THE METHOD OF A SMALL PARAMETER FOR THE NON-LINEAR THEORY OF SHALLOW SHELLS

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1. A complete system of non-linear of equation of the three-dimensional shallow shell-type elastic bodies can be written as [1]:

a) Equilibrium equations have forms:

$$\begin{cases} \nabla_\alpha \sigma^{\alpha\beta} - b_\alpha^\beta \sigma^{\alpha 3} + \partial_3 \sigma^{3\beta} + \Phi^\beta = 0, \\ \nabla_\alpha \sigma^{\alpha 3} + b_{\alpha\beta} \sigma^{\alpha\beta} + \partial_3 \sigma^{33} + \Phi^3 = 0, \end{cases} \tag{1}$$

$$\left(\alpha, \beta = 1,2; \quad \partial_3 = \frac{\partial}{\partial x_3} \right),$$

where $\sigma^{ij} = \boldsymbol{\sigma}^i \mathbf{r}^j$ $(i, j = 1,2,3)$ are contra variant components of the stress tensor, $b_{\alpha\beta} (b^{\alpha\beta}, b_\alpha^\beta)$ -covariant (contra variant, mixed) components of the curvature tensor of the mid surface S of the shell Ω, $\Phi^i (i = 1,2,3)$ are contra variant components of the external force, $\nabla_\alpha (\alpha = 1,2)$ - symbols of covariant derivatives on the surface S, x_3 is a thickness coordinate. Then $-h \le x_3 \le h$ and h is a semi-thickness of the shell Ω.

b) The equation of state (dependence between the stress σ^{ij} and the strain e_{ij} tensors) takes the form [2]

$$\sigma^{ij} = E^{ikpq} e_{pq} (a_k^i + \mathbf{r}^j \partial_k \mathbf{U}), \tag{2}$$

where $e_{pq} = \frac{1}{2} (\mathbf{r}_q \partial_p \mathbf{U} + \mathbf{r}_p \partial_q \mathbf{U} + \partial_p \mathbf{U} \partial_q \mathbf{U})$ are covariant components of the strain tensor, $\mathbf{r}_i$ and $\mathbf{r}^i$ are covariant and contra variant base vectors of mid surface S, moreover $\mathbf{r}^3 = \mathbf{r}_3 = \mathbf{n}$ is a normal of S, $\mathbf{U}$ is the displacement vector, E^{ikpq} -contra variant components of the elasticity tensor, defined by the formula

$$E^{ikpq} = \lambda a^{ik} a^{pq} + \mu (a^{ip} a^{kq} + a^{iq} a^{kp})$$

$$(a^{ij} = \mathbf{r}^i \mathbf{r}^j, a_k^j = \mathbf{r}_k \mathbf{r}^j).$$

c) The stress vector $\sigma_{(l)}$, acting on the lateral surface with tangential normal $\mathbf{l}$ has the form

$$\boldsymbol{\sigma}_{(\mathbf{l})} = \sigma_{(ll)}\mathbf{l} + \sigma_{(ls)}\mathbf{s} + \sigma_{(ln)}\mathbf{n} \Rightarrow$$

$$\sigma_{(ll)} = \sigma^{\alpha\beta} l_\alpha l_\beta, \quad \sigma_{(ls)} = \sigma^{\alpha\beta} l_\alpha s_\beta, \quad \sigma_{(ln)} = \sigma^{\alpha 3} l_\alpha \tag{3}$$

$$(l_\alpha = \mathbf{l}\mathbf{r}_\alpha, \quad s_\alpha = \mathbf{s}\mathbf{r}_\alpha, \quad \mathbf{l}\times\mathbf{s} = \mathbf{n})$$

2. There exist several methods of reduction of the three-dimensional problems to the two-dimensional one (Kirchoff-Love, E. Reissner, K. Fridrixs, A. Green, A. Goldenveizer, I. Vorovich, I. Vekua, etc).

