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Variational Approach for Construction and Investigation of Hierarchical Models of Thermoelastic Piezoelectric Structures

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In this talk we present results of investigation of mathematical models of thermoelastic piezoelectric shells and curvilinear bars consisting of inhomogeneous anisotropic material with regard to magnetic field. We consider multilayer shells with variable thickness, which may vanish on a part of the lateral boundary, and multilayer curvilinear bars with variable rectangular cross-section, which may degenerate into segment or point at the butt ends of bar, and obtain variational formulations of the boundary and initial-boundary value problems in curvilinear coordinates corresponding to the static and dynamical linear three-dimensional models of piezoelectric thermoelastic solids with regard to magnetic field when density of surface force, and normal components of electric displacement, magnetic induction and heat flux vectors are given along certain parts of the boundary and on the remaining parts of the boundary mechanical displacement, electric and magnetic potentials, and temperature vanish. By applying extension and generalization of I. Vekua dimensional reduction method and the variational formulations we construct hierarchies of static and dynamical two-dimensional models for shells and one-dimensional models for curvilinear bars. For the boundary and initial-boundary value problems corresponding to the constructed two-dimensional and one-dimensional models we give the results on the existence and uniqueness of solutions in suitable weighted Sobolev spaces or spaces of vector-valued distributions. Moreover, we present the results on the relationship between the constructed two-dimensional and one-dimensional models and the original three-dimensional problems.

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Hierarchical Models for Thermoelastic Kelvin-Voight Piezoelectric Prismatic Shells

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The present work is devoted to construction of hierarchical models for thermoelastic Kelvin-Voight piezoelectric prismatic shells. Using I. Vekua's dimension reduction method, governing systems are derived and in the Nth approximation of hierarchical models boundary value problems (BVPs) and initial boundary value problems (IBVPs) are set. In the $N = 0$ approximation of hierarchical models antiplane deformation of transversaly isotropic piezoelectric materials are considered, Dirichlet and Keldysh type problems (BVP) are set and investigated.

New Seismic Insulation Systems for Buildings to Protect Against Catastrophic Earthquakes

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Georgia is well known for being in the zone of high seismic activity. The magnitude and number of the recent earthquakes have also been specifically listed in our book [1]. How our society of engineering has been and still is fighting the disaster, these past six decades of scientific-experimentation, design and construction work showed us.

Reducing the impact of seismic forces on the load bearing structures causes the inertial forces to decrease, resulting in a decrease in the rate of failure frequency of the structural elements during the earthquake. The reduction of inertial forces results in a reduction in the displacement of the building's load bearing structures that is very important in calculating the seismic stability of the building.

The decrease in displacement is related to the reduction of forces in the load bearing structural elements, and the decrease in force causes a decrease in the constriction of the structures; The cost of concrete and reinforcement decreases. We believe that special attention should be paid to immediate use of seismic insulation systems during the construction of kindergartens, schools, and sanitation facilities. This system should also be used in large human settlements: Sporting and cultural venues, museums, government, office, water supply and fire protection facilities.

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Progress in Mathematical and Numerical Modelling of Piezoelectric Smart Structures

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Based on the author's works, this plenary lecture discusses developed mathematical and numerical models for piezoelectric smart structures analysis. After recalling some early (late 1990s) asymptotic theories-based mathematical results regarding the mechanical displacements and electric potential and displacements variations through the thickness of thin piezoelectric smart structures, the related (around 2000s) displacement-potential finite elements (FE) and advanced partial mixed electromechanical variational formulations are briefly presented. The focus is then put on the developments regarding the more recent (2010s) Hamiltonian partial mixed FE-state space symplectic approach and semi-analytical distributed transfer function and iterative extended Kantorovich methods for the piezoelectric smart multilayer cross-ply and angle-ply composite plates and sandwich beams static and free-vibration analyses under theoretical and realistic mechanical and electric boundary conditions.

Connections Between the Dirichlet and the Neumann Problem for Integrable Boundary Data

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We provide an explicit solution of the generalized solution of the Neumann problem for the Laplace operator, based on a representation of the solution on the unit ball in $\boldsymbol{R}_n, n \geq 1$, in terms of the solution of an associated Dirichlet problem, in the case of integrable boundary data. We also provide a new approach to Brosamler's formula which gives a probabilistic representation of the solution of the Neumann problem for the Laplacian in terms of the reflecting Brownian motion. The talk is based on joint works with Mihai N. Pascu (Brasov, Romania) and Nicoale R. Pascu (Kennesaw State University, USA).

