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NUMERICAL INVESTIGATIONS OF SPREADING OIL FILTRATION IN SOILS FOR CAUCASIAN REGION

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Introduction

As known the European Union (EU) is one of the main ideologists and sponsors of the transport corridor Europe-Caucasus-Asia (TRACECA). According to data of European transit countries besides of great political and economical benefits the transit of strategic materials causes great losses to the ecological situation in these countries. Besides of the ordinary pollution of environment there can arise non-ordinary situations as well – accidents on pipes, depositories, possibilities of terrorist attacks and sabotage and oil and oil-products spilling on the territory of Georgia which are followed by the sharp deterioration of the ecological situation in the neighbouring regions. For an example: as known, the Western Export Pipeline Route fulfils a transportation of oil by pipeline from the expanded Sangachal terminal through Georgia to Supsa terminal, which is located between Poti and Batumi. Western Export pipeline almost fully was blasted by terrorist attacks and sabotage in 1991-1992. Almost all content of oil in the pipes was spilled. Since 1998 to 2000 80% of Baku-Supsa Western Export Pipeline Route pipeline was restored and 20% - was constructed all over again [1-3]. Thus investigation and assessment of environmental pollution along the TRACEKA route is one of the urgent problems of the present days.

1. Environmental Baseline of Georgian Section of Baku-Tbilisi- Ceyhan Pipeline Route.

Gas and oil transportation by pipeline from the Caspian Sea to Turkey via Azerbaijan and Georgia was defined as the most acceptable commercial and environmental solution. Pipeline is generally considered to be most cost effective environmentally safest of transporting hydrocarbons and the route through Georgia was found to be commercially competitive. Baku-Tbilisi-Ceyhan oil pipeline will transport up 50 million ton fresh oil from the Caspian Sea to Ceyhan on the Mediterranean coast in Turkey [3].

Pipeline trenches will be excavated to a nominal depth of 2.2m. This will vary according to the severity of the terrain and local topography in order to ensure, that the pipeline is buried with a minimum depth of cover of 1m in soil and 0.6m in rock. Deeper installation will be required at river, road, rail and other crossings [3]. But there are able to take place non ordinary situations too.

As foreign practice of pipeline exploitation shows, that the main reason of crashes and spillages (and fires as a consequence) are destruction of pipes as a result of corrosion, defects of welding, natural phenomena and so on (including terrorist attacks and sabotage).

Ground water along the route is also abundant and generally of high quality. The eastern part of the proposed route is characterized by a shallow water table and by localized poor quality, owing to either high salinity, biological and chemical contamination. The central part of the route is characterized by drinking water used by the local population as the main source of water supply. The western part of the route is characterized by low permeability rocks that locally overlay pressure sod-mineral water aquifers, including the famous therapeutic water associated with the Borjomi springs.

2. Numerical Investigation of Spreading of Spilled Along the Baku-Tbilisi-Ceyhan Pipeline

In connections with construction of the new lines of oil pipelines and increase a rail transportation of oil across the territory of Georgia, there is necessity to develop the calculation methods of diffusion, of the spilled oil ingredients. At the present time only a few works are devoted to the investigation of the mechanisms of a filtration and infiltration of oil and polluting substances for the Georgian territory [4-6]. With the help numerical integration of nonlinear filtration equation of a liquid, we have studied distributions of petroleum and mineral oil to the soils in case of their emergency spilling. Generally there are ten types of soils along the Baku-Tbilisi-Ceyhan pipeline rout for the Georgian section [3]. But mainly there are four types of soils which occur more frequently along the pipeline rout for the Georgian section. These four types of soils are follows: meadow-alluvial, sandy and sub-sandy; meadow-marshy; brown- black, peat; grey-brown, solt.

According to Polubarinova-Kochina the process of oil-products spreading in soil can be described by the following equation [7]:

$$\frac{\partial W}{\partial t}=\frac{\partial}{\partial x}\left(\frac{K(W)}{\gamma}\frac{\partial P}{\partial W}\frac{\partial W}{\partial x}\right)+\frac{\partial}{\partial y}\left(\frac{K(W)}{\gamma}\frac{\partial P}{\partial W}\frac{\partial W}{\partial y}\right)+\frac{\partial}{\partial z}\left(\frac{K(W)}{\gamma}\frac{\partial P}{\partial W}\frac{\partial W}{\partial z}\right)+\frac{\partial K(W)}{\partial z}+Q, \quad (1)$$

where W is a saturation of the soil; t - is time; $K(W)$ is a coefficient of filtration; P is a pressure; $\gamma = g\rho$ is a unit weight of liquid. ρ is density; g is gravitational acceleration; .an axis Oz is directed vertically down.