Following I. Vekua [1] we assume the validity of the expansions

$$\left(\sigma^{ij}, U^i, \Phi^i\right) = \sum_{m=0}^{\infty} \left(\overset{(m)}{\sigma}{}^{ij}, \overset{(m)}{U}{}^i, \overset{(m)}{\Phi}{}^i\right) P_m\left(\frac{x_3}{h}\right) \qquad \Rightarrow$$

$$\left(\overset{(m)}{\sigma}{}^{ij}, \overset{(m)}{U}{}^i, \overset{(m)}{\Phi}{}^i\right) = \frac{2m+1}{h}\int_{-h}^{h}\left(\sigma^{ij}, U^i, \Phi^i\right) P_m\left(\frac{x_3}{h}\right) dx_3 \tag{4}$$

where P_m are Legendre polynomials of order m.

Substituting the above expansions in the relations (1),(2) and (3), having satisfied beforehand the conditions of the face surfaces $x_3 = \pm h$

$$\boldsymbol{\sigma}^3(x^1, x^2, \pm h) = \overset{(\pm)}{\boldsymbol{\sigma}}{}^3,$$

we obtain the following infinitive complete system of two-dimensional equations [3]:

a) Equilibrium equations take the form

$$\begin{cases} h\nabla_\alpha \overset{(m)}{\sigma}{}^{\alpha\beta} - \varepsilon R b_\alpha^\beta \overset{(m)}{\sigma}{}^{\alpha\beta} - (2m+1)\left(\overset{(m-1)}{\sigma}{}^{3\beta} + \overset{(m-3)}{\sigma}{}^{3\beta} + \ldots\right) + h \overset{(m)}{F}{}^\beta = 0, \\ h\nabla_\alpha \overset{(m)}{\sigma}{}^{\alpha 3} + \varepsilon R b_\alpha^\beta \overset{(m)}{\sigma}{}^{\alpha 3} - (2m+1)\left(\overset{(m-1)}{\sigma}{}^{33} + \overset{(m-3)}{\sigma}{}^{33} + \ldots\right) + h \overset{(m)}{F}{}^3 = 0, \\ (m = 0,1,\ldots) \end{cases} \tag{5}$$

where $\varepsilon = \dfrac{h}{r}$ is a small parameter, with characteristic radius R of curvature; then

$$\overset{(m)}{\mathbf{F}} = \overset{(m)}{\boldsymbol{\Phi}} + \frac{2m+1}{2h}\left(\overset{(+)}{\boldsymbol{\sigma}}{}^3 - (-1)^m \overset{(-)}{\boldsymbol{\sigma}}{}^3\right).$$

b) The equation of the state can be written as []

$$h^3 \overset{(m)}{\sigma}{}^{ij} = E^{ikpq}\Big\{h^2 (h\nabla_p \overset{(m)}{U}_q - \varepsilon R \overset{(m)}{\mathbf{U}} \cdot \nabla_p \mathbf{r}_q) a_k^j + h \sum_{m_1, m_2 = 0}^{\infty} \Big[h(\nabla_p \overset{(m_{11})}{U}_q - \varepsilon R \overset{(m_1)}{\mathbf{U}} \cdot \nabla_p \mathbf{r}_q) \times$$

$$\times (h\nabla_k \overset{(m_2)}{U}{}^j - \varepsilon R \overset{(m_2)}{\mathbf{U}} \cdot \nabla_k \mathbf{r}^j) + \frac{1}{2}(h\nabla_p \overset{(m_1)}{U}_{p_1} \mathbf{r}^{p_1} + \varepsilon R \overset{(m_1)}{U}_{p_1} \nabla_p \mathbf{r}^{p_1}) \times$$