Some Problems for Elastic Double-Layer Bodies

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In the fifties of the XX century investigations of cusped elastic prismatic shells actually takes its origin, namely, in 1955 I.Vekua raised the problem of investigation of elastic cusped prismatic shells, whose thickness on the prismatic shell entire boundary or on its part vanishes.In practice, such cusped prismatic shells, in particular, cusped plates, and cusped beams (i.e., beams whose cross-sections area vanishes at least at one end of the beam) are often encountered in spatial structures with partly fixed edges, e.g., stadium ceilings, aircraft wings, submarine wings etc., in machine-tool design, as in cutting-machines, planning-machines, in astronautics, turbines, and in many other application fields of engineering. Investigation of elastic cusped prismatic shells, considered as $3D$ ones, may occupy $3D$ domains with, in general, non-Lipschitz boundaries. The problem mathematically leads to the question of setting and solving of boundary value problems for even order equations and systems of elliptic type with the order degeneration in the static case and of initial boundary value problems for even order equations and systems of hyperbolic type with the order degeneration in the dynamical case. At the same time I.Vekua introduced a new mathematical model for elastic prismatic shells which was based on expansions of the three-dimensional displacement vector fields and the strain and stress tensors in linear elasticity into orthogonal Fourier-Legendre series with respect to the variable plate thickness. By taking only the first $N + 1$ terms of the expansions, he introduced the so called N-th approximation. Each of these approximations for $N = 0; 1; ...$ can be considered as an independent mathematical model of plates. In particular, the approximation for N = 1 corresponds to the classical Kirchhoff plate model. In the sixties, I. Vekua developed the analogous mathematical model for thin shallow shells. Works of I. Babuska, D. Gordeziani, T. Vashakmadze, V. Guliaev, I. Khoma, A. Khvoles, T. Meunargia, C. Schwab, V. Zhgenti, G. Jaiani, G. Tsikarishvili, M. and G. Avalishvili, W. Wendland, D. Natroshvili, S. Kharibegashvili, N. Chinchaladze, R. Gilbert, and others are devoted to further analysis of I.Vekua's models (rigorous estimation of the modeling error, numerical solutions, etc.) and their generalizations.

The analogues system in the case of (N_3, N_2) approximation of hierarchical models for cusped beams, in general, beams with variable rectangular cross-sections are derived by G. Jaiani. His goal was to find out the peculiarities of setting the boundary conditions for primarily cusped bars and to determine its geometric-mechanical seuse. In the present work we consider a two-layer prismatic shell consisting of two cusped elastic bars. In introduction is devoted to review of the literature. The first chapter provides auxiliary material, and the second chapter deals with the main results of the master's thesis.

Analysis on Strengths of Isotropic Cusped Beams

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Since demand for isotropic materials is gradually decreasing since their density is big, they are also expensive and hard-working, the task of minimizing mass of isotropic material has been on the agenda. A clear example of this is the construction of aircraft where the most important requirement is the maximum lifting force in the minimum mass. This challenge is answered by the variable geometric beams, which are very small compared to the prisms. The main purpose of the work is to optimize the geometry of cusped beams and their complex analysis in terms of strength. The work deals with two different cases of the variable beams, in one case we discussed the constant thickness and the variable width, and in the second case a constant width and variable thickness are considered. Based on a theoretical analysis, we have optimized their geometry and as a result, if the thickness of the variable geometric beams is constant, the width is changing as a linear function, and in the second case it is found out that if the cusped isotropic width is constant, then the thickness is changing as a square function. After optimization, we conducted a comparative analysis of the above-mentioned cusped beams with the same geometric characteristics of the prismatic beam with the constant cross-section and as a result we identified the percentage of the material economy. Namely, in one case, the savings of the isotropic cusped beam (with constant thickness and the width changing as a linear function) is 50%. In the second case, when the width of the beam is constant and the thickness changing as a square function, the material savings are - 33%.

Deep Residual Networks and Numerical Linear Advection

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Some deep neural networks can be viewed as a numerical approximation of partial differential equations and dynamical systems. Motivated by numerical partial differential equations several new architectures of neural networks are proposed. Some results also exist regarding connections between neural networks and linear advection equation. An award-winning deep residual network for image classification is interpreted as a numerical solution of the linear transport equation using the method of characteristic with the specific velocity vector. Some numerical schemes for the linear advection equation with a constant coeffcient are considered as a deep neural network with specific architecture and the approach is used for accelerating numerical computations. Here we consider different aspects and study deep convolutional residual network with the help of numerical linear advection. We prove universal approximation property of considered deep residual network. In particular, we prove that deep residual network with specific convolutional kernel, depth and width can approximate continuously differentiable functions at arbitrary precision. We consider several residual networks motivated by finite volume discretization and provide error estimates for them. We formulate classification and regression tasks as a parameter identification problem for the linear advection equation. We show how to incorporate stable adjoint numerical solver in backpropagation algorithm for these tasks.

Anomalous Transport in the Space Plasmas

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Development of mathematical methods of investigation of the near Earth plasma processes and appropriate technical resources, gives possibility to reveal more interesting processes in the media and deep understanding of their characteristics, influence of these processes on the Earth dynamics. Development of the mathematical approaches make possible more adequate analytical description of the experimentally revealed plasma processes. One of such interesting phenomena represents anomalous transport of the energetical particles in the space plazma at existance of the magnetic turbulence.