Averianov S. F. has investigated a subject of dependence of filtration coefficient on the soil saturation and obtained the following theoretical dependence [6,7]:

$$K(W)=K_1\left(\frac{W-W_0}{\sigma-W_0}\right)^n\cdot\frac{y_W}{y_{oil}}, \quad (2)$$

where K_1 is a coefficient of water filtration; when soil is saturated (experiments verified this dependence at $n = 3.5$); W_0 is a comparative volume related with water; σ is a porosity of the soil; y_W, y_{oil} are kinematics coefficients of viscosity of water and oil, respectively.

If we use denotation

$$D(W)=\frac{K(W)}{\gamma}\frac{\partial P}{\partial W}, \quad (3)$$

then (1) can be rewritten in the following diffusive form:

$$\frac{\partial W}{\partial t}=\frac{\partial}{\partial x}\left(D(W)\frac{\partial W}{\partial x}\right)+\frac{\partial}{\partial y}\left(D(W)\frac{\partial W}{\partial y}\right)+\frac{\partial}{\partial z}\left(D(W)\frac{\partial W}{\partial z}\right)+\frac{\partial K(W)}{\partial z}+Q, \qquad (4)$$

In the theory of soil liquid motion it is accepted, that pressure (and coefficient of water permeability) are the given functions of saturation W and this dependence is given due to empirical reason. From (1) and (3) it is clear, that at different forms of $K(W)$ and $\frac{\partial P}{\partial W}$ we obtain different representations of equation (4), however there exist some evidence from experimental observations. It is exactly a set of experiments that have shown a direct proportion between diffusion coefficient and filtration velocity. According to Averianov S.F., dependence between pressure P and saturation W can be given in the following form [4]:

$$P=P_0\left(\frac{W-W_0}{\sigma-W_0}\right)^{m+1}\cdot. \qquad (5)$$

Here P_0 is a liquid pressure at full saturation of vapors, i.e. when $W=\sigma,\ m\in[-1,+\infty]$.

Taking into account (5), (3) can be rewritten in the following form

$$D(W)=\frac{K(W)\cdot\alpha_1}{\gamma}\left(\frac{W-W_0}{\sigma-W_0}\right)^{m}, \qquad (6)$$

where $\alpha_1=\frac{P_0 y_W}{(\sigma-W_0)y_{oil}}$,

equation (4) can be rewritten in the following form [4]:

$$\frac{\partial S}{\partial t}+K_{11}\frac{\partial S^n}{\partial z}=D_{11}\frac{\partial^2 S^{n+m+1}}{\partial x^2}+D_{11}\frac{\partial^2 S^{n+m+1}}{\partial y^2}+D_{11}\frac{\partial^2 S^{n+m+1}}{\partial z^2}+\mathrm{Q}, \qquad (7)$$

where

$$K_{11}=\frac{-K_1}{(\sigma-W_0)^n}\cdot\frac{y_W}{y_{oil}},\ D_{11}=\frac{K_1\alpha_1}{(n+m+1)\gamma(\sigma-W_0)^m},\ S=W-W_0. \qquad (8)$$

Now we solve the following practical problem. Suppose a big amount of oil is spilled on the Earth surface which has a cylindrical form with a basis Ω_1 and height H_0. And takes the area $\Omega_1=\Omega_1 * H_0$. We are interested in the problem of oil filtration and diffusion in the soil, so we seek for solution of the problem (7) in the rectangular parallelepiped $G=\{0\le x\le l_1,\ 0\le y\le l_2,\ 0\le z\le l_3\}$,

until the following condition is fulfilled

$$H(t)=H_0-\int\limits_t\iiint\limits_G S\,dG,\quad H(t)\ge 0. \qquad (9)$$

We suppose that axis Ox and Oy are oriented in the way, that the upper boundary coincides with the Earth surface, and axis Oz is directed vertically down. See Fig. 1