$$\times(h\nabla_q \overset{(m_2)}{U}_{q_1}\cdot \mathbf{r}^{q_1} + \varepsilon R \overset{(m_2)}{U}_{q_1}\cdot \mathbf{r}^{q_1})a_k^j \Big] I_m^{m_1m_2} + \frac{1}{2}\sum_{m_1,m_2,m_3=0}^{\infty}(h\nabla_p \overset{(m_{1_1})}{U}_{p_1}\mathbf{r}^{p_1} + \varepsilon R \overset{(m_1)}{\mathbf{U}}\cdot\nabla_q \mathbf{r}^{q_1})\times$$

$$\times(h\nabla_k \overset{(m_2)}{U}_{q_1}\mathbf{r}^{q_1} + \varepsilon R \overset{(m_2)}{U}_{q_1}\cdot\nabla_q \mathbf{r}^{q_1})(h\nabla_k \overset{(m_3)}{U}{}^j - R \overset{(m_3)}{\mathbf{U}}\cdot\nabla_k \mathbf{r}^j) I_m^{m_1m_2m_3} \Big\} \quad (6)$$

where

$$\nabla_p \overset{(m)}{U}_q = \begin{cases} \nabla_p \overset{(m)}{U}_q, & p=\alpha \\ \nabla_3 \overset{(m)}{U}_q = \dfrac{2m+1}{h}\left(\overset{(m+1)}{U}_q + \overset{(m+3)}{U}_q + \ldots\right), & p=3; \end{cases}$$

$$\nabla_p \mathbf{r}_q = \begin{cases} \nabla_\alpha \mathbf{r}_\beta = b_{\alpha\beta}\mathbf{n}, & p=\alpha,\ q=\beta, \\ \nabla_\alpha \mathbf{n} = -b_\alpha^\beta \mathbf{r}_\beta, & p=\alpha,\ q=3, \\ \nabla_3 \mathbf{r}_q = 0, & p=3; \end{cases}$$

$$I_m^{m_1m_2} = \frac{2m+1}{2h}\int_{-h}^{h} P_{m_1}P_{m_2}P_m dx_3 = \sum_{r=0}^{\min(m_1,m_2)} \alpha_{m_1m_2r}\delta_m^{m_1m_2-2r}, \qquad \left(\delta_m^n = \begin{cases} 0, & m\neq n \\ 1, & m=n \end{cases}\right)$$

$$I_m^{m_1m_2m_3} = \frac{2m+1}{2h}\int_{-h}^{h} P_{m_1}P_{m_2}P_{m_3}P_m dx_3 = \sum_{r_1=0}^{\min(m_1,m_2)} \alpha_{m_1m_2r_1} \sum_{r_2=0}^{\min(m_1,m_2m_3)} \alpha_{m_1+m_2-2r_1,m_3,r_2}\delta_m^{m_1+m_2+m_3-2(r_1+r_2)},$$

$$\alpha_{mnr} = \frac{A_{m-r}A_rA_{n-r}}{A_{m+n-r}}\frac{2(m+n)-4r+1}{2(m+n)-2r+1}, \quad A_m = \frac{1\cdot 3\cdots 2m-1}{m!}.$$

c) The vector $\sigma_{(l)}$ has the form

$$\overset{(m)}{\sigma}_{(ll)} = \overset{(m)}{\sigma}{}^{\alpha\beta} l_\alpha l_\beta, \quad \overset{(m)}{\sigma}_{(ls)} = \overset{(m)}{\sigma}{}^{\alpha\beta} l_\alpha s_\beta, \quad \overset{(m)}{\sigma}_{(ln)} = \overset{(m)}{\sigma}{}^{\alpha 3} l_\alpha n. \quad (7)$$

3. The passage to the finite systems is performed by considering a finite expansions (4), where m=0,1,....N (N=0,1,...).

Another basic assumption in the validity of the following expansions:

$$\left(\overset{(m)}{\sigma}{}^{ij}, \overset{(m)}{U}{}^i, \overset{(m)}{F}{}^i\right) = \sum_{n=1}^{\infty}\left(\overset{(m,n)}{\sigma}{}^{ij}, \overset{(m,n)}{U}{}^i, \overset{(m,n)}{F}{}^i\right)\varepsilon^n . \quad (8)$$