The anomalous transport has gained growing attention during the last two decades in many fields including laboratory plasma physics, and recently in astrophysics and space physics. Here the examples, both from laboratory and from astrophysical plasmas are shown, where superdiffusive transport has been identified, with a focus on what could be the main influence of superdiffusion on fundamental processes like diffusive shock acceleration and heliospheric energetic particle propagation. The use of fractional derivatives in the diffusion equation is also discussed, and directions of future investigations are indicated.

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Nonlinear Mathematical Models of Settlement of Political Conflicts and Problems of Minimization of Resources

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Mathematical modeling got already into the areas of the social sphere, such as, processes of linguistic globalization [1].

Prof. T. Chilachava offered [2] and we discussed nonlinear mathematical models of economic cooperation for resolution existing political conflict between two opposing political sides [3 6].

In the given work two nonlinear mathematical models are discussed: in the first one the process of economic cooperation is free from political pressure, in the second -one the governments of both sides encourage the process of economic cooperation.

In case of some ratios between constant coefficients of mathematical models exact analytical solutions of Cauchy's problems for nonlinear two-dimensional dynamic systems are found. Through management parameters existing in mathematical models are possible to determine the conditions for which the conflicts are peacefully solvable, i.e. for fixed coefficient of aggressiveness, for quantities of the population of sides (zero demographic factor), minimum values of coefficients of cooperation and coercion for cooperation (minimum economic expenses for peaceful settlement of the conflict) are found.

In case of variability of parameters of models, by means of computer modeling, for the set functions of coefficients of aggression of the sides, the minimum values for variable coefficients of the models characterizing aspiration of parts of the population of the sides to cooperation are found.

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Modeling of Extreme Events and Regional Climate Variability on the Territory of the Caucasus (Georgia)

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At present the climate change problem is associated with a growing crisis in food production and health safety owing to environment pollution, more frequent heavy precipitations, hails, floods and droughts with a growing desertification processes in many regions of the earth. Climate change process is in progress in the South Caucasus region and in Georgia too where increased trends in mean annual temperature with heavy precipitations, hails, floods and droughts are more frequent. So for prevention of accidents to take more efficient steps in provision with scientific information (regional and local scale extreme weather prediction, climate change tendencies) against the freaks of nature is an urgent issue on the territory of Georgia.

For regional weather forecasting and climate trends prediction one of the most essential means represents creation an ensemble of the regional climate forecasting system (WRF Chem, WRF Climate, RegCM) for the Caucasus (Georgia) territory for the purpose of studding the different climate change scenarios and for future climate projections.

In this article a high resolution modeling ensemble system is developed for studying extreme natural phenomenon, climate variability and the human impact on climate on the territory of Georgia. The climate ensemble system is constructed by existing global and regional climate earth-system models which are based on numerical solution of nonlinear hydro-thermodynamics system of equations using high performance computation technologies. For achievement of these approaches the statistical and dynamical downscaling methods and Regional Climate Model Evolution System (RCMES) are used. Some results of numerical calculations are analyzed and presented.

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Strategic Graph Rewriting in an Interactive Modelling Framework

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In this talk I will describe the use of strategic port graph rewriting as a basis for the implementation of visual modelling tools. The goal is to facilitate the specification and analysis of complex systems, in particular programs. A system is represented by an initial graph and a collection of graph rewrite rules, together with a user-defined strategy to control the application of rules. The traditional notions of strategies for functional languages and term-rewriting languages have been adapted to deal with the more general setting of graph rewriting, and some new constructs have been included in the strategy language to deal with graph traversal and management of rewriting positions in the graph. We give a formal semantics for the language, examples of application, and a brief description of its implementation: the graph transformation and visualisation tool PORGY.

Metric Lie Algebras and Applications

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In my talk I will consider Lie algebras with an invariant symmetric nondegenerate bilinear form. These are important in many areas of mathematical physics. They include beside semisimple Lie algebras other interesting algebras, like the diamond and oscillator algebra. I will chatacterize them in low dimension, and present their metric deformations.

On the Strong Angular Summability

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The method of strong angular summability is introduced. Its approximation properties are studied.

Some Basic Boundary Value Problems for Plane Theory of Elasticity for Materials with Voids

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The static equilibrium of elastic materials with voids is considered. The corresponding three-dimensional system of differential equations is derived. Detailed consideration is given to the case of plane deformation. A two-dimensional system of equations of plane deformation is written in the complex form and its general solution is represented by means

of two analytic functions of a complex variable and one solutions of Helmholtz equations. The constructed general solution enables one to solve analytically a sufficiently wide class of plane boundary value problems of the elastic equilibrium of body with voids. The concrete boundary value problems are solved.