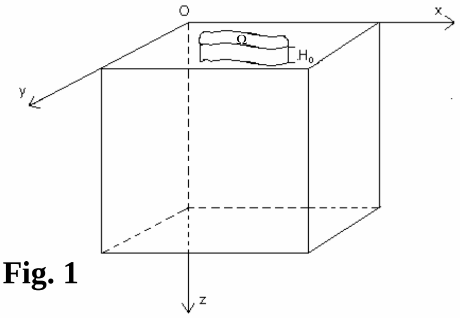

Fig. 1

With the following initial conditions

$$S(O,x,y,z) = -W_0 \text{ at } \quad x,y,z \notin \Omega, \tag{10}$$

$$S(O,x,y,z) = \sigma - W_0 \text{ at } \quad x,y,z \in \Omega,$$

and boundary conditions

$$S(x,y,O,t) = \sigma - W_0 \text{ at } \quad x,y \in \Omega, \tag{11}$$

$$S(x,y,O,t) = -W_0 \text{ at } \quad x,y \notin \Omega, \tag{12}$$

$$\frac{\partial S}{\partial z} = 0, \text{ at } \ z = l_3 \tag{13}$$

$$\frac{\partial S}{\partial x} = 0, \text{ at } \ x = l;\ x = 0; \tag{14}$$

$$\frac{\partial S}{\partial y} = 0, \text{ at } \ y = 0,\ y = l_2. \tag{15}$$

In general, problem (7), with initial (10) and boundary conditions (11)-(15) is solved numerically[4].

We have carried out numerical experiments for these four type of soils along Baku-Tbilisi-Ceyhan pipeline route on the bassis of a new filtration equation. Numerical integration was carried out during *t* = 240 days for the meadow-alluvial, sandy and sub-sandy soils, *t* = 265 days for the meadow-marshy soils, *t* = 424 days for the brown- black, peat soils and *t* = 270 days for the grey-brown, solt soils. For an example some of the received results are given in Fig. 2 and 3.

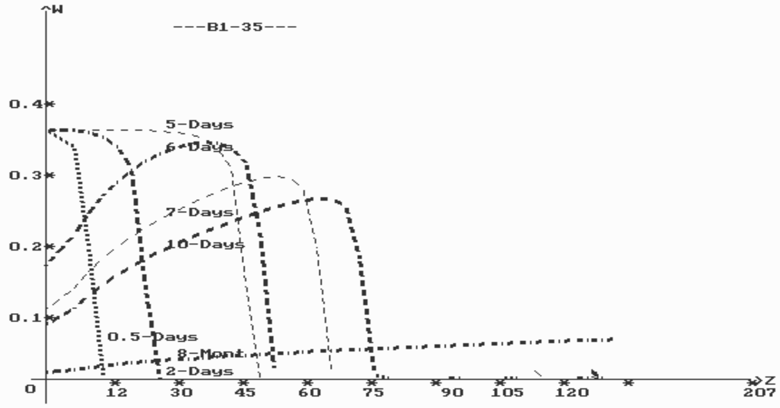

Fig. 2 The distribution of the concentrations *W(z)* of the oil at *t=0.5 2, 5, 7, 10, and 240 days* for the meadow-alluvial, sandy and sub-sandy soils.

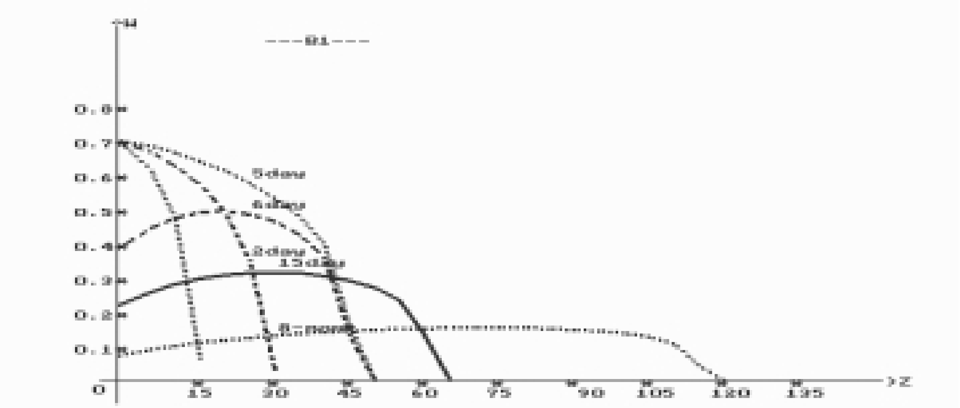

Fig. 3. The distribution of the concentrations *W(z)* of the oil at t=0.5 2, 5, 6, 15, and 265 days for the meadow-marshy soils.