Substituting expansions (8) in the equations (5) and (6), we obtain the following system of equations of the equilibrium:

$$\begin{cases} h\nabla_\alpha \overset{(m,n)}{\sigma^{\alpha\beta}} - (2m+1)\left(\overset{(m-1,n)}{\sigma^{3\beta}} + \overset{(m-3,n)}{\sigma^{3\beta}} + \cdots\right) = -h\overset{(m,n)}{F^{\beta}} + Rb_\alpha^\beta \overset{(m,n-1)}{\sigma^{\alpha 3}}, \\ h\nabla_\alpha \overset{(m,n)}{\sigma^{\alpha 3}} - (2m+1)\left(\overset{(m-1,n)}{\sigma^{33}} + \overset{(m-3,n)}{\sigma^{33}} + \cdots\right) = -h\overset{(m,n)}{F^{3}} - Rb_{\alpha\beta} \overset{(m,n-1)}{\sigma^{\alpha\beta}}, \\ (m = 0,1,\dots,N;\ n = 1,2,\dots), \end{cases} \tag{9}$$

where

$$\begin{aligned} h^3 \overset{(m,n)}{\sigma^{ij}} = E^{ikpq}\Bigg\{ & h^2(h\nabla_p \overset{(m,n)}{U_q} - R\overset{(m,n-1)}{\mathbf{U}} \cdot \nabla_p \mathbf{r}_q) a_k^j + h\sum_{m_1,m_2=0}^{N}\sum_{n_1=1}^{n-1}\Bigg[h(\nabla_p \overset{(m_1,n_1)}{U_q} - R\overset{(m_1,n_1-1)}{\mathbf{U}} \cdot \nabla_p \mathbf{r}_q) \times \\ & \times(\nabla_k \overset{(m_1,n-n_1)}{U^j} - R\overset{(m_2,n-n_1-1)}{\mathbf{U}} \cdot \nabla_k \mathbf{r}^j) + \frac{1}{2}(h\nabla_p \overset{(m_1,n_1)}{U_{p_1}} \mathbf{r}^{p_1} + R\overset{(m_1,n_1-1)}{U_{p_1}} \nabla_p \mathbf{r}^{p_1}) \times \\ & \times(h\nabla_q \overset{(m_2,n-n_1)}{U_{q_1}} \cdot \mathbf{r}^{q_1} + R\overset{(m_2,n-n_1-1)}{U_{q_1}} \cdot \mathbf{r}^{q_1}) a_k^j \Bigg] I_m^{m_1 m_2} + \\ & + \frac{1}{2}\sum_{m_1,m_2,m_3=0}^{N}\sum_{n_1=1}^{n-n_2-1}\sum_{n_2=1}^{n-2}(h\nabla_p \overset{(m_1,n_1)}{U_{p_1}} \mathbf{r}^{p_1} + R\overset{(m_1,n_1-1)}{U_{p_1}} \nabla_p \mathbf{r}^{p_1}) \times \\ & \times(h\nabla_q \overset{(m_2,n-n_1-n_2)}{U_{q_1}} \mathbf{r}^{q_1} + R\overset{(m_2,n-n_1-n_2-1)}{U_{q_1}} \cdot \nabla_q \mathbf{r}^{q_1})(h\nabla_k \overset{(m_3,n_2)}{U^j} - R\overset{(m_3,n_2-1)}{\mathbf{U}} \cdot \nabla_k \mathbf{r}^j) I_m^{m_1 m_2 m_3} \Bigg\} \end{aligned} \tag{10}$$

$$\left(\overset{(m,-k)}{\mathbf{U}} = 0,\ \ k = 0,1,\dots, \text{or } m > N\right).$$