On Hierarchical Models for Piezoelectric Bars

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The present talk is devoted to construction and investigation of hierarchical models for piezoelectric termo-viscoelastic Kelvin-Voigt bars with rectangular cross-sections. In particular, in $(0,0)$ approximation static and oscillation problems are discussed. A special attention is given to analysis of peculiarities of nonclassical setting boundary conditions (BCs) in the case of cusped bars. Namely, the criteria are established for piezoelectric transversely isotropic cusped bars when on one end or on both ends of the bar no data need to be prescribed. Weighted BCs are set as well. On the face surfaces of the bar under consideration stress vectors and outward normal components of the electric displacement vectors are prescribed, while at the ends of the bar all the admissible (in sense of well posedness of boundary value problems) BCs, including mixed ones, with respect to weighted $(0,0)$ moments of the components of the mechanical displacement vectors and electric potential, and $(0,0)$ moments of the components of the stress and electric displacement vectors are prescribed.

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On a Numerical Solution of One Two-Dimensional Nonlinear Model

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In order to construct an approximate solution of initial-boundary value problem for one system of non-linear two-dimensional equations, two different approaches are used. In particular, decomposition methods based on variable directions difference scheme and on the averaged model are investigated. Algorithms which are necessary for realization are described for both methods. Software code is developed and calculations are made for different test cases. The number of operations is defined for both methods. The time necessary for the realization of algorithms and the accuracy of the numerical experiments are compared. Obtained results are analyzed and relevant conclusions are made.

Recent Developments on Numerical Solutions for Hyperbolic Systems of Conservation Laws

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In 1757 Euler developed the famous Euler equations describing the flow of a compressible gas. This is a system of hyperbolic conservation laws in three space dimensions. However until recently one could not show convergence of numerical schemes to the 'classical' weak entropy solutions. By adapting the concept of measure-valued and statistical solutions to multidimensional systems Siddhartha Mishra and his coauthors could recently show convergence of numerical schemes. Mishra has presented these results at the ICM 2018 in Rio de Janeiro. I will present even newer developments which he presented at the ICIAM Congress 2019. After a brief introduction to the field these developments will be described.

The Flow of Nonlinear Viscous Strain-Hardening Material in Variable Section Passages

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The radial flow of material in conic and wedge-shaped passages is considered, the entrance and exit of which are bounded by surfaces with variable curvature. The curvature of the surfaces depends on distribution of flow velocity in the passage entrance. The constitutive equation of material was obtained on the basis of the work-hardening theory. The solution of a two-dimensional problem is reduced to the integration of a system of ordinary nonlinear differential equations with the boundary conditions:$X' = F(\alpha, X, q)$; $H_0(X(0)) = 0$, $H(X(\alpha_1)) = 0$; where: X is the vector of unknowns, α is an angular coordinate, q parameter was entered artificially. Integration of the nonlinear differential equations with boundary conditions is carried out numerically by the method of movement towards parameter shooting. The use of components allows to determine flow velocity, components of a tensor of velocities of deformations, the components of strain velocity tensor and components.

Mathematical Model of Project Management

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Planning of projects in the longer term by a particular company depends on existing resources (especially financial). The optimal allocation and management of resources is a complex task. For the stability and economic development of the company, the main importance is attached to the correct decision making. In the main, decision making depends on the experience of the manager and does not depend on any reasoned reasoning. It is therefore very important to use the correct calculation and methodology for making decisions that will allow us to make a more competent and informed decision.

One of such methods is the construction and management of a mathematical model. Obviously, the more accurate the mathematical model, the more correct and effective the decision will be made. However, real economic processes are characterized by many parameters and factors, and therefore, in some cases, the construction of exact mathematical models is impossible, or the constructed model is so complex that it is impossible to choose a method for its solution. Because of this, in some cases it is more acceptable to

create an interactive model in which the decision maker (expert) can observe, intervene and modify the data if it's necessary. The paper will consider one of these problems and the mathematical model of this problem.

Formulation of the problem. Suppose the company plans to draw up a project implementation schedule during the year. In projects we can infer new projects, the continuation of existing projects, the planning of charity events, the planning of advertising campaigns and much more. The implementation of each project requires some financial costs. The income of the company before the beginning of each month is known. The number of projects is so large that the total costs of implementing all projects exceed the company's total revenue for the year. We need:

1. Select the maximum number of projects that the company can implement during the year;

2. Create a continuous plan for the implementation of selected projects based on their financial costs.

Proximity Constraint Solving

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Proximity relations are binary fuzzy relations that satisfy reflexivity and symmetry properties, but are not transitive. This relation induces the proximity measure between function symbols, which is further extended to terms. The problem that we discuss in this talk is about solving unification problems modulo proximity: given two terms, find a substitution that brings the terms "sufficiently close" to each other, i.e., the proximity measure between substitution instances of the terms exceeds a predefined threshold. We impose no extra restrictions on proximity relations, allowing a term in unification to be close to two terms that themselves are not close to each other.

The unification problem has finite minimal complete set of unifiers. We designed an algorithm that computes this set. It works in two phases: first reducing the equation solving problem to neighborhood constraints over sets of function symbols, and then solving the obtained constraints. We present the algorithm, illustrate it on examples, and discuss its properties and applications. This is joint work with Cleo Pau.