The numerical calculations showed, that the process of oil infiltration in all considered soils proceeds qualitatively equally i.e. in all considered soils it is possible to distinguished a stage of absorption of the oil in the soil and a stage of distribution of the oil to depth and width of soils[4,6]. In our case the stage of the oil absorption proceeds about 5 days in the all types of soils (with the exception of the grey-brown, and solt soils when the stage of the oil absorption proceeds about 6.3 days). For this time, an oil spillage with thickness 5cm is fully absorbed by a surface layer of soil. During this stage the value of W (oil concentration) (for the meadow-alluvial, sandy and sub-sandy type of soils) is maximal on the soil surface, and it quickly falls with depth i.e. on the depth of 62cm the concentration of oil- W is minimal and extended with width. The front of pollution at *t =0.5, 2, 5* and *10* days reaches the depths *z =9, 27, 48, 75* cm, respectively. During the second stage of infiltration, concentration of oil-W on a surface of soil gradually decreases (see Fig.3) .

The process of infiltration in the brown- black, peat soils is the most intensively and the maximal values of concentration at *t = 5, 10* days reaches on the depths *Z*=26, 50cm respectively. The process of infiltration in the grey-brown and solt soils is the least intensively. The maximal values of concentration at *t = 5, 10* days reaches on the depths *Z*=12, 21cm respectively the front of oil pollution at t=5, 10 days reaches the depths *Z*=25, 37cm. The process of infiltration in the meadow-alluvial, sandy and sub-sandy soils proceded qualitatively equally of the brown- black, peat soils but this process is less intensive than in brown- black, peat soils. The front of oil pollution at t=5, 10 days reaches the depths *Z*=153, 203cm respectively and the maximal values of concentration at *t = 5, 10* days reaches on the depths *Z*=24, 45cm respectively. The process of infiltration in the meadow-marshy soils proceeded qualitatively equally of the meadow-alluvial, sandy and sub-sandy soils but this process is less intensive than in meadow-alluvial, sandy and sub-sandy soils. The front of oil pollution at *t*=1,2 and 5 days reaches the depths *Z*=15, 30 and 48cm respectively and the maximal values of concentration at *t = 15* and 265 days reaches on the depths *Z*=63, 120cm respectively (see Fig.3).

Beside this we studied a filtration property of equation (7) at different values of n and m; some results of these investigations are shown on Table 1 for the meadow-alluvial, sandy and sub-sandy type of soils.

Table 1

$n=1,\ m=1$	$t=6\,\text{h}$	$t=12\,\text{h}$	$t=24\,\text{h}$	$t=48\,\text{h}$	$t=96\,\text{h}$	$t=192\,\text{h}$
	$z=50$	$z=75$	$z=95$	$z=125$	$z=210$	$z=315$
$n=2,\ m=1$	$z=39$	$z=58$	$z=83$	$z=109$	$z=195$	$z=280$
$n=3,\ m=1$	$z=23$	$z=39$	$z=57$	$z=62$	$z=153$	$z=253$

Analysis of Table 1 shows, that at the same other values of parameters increase of parameter n from 1 to 3 (increase of non-homogeneity of the problem) reduces a filtration property of equation (7).

So we are able to make a brief conclusion that environmental and social-economical baseline of the Georgian section of TRACECA have not studied enough (we are performing works in that direction) and it is necessary to study this problem more comprehensive. Namely it is necessary to evaluate existing numerical schemes and mathematical models describing oil distribution to environment in case of pipeline damage. They will be used : to study and forecast possible emergency situations, in the air, soils, rivers and under ground water along the TRACECA; to obtain time-space distributions of harmful substances in the air along the TRACECA in the general case and in extra-ordinary situations; to obtain time-space distribution of harmful substances to the soil with account of specific characters of each sort of soils meeting along the TRACECA; to develop the most important task of the river water-monitoring problem - the identification of emergency release sources and the development of operative algorithms and unified computer programs for the common types of water objects and pollution processes; to investigate the impact of pollution on the population in order to prevent harmful contamination and to support a design of sensible economical, social and political abatement strategies.

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