Approximation of order N=0: By introducing the following notations

$$\overset{(0,n)}{U^i} = \overset{(n)}{u^i},\quad \overset{(0,n)}{\sigma^{ij}} = \overset{(n)}{T^{ij}},\quad \overset{(0,n)}{F^j} = \overset{(n)}{X^j},$$

we obtain the equation of equilibrium for approximation of order N=0:

$$\begin{cases} h\nabla_\alpha \overset{(n)}{T^{\alpha\beta}} = -h\overset{(n)}{X^{\beta}} + Rb_\alpha^\beta \overset{(n-1)}{T^{\alpha 3}}, \\ h\nabla_\alpha \overset{(n)}{T^{\alpha 3}} = -h\overset{(n)}{X^{3}} - Rb_{\alpha\beta} \overset{(n-1)}{T^{\alpha\beta}}, \end{cases} \tag{11}$$

where

$$\begin{aligned} h^3 \overset{(n)}{T^{ij}} = E^{ikpq}\Bigg\{ & h^2(h\nabla_p \overset{(n)}{u_q} - R\overset{(n)}{\mathbf{u}} \cdot \nabla_p \mathbf{r}^q) a_k^j + h\sum_{n_1=1}^{n-1}\Bigg[h(\nabla_p \overset{(n_1)}{u_q} - R\overset{(n_1-1)}{\mathbf{u}} \cdot \nabla_p \mathbf{r}_q) \times \\ & \times(h\nabla_k \overset{(n-n_1)}{u^j} - R\overset{(n-n_1-1)}{\mathbf{u}} \cdot \nabla_k \mathbf{r}^j) + \frac{1}{2}(h\nabla_p \overset{(,n_1)}{u_{p_1}} \mathbf{r}^{p_1} + R\overset{(n_1-1)}{u_{p_1}} \nabla_p \mathbf{r}^{p_1}) \times \\ & \times(h\nabla_q \overset{(n-n_1)}{u_{q_1}} \cdot \mathbf{r}^{q_1} + R\overset{(n-n_1-1)}{u_{q_1}} \cdot \mathbf{r}^{q_1}) a_k^j \Bigg] + \frac{1}{2}\sum_{n_1=1}^{n-n_2-1}\sum_{n_2=1}^{n-2}(h\nabla_p \overset{(n_1)}{u_{p_1}} \mathbf{r}^{p_1} + R\overset{(n_1-1)}{u_{p_1}} \nabla_p \mathbf{r}^{p_1}) \times \end{aligned} \tag{12}$$

$$\times(h\nabla_q \overset{(n-n_1-n_2)}{u_{q_1}} \mathbf{r}^{q_1} + R \overset{(n-n_1-n_2-1)}{u_{q_1}} \cdot \nabla_q \mathbf{r}^{q_1})(h\nabla_k \overset{(n_2)}{u^j} - R \overset{(,n_2-1)}{\mathbf{u}} \cdot \nabla_k \mathbf{r}^j)\Big\}.$$

Substituting (12) in (11) we obtain the system of differential equations in terms of the displacement vector components

$$\begin{cases} h\nabla_\alpha \nabla^\alpha \overset{(n)}{u_\beta} + (\lambda+\mu)\nabla_\beta \overset{(n)}{\theta} = \overset{(n)}{M_\beta}\left(\overset{(1)}{\mathbf{u}}, \overset{(2)}{\mathbf{u}},..., \overset{(n-1)}{\mathbf{u}}\right), \\ h\nabla_\alpha \nabla^\alpha \overset{(n)}{u_3} = \overset{(n)}{M_3}\left(\overset{(1)}{\mathbf{u}}, \overset{(2)}{\mathbf{u}},..., \overset{(n-1)}{\mathbf{u}}\right), \end{cases} \tag{13}$$

$$\left(\overset{(n)}{\theta} = \nabla_\alpha \overset{(n)}{u^\alpha},\ \nabla^\alpha = a^{\alpha\beta}\nabla_\beta,\ \ n = 1,2,...\right),$$