Specification and Analysis of ABAC Policies in a Rule-Based Framework

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Access control is a fundamental security requirement for computing environments: It controls the ability of a subject to use an object in some specific manner. Attribute-based access control (ABAC) is a logical access control with great flexibility to specify access control policies as rules which get evaluated against the attributes of participating entities (user/subject or subject/object), operations, and the environment relevant to a request. The access control policies that can be implemented in ABAC are limited only by the computational language and the richness of the available attributes. Considerable work has been done and a number of formal models have been proposed recently for ABAC, with minimal sets of features that are sufficient to implement many desirable capabilities.

In this talk, we discuss the popular access control models ABAC_alpha and ABAC_beta, and propose to study and analyse them in RhoLog, a rule-based framework developed by us. We specify the logical and operational semantics of their policies in our framework, and show how to use the RhoLog system to decide some properties of interest.

Modelling of Elastic Composites with Several Small Parameters

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In addition to the thinness ε of a structure, we introduce another positive small parameter δ in order to take into account some characteristics of the geometry, or of the heterogeneities of an elastic body. When both parameters vanish simultaneously, new models are obtained as limit of an asymptotic approach (in the framework of linearised elasticity). Three examples are discussed. With Dr Eduard Rohan (Pilsen University, Czech Republic) we considered a composite body made of periodic elastic inclusions of size ε; and δ is the ratio between the elastic tensor components of the inclusions and those of the matrix. We proved that for a strongly heterogeneous composite (with $\delta{=}\varepsilon 2$) the limit homogeneous model, obtained by letting ε go to 0, presents a negative "mass density" tensor implying the existence of band-gaps in the propagation of elastic waves. The cases of thin plates ruled by Reissner-Mindlin or Kirchhoff-Love equations are compared. With Pr Georges Griso (Universite Pierre et Marie Curie, Paris, France) we considered a thin beam (ε is the ratio between its thickness and its length) made of a periodic distribution of small elastic inclusions along its length (δ is the size of each inclusions). When both the thickness of the beam and the size of the heterogeneities tend simultaneously to zero, we obtain three different one-dimensional models of beam depending upon the limit of the ratio $\varepsilon\delta$ of these two small parameters. With Dr Patrick Ballard (Institut Jean le Rond D'Alembert, Paris, France) we considered the case of a slender beam (ε is, as before, the ratio between its thickness and its length) whose cross-section is also slender (δ measures this slenderness). When both parameters are of the same order of magnitude we recover Vlassov's one-dimensional model for thin-walled beams (for special profiles of the cross-sections and specific loadings) thanks to an appropriate Korn's inequality.

An Investigation of Electromagnetic Force Models: Electromagnetic Forces and Moments Acting on Spherical Magnets

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From Maxwel's equations balance laws for the electromagnetic linear momentum, angular momentum, and energy can be found after recasting and using several identities of vector calculus. Therefore, the obtained equations are not "new results but rather identities having the form of a balance law. However, there is some degree of freedom, (a) during construction of a particular identity and (b) for the choice of the to-be-balanced quantity, the non-convective flux, and the production term. In short, one is insecure which of the various forms is correct under which circumstances. This conundrum is referred to as the AbrahamMinkowski controversy, who first proposed different expressions for the electromagnetic linear momentum. The proper choice of electromagnetic force and torque expressions is of particular importance in matter where the mechanical and electromagnetic fields couple. The question arises as to whether a comparison between the predicted deformation behavior and the observed one can help to decide which electromagnetic force model is suitable for a material of interest. In this talk we shall briefly review the controversy and suggest new approaches for its solution on the continuum level. We will learn from the example of total force and moment calculation of two permanent magnets interacting with each other.

On the Limit Distribution of the Integral Square Deviation of a Nonparametric Estimator of the Bernoulli Regression Function for One Sample and Two Independent Samples

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The limiting distribution of the integral square deviation of a kernel-type nonparametric estimator of the Bernoulli regression function is established. The criterion of testing the hypothesis about the Bernoulli regression function is constructed. The question as to its consistency is studied. The asymptotic power of the constructed test is also studied for certain types of close alternatives. The question is investigated for one sample and two independent samples.

A FEM-Based Algorithm for Solving Natural Vibration Problems for Piezoelectric-based Smart-Systems and its Application to the Problem of Multimodal Vibration Damping

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One of the efficient approaches which can be applied to searching for optimal configurations of complex compound structures with a large number of parameters is mathematical modeling. The capabilities of dynamic behavior control of piezoelectric-based smart-systems can be substantially expanded by connection of external impedance or the so-called shunting electric circuits. In this case the search for optimal configuration of such a smart-system is reduced to the finding of optimal parameters of elements of shunting circuits which provide maximal damping properties.