which is isometric coordinates $(ds^2 = \Lambda(z,\bar{z})dzd\bar{z},\ \ z = x^1 + ix^2)$ written in a complex form looks as follows:

$$\begin{cases} 4\mu \dfrac{\partial}{\partial \bar{z}}\left(\dfrac{1}{\lambda}\dfrac{\partial \overset{(n)}{u_+}}{\partial z}\right) + 2(\lambda+\mu)\dfrac{\partial \overset{(n)}{\theta}}{\partial \bar{z}} = \overset{(n)}{M_+}, \\ 4\mu \dfrac{1}{\Lambda}\dfrac{\partial^2 \overset{(n)}{u_3}}{\partial z \partial \bar{z}} = \overset{(n)}{M_3}, \end{cases} \tag{14}$$

$$\left(\overset{(n)}{u_+} = \overset{(n)}{u_1} + i\overset{(n)}{u_2},\quad \overset{(n)}{\theta} = \frac{1}{\Lambda}\left(\frac{\partial \overset{(n)}{u_+}}{\partial z} + \frac{\partial \overline{\overset{(n)}{u_+}}}{\partial \bar{z}}\right),\qquad \overset{(n)}{M_+} = \overset{(n)}{M_1} + i\overset{(n)}{M_2},\right.$$

$$\left. 2\frac{\partial}{\partial z} = \frac{\partial}{\partial x^1} - i\frac{\partial}{\partial x^2},\quad 2\frac{\partial}{\partial \bar{z}} = \frac{\partial}{\partial x^1} + i\frac{\partial}{\partial x^2}\right)$$

The general solution of the homogeneous system (14) is expressed by three arbitrary analytic functions of z:

$$\begin{cases} \overset{(n)}{u_+} = \dfrac{\lambda+3\mu}{\lambda+\mu}\left[\Lambda(z,\bar{z})\varphi(z) + \dfrac{1}{\pi}\iint_G \dfrac{\partial\Lambda}{\partial\zeta}\dfrac{\varphi'(\zeta)d\xi d\eta}{\bar{\zeta}-\bar{z}}\right] + \left(\dfrac{1}{\Lambda}\iint_G \dfrac{\Lambda d\xi d\eta}{\bar{\zeta}-\bar{z}}\right)\overline{\varphi'(z)} - \overline{\psi(z)}, \\ \overset{(n)}{u_3} = f(z) + \overline{f(z)},\quad (\zeta = \xi_1 + i\eta \in G). \end{cases} \tag{15}$$

Approximation of order N=1: By introducing the following notations

$$\overset{(0,n)}{U^i} = \overset{(n)}{u^i},\ \overset{(1,n)}{U^i} = \overset{(n)}{v^i},\ \overset{(0,n)}{\sigma^{ij}} = \overset{(n)}{T^{ij}},\ \overset{(1,n)}{\sigma^{ij}} = \overset{(n)}{S^{ij}},$$

we obtain the system of equations in a complex form:

$$\begin{cases} 4\mu h^2 \dfrac{\partial}{\partial \bar{z}}\left(\dfrac{1}{\lambda}\dfrac{\partial \overset{(n)}{u_+}}{\partial z}\right) + 2(\lambda+\mu)h^2\dfrac{\partial \overset{(n)}{\theta}}{\partial \bar{z}} + 2\lambda h\dfrac{\partial \overset{(n)}{v_3}}{\partial \bar{z}} = \overset{(n)}{N_+}, \\ 4\mu h^2 \dfrac{1}{\Lambda}\dfrac{\partial^2 \overset{(n)}{v_3}}{\partial z\partial \bar{z}} - 3\left[\Lambda h\overset{(n)}{\theta} + (\lambda+2\mu)\overset{(n)}{v_3}\right] = \overset{(n)}{N_3}, \end{cases} \tag{16}$$