Within frameworks of this work it is proposed to apply the problem of natural vibrations to construction of an algorithm for searching the optimal parameters of a shunting circuit. The mathematical statement of the problem for piecewise-homogeneous electro-viscoelastic bodies with connected passive external electric circuits is formulated. In order to obtain the solution of the stated problem a novel FEM-based algorithm which implies using capabilities of the ANSYS software package is developed. The originality of the algorithm is in exploiting of global stiffness and mass matrices assembled in ANSYS. After exporting from ANSYS these matrices are decomposed into required constituents

for importing into program which realizes the Muellers method for solving eigenvalue problems.

The results of the solution to the problem of natural vibrations are complex natural vibration frequencies which characterize circular natural vibration frequencies and exponential damping rate of vibrations. These results are very convenient to be used for building the optimization algorithm since they are not dependent on loading conditions. Within frameworks of this work the proposed algorithm is applied for searching for the parameters of elements of single RL-circuit which shunts piezoelectric element, providing damping of vibrations at several vibration modes (multimodal vibration damping).

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Effect of Magnetic Declination on Refractive Index and Wave Polarization Coefficients of Electromagnetic Waves in Mid-Latitude Ionosphere

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A lot of researchers developed a theory for the propagation of electromagnetic waves in the ionospheric plasma (1-5). However, they made certain assumptions such as that the ambient magnetic field is vertical and the magnetic declination is zero, which is unrealistic in mid-latitude ionosphere. In this study, the general forms of conductivity tensor, refractive index and polarization coefficients for ionospheric plasma have been obtained whit including the magnetic declination and dip angle by solving the equation

$$m_\alpha \frac{dV_\alpha}{dt} = q_\alpha[E + V_\alpha \times B] - m_\alpha v_\alpha V_\alpha \tag{1}$$

Calculations show that polarization coefficients become complex numbers when magnetic declination is included, while they are pure imaginary in the absence of declination. The declinational effect on the real parts of the polarization coefficients is more pronounced around plasma frequency than at other frequencies. However, the imaginary parts and refractive index are not affected at the plasma frequency [1, 2, 3, 4].

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On Stability and Convergence of Three-Layer Semi-Discrete Scheme for Nonlinear Second Order Evolution Equation

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The Cauchy problem for the second order nonlinear evolution equation in the Hilbert space is investigated. This equation represents generalization of J. Ball non-linear integro-differential equation. Approximate solution of the stated problem is searched using three-layer semi-discrete scheme. Convergence of the proposed scheme is proved and approximation error of the solution is obtained.

Induced Operators by Subgaussian Random Elements

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To a (real-valued) random variable ξ given on a probability space (Ω, A, P) let us associate a quantity $\tau(\xi) \in [0, +\infty]$ defined by the equality: $\tau(\xi) = \inf\{a \geq 0 : \mathbf{E}e^{t\xi} \leq e^{1/2t^2a^2}$ for every $t \in \mathbf{R}\}$. A random variable ξ is called

* Subgaussian if $\tau(\xi) < +\infty$,

* strictly Subgaussian if it is Subgaussian and $\tau^2(\xi) = \mathbf{E}\xi^2$. Let $SG(\Omega)$ be the set of all Subgaussian random variables $\xi : \Omega \to \mathbf{R}$. It is known that $SG(\Omega)$ is a vector space with respect to the natural point-wise operations, the functional $\tau(\cdot)$ is a norm on $SG(\Omega)$ (provided the random variables which coincide a.s. are identified) and, moreover, $(SG(\Omega), \tau(\cdot))$ is a Banach space. Let X be a real Banach space and X^* be its dual. We say that a random element $\eta : \Omega \to X$

* is weakly Subgaussian, if for every $x^* \in X^*$ the random variable $x^*(\eta)$ is Subgaussian;

* is strictly Subgaussian, if for every $x^* \in X^*$ the random variable $x^*(\eta)$ is strictly Subgaussian;

* is Subgaussian in Fukuda's sense, or F-Subgaussian, if for every $x^* \in X^*$ the random variable $x^*(\eta)$ is Subgaussian and there is a finite constant $C \geq 0$ such that $\tau(x^*(\eta)) \leq C(E|x^*(\eta)|^2)^{1/2}$ for every $x^* \in X^*$;

* T-Subgaussian, if there exists a Gaussian random element $\gamma : \Omega \to X$ such that $\mathbf{E}e^{x^*(\eta)} \leq \mathbf{E}e^{x^*(\gamma)}$ for all $x^* \in X^*$.

For every weakly Subgaussian random element $\eta : \Omega \to X$ let us define the induced operator $T_\eta : X^* \to SG(\Omega)$ by the equality $T_\eta x^* = x^*(\eta)$ for all $x^* \in X^*$.

We will discus a relationship between these notions together with a proof of the following statement: For any F-Subgaussian random element η in a Banach space X the induced operator $T_\eta : X^* \to SG(\Omega)$ is a 1-summing operator.

Well-posedness for a Class of Degenerate Itô-SDEs with Fully Discontinuous Coefficients

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We show uniqueness in law for a general class of stochastic differential equations in $\mathbb{R}^d$, $d \geq 2$, with possibly degenerate and/or fully discontinuous locally bounded coefficients among all weak solutions that spend zero time at the points of degeneracy of the dispersion matrix. The points of degeneracy have d-dimensional Lebesgue-Borel measure zero. Weak existence is obtained for more general, not necessarily locally bounded drift coefficient. This is joint work with Haesung Lee (Seoul National University).

To the Theory and Practice of Thin Walled Structures

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The classical theory for elastic thinwalled beams, plates and shells (tpthwalstr) may give rough results different from models according to [1]. These models, in particular, contain the refined theories but not only in elastic case. On the other hand, we constructed in [2] uniform 3D mathematical models which as a particular case contain the systems of Navier-Stokes, Euler, nonlinear PDEs of Solid Mechanics, Maxwell's dynamical system, principles of the mass and energy conservations, Saint-Venant-Beltrami continuity conditions, Hooke's and Newton's relations. Differing from the Truesdell theory [3], we [1, 2] present both the balance equations of continuum mechanics and experimental laws uniformly by means of a parameter. Thus we formulate the following principle: any phenomenon discovered for the separate matter has the universal nature for any form of continuum mechanics.

We consider three examples:

1. The anisotropic inhomogeneous elastic structures. For this case the classical topic "tpthinwalstr" presents as fight of opposites, while really there is the presence of the classical category of unity of opposites. In particular, this way gives the possibility to construct easily the mixed theory for the inhomogeneous case.

2. Piezoelectric materials and electrical conductivity. [1,2] give for basic relations the possibility of constructing models describing also the shock, 2D soliton and Rayleigh-Lamb's waves.

3. Poroelasticity. Application of our methodology for this case refined the basic equations while Biot's and Reissner's models for porosity part used the Pascal-Darcy law having for symbolic determinant the parabolic degeneracy.

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The Ecliptic Polarization of the Characteristic Wave for the Cold Ionosphere Plasma in the Northern Hemisphere

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The polarization of the characteristic wave for the cold ionosphere plasma in the northern hemisphere is theoretically investigated. These wave is composed of two parts both real and imaginary as the ionosphere becomes double-refractive. As a result the characteristic wave has ecliptic polarization due to the Earths magnetic field as theoretical [1, 2, 3, 4].

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Lectures of the Advanced Courses on Mathematical Models of Piezoelectric Solids and Related Problems

On Variational Methods of Investigation of Mathematical Problems for Thermoelastic Piezoelectric Solids

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In this lecture, we present the results of investigation of boundary and initial-boundary value problems corresponding to mathematical models of thermoelastic piezoelectric solids with regards to magnetic field. We consider three-dimensional static and dynamical models of multi-domain general inhomogeneous anisotropic thermoelastic piezoelectric solids with mixed boundary conditions, when on certain parts of the boundary density of surface force, and normal components of electric displacement, magnetic induction and heat flux are given, and on the remaining parts of the boundary mechanical displacement, temperature, electric and magnetic potentials vanish. We obtain variational formulations of the boundary and initial-boundary value problems in suitable function spaces and present existence, uniqueness and continuous dependence results. Moreover, we construct and investigate hierarchical models of thermoelastic piezoelectric thin structures applying extensions and generalizations of dimensional reduction method, which was suggested by I. Vekua in the classical theory of elasticity for plates with variable thickness. This work was supported by Shota Rustaveli National Science Foundation (SRNSF) [Grant number 217596, Construction and investigation of hierarchical models for thermoelastic piezoelectric structures].

Piezoelectric Material, Effective and Structural Behaviours

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Piezoelectric materials are popularly used for sensing, actuation and transduction in the framework of smart structures applications such as for structural noise, vibration, shape, control and health monitoring or energy harvesting for autonomous and wireless smart, internet of things or communication devices. Therefore, mastering the behaviours of such smart materials is the key issue for the correct mathematical modelling of piezoelectric solids and related problems. Piezoelectric materials exist in monolithic and composite forms for which the main representatives of the former are piezoelectric ceramics (or piezoceramics) and polymers (or piezo-polymers), while those of the latter are piezoelectric fibre composites. The latter use particles and short or long fibres as reinforcements of polymer-type (usually epoxy) matrices. Besides, the long fibres can have micro circular or macro rectangular cross sections leading, respectively, to the so-called active fibre composites or macro-fibre composites (MFC). After a short introduction, this advanced course starts with recalling the material behaviours of monolithic piezoelectric patches. For this purpose, the coupled response modes are given first; then, the dominant ones under electrodes, polarization and electro-mechanical loads configurations are presented. After that, the effective behaviours of piezoelectric composites in transverse (d31) and shear (d15) response modes are discussed for MFC patches. Focus will be made on the properties identification, through analytical and numerical homogenizations of the active core only and multilayer (7 for the concept or 5 for prototypes) stacks. The last part of the course will focus on the recently introduced concept of structural behaviours of piezoelectric patches that are met once integrated (surface-bonded or embedded) onto a support or into a host. Here, the focus will be on surface-bonded (to a support) shear MFC patches numerical (finite element) and experimental global (displacement) response simulations and measurements and their correlations. Depending on the available time, the case of the transverse response of monolithic piezoceramic (PZT PIC255) patches, surface-bonded on or embedded in host composites (Carbon or Glass Fibre Reinforced Polymers), can be presented and discussed.