$$\begin{cases} 4\mu h^2 \dfrac{\partial}{\partial \bar{z}}\left(\dfrac{1}{\lambda}\dfrac{\partial \overset{(n)}{v}_+}{\partial z}\right) + 2(\lambda+\mu)h^2 \dfrac{\partial \overset{(n)}{\rho}}{\partial \bar{z}} - 3\mu\left(2\lambda h \dfrac{\partial \overset{(n)}{u}_3}{\partial \bar{z}} + \overset{(n)}{v}_+\right) = \overset{(n)}{Q}_+, \\ \mu h^2\left(\dfrac{4h}{\Lambda}\dfrac{\partial^2 \overset{(n)}{u}_3}{\partial z \partial \bar{z}} + \overset{(n)}{\rho}\right) = \overset{(n)}{Q}_3, \end{cases} \tag{17}$$

$$\left(\overset{(n)}{v}_+ = \overset{(n)}{v}_1 + i\overset{(n)}{v}_2, \quad \overset{(n)}{\rho} = \frac{1}{\Lambda}\left(\frac{\partial \overset{(n)}{v}_+}{\partial z} + \frac{\partial \overline{\overset{(n)}{v}_+}}{\partial \bar{z}}\right)\right).$$

The general solution of the homogeneous systems (16) and (17) are expressed as follows

$$\begin{cases} \overset{(n)}{u}_+ = -\dfrac{\lambda h}{6(\lambda+\mu)}\dfrac{\partial \omega}{\partial \bar{z}} - \dfrac{5\lambda+6\mu}{3\lambda+2\mu}\dfrac{1}{\pi}\iint\limits_G \dfrac{\Lambda \varphi'(\zeta) d\xi d\eta}{\bar{\zeta}-\bar{z}} + \left(\dfrac{1}{\pi}\iint\limits_G \dfrac{\Lambda d\xi d\eta}{\bar{\zeta}-\bar{z}}\right)\overline{\varphi'(z)} - \overline{\psi(z)}, \\ \overset{(n)}{v}_3 = \omega - \dfrac{2\lambda h}{3\lambda+2\mu}(\varphi'(z) + \overline{\varphi'(z)}), \end{cases} \tag{18}$$

$$\left(\frac{4}{\Lambda}\frac{\partial^2 \omega}{\partial z \partial \bar{z}} - \frac{3(\lambda+\mu)}{(\lambda+2\mu)h^2}\omega = 0\right),$$

$$\begin{cases} \overset{(n)}{v}_+ = i\dfrac{\partial \chi}{\partial \bar{z}} - 2h\overline{\Psi'(z)} - \dfrac{1}{\pi}\iint\limits_G \dfrac{\Lambda(\Phi'(\zeta) + \overline{\Phi'(\varsigma)}) d\xi d\eta}{\bar{\zeta}-\bar{z}} + \dfrac{4(\lambda+2\mu)h^3}{3\mu}\overline{\Phi''(z)}, \\ \overset{(n)}{u}_3 = \Psi(z) + \overline{\Psi(z)} - \dfrac{1}{\pi}\iint\limits_G \Lambda(\varsigma,\bar{\varsigma})[\Phi'(\zeta) + \overline{\Phi'(z)}]\ln|\varsigma - z| d\xi d\eta. \end{cases} \tag{19}$$

$$\left(\frac{4}{\Lambda}\frac{\partial^2 \chi}{\partial z \partial \bar{z}} - \frac{3}{h^2}\chi = 0\right),$$

where $\varphi(z)$, $\psi(z)$, $\Phi(z)$ and $\Psi(z)$ are analytic functions of z.

REFERENCES

1. I. Vekua N. Shell Theory: General Methods of Construction. Pitman Advanced Publishing Program, Boston-London-Melbourne, 1986.
2. Ciarlet P. G. Three-Dimensional Elasticity. "Mir", Moscow, 1992, (In Russian).
3. Meunargia T. V. The Method of Small Parameter for Non-Linear Sperical Shells. Reports of Enlarged Session of the Seminar of I. Vekua Institute of Applied Mathematics, V. 14, N 2, 1999.