Piezoelectric Cusped Prismatic Shells

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The present lecture course is devoted to construction of differential hierarchical models for piezoelectric nonhomogeneous porous elastic and viscoelastic Kelvin-Voigt prismatic shells on the basis of linear theories. Using I. Vekua's dimension reduction method, governing systems are derived and in the Nth approximation of hierarchical models boundary value problems (BVPs) and initial boundary value problems (IBVPs) are set. In the N=0 approximation, considering, e.g., elastic, plates of a constant thickness, governing systems mathematically coincide with the governing systems of the plane strain corresponding to the basic three-dimensional (3D) linear theory up to a separate equation for the out of plane component of the displacement vector. The ways of investigation of BVPs and IBVPs, including the case of cusped prismatic shells, are indicated and some preliminary results are presented. Antiplane deformation of piezoelectric nonhomogeneous materials in the three-dimensional formulation and in N=0 approximation is analyzed. Well-posedness of Dirichlet and Keldysh type problems (BVP) are studied in the N=0 order approximation of hierarchical models for cusped prismatic shells. Some BVPs are solved in explicit forms in concrete cases. This work was supported by Shota Rustaveli National Science Foundation (SRNSF) [Grant number 217596, Construction and investigation of hierarchical models for thermoelastic piezoelectric structures].

Homogenization and Control of a Piezoelectric Body

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In this lecture, focused on the modelling of piezoelectric materials we present two examples in the general framework of linearized three-dimensional evolution equations. - In the first one we consider an heterogeneous composite made of inclusions periodically distributed in a matrix. And we show that when each microstructure presents "strong" heterogeneities, (i.e., the characteristics, such as mass density, elasticity, dielectric and coupling tensors of the inclusions and of the matrix are "strongly" different in terms of the size of the microstructures), then a band-gaps phenomenon may appears, namely in some intervals of frequency where there is no waves propagation. This modelling is obtained through the "homogenization"' technique. - The second one addresses the exact controllability of an homogeneous piezoelectric body. More precisely, we investigate the existence of boundary controls (such that elastic displacement and / or electric field) which drive the body to rest after a finite time.

An Examination of Elastic Deformation Predictions of Polarizable Media due to Various Electromagnetic Force Models

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This study investigates the implications for the deformation of electrets due to various electromagnetic force models. This deformation of dielectric materials due to electromagnetic forces is called electrostriction. Analytical solutions for the electrostriction problem of spherical electrets are derived in five different situations. First, an affine linear dielectric sphere with surface charges in an external field is considered. The electric field is computed analytically in a stationary situation. From this solution several model simplifications yield the electric fields of: a linear dielectric in an external field without surfaces charges, a real-charge electret without polarization, an oriented-dipole electret and a real-charge electret with linear dielectric material. With the electric field solutions the electromagnetic forces can be computed for selected force models. Expanding the forces conveniently in terms of Legendre polynomials, the method of Hiramatsu and Oka is applied to obtain the elastic deformation of the spheres.

Dynamical Problems of Generalized Thermo-Electro-Magneto-Elasticity Theory

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The lecture course is dedicated to the theoretical investigation of basic, mixed and crack type three-dimensional initial-boundary value problems of the generalized thermo-electro-magneto-elasticity theory associated with Green-Lindsay's model. The essential feature of the generalized model under consideration is that heat propagation has a finite speed. We analyse dynamical initial-boundary value problems and the corresponding boundary value problems of pseudo-oscillations, which are obtained from the dynamical problems by the Laplace transform. The dynamical system of partial differential equations generate a nonstandard six dimensional matrix differential operator of second order, while the system of partial differential equations of pseudo-oscillations generates a second order strongly elliptic formally non-selfadjoint six dimensional matrix differential operator depending on a complex parameter. First, we prove uniqueness theorems of dynamical initial-boundary value problems under reasonable restrictions on material parameters and afterwards we apply the Laplace transform technique to investigate the existence of solutions. This approach reduces the dynamical problems to the corresponding elliptic problems for pseudo-oscillation equations. The fundamental matrix of the differential operator of pseudo-oscillations is constructed explicitly by the Fourier transform technique, and its properties near the origin and at infinity are established. By the potential method, the corresponding three-dimensional basic, mixed and crack type boundary value problems and the transmission problems for composite elastic structures are reduced to the equivalent systems of boundary pseudodifferential equations. The solvability of the resulting boundary pseudodifferential equations are analysed in appropriate Sobolev-Slobodetskii, Bessel potential, and Besov spaces and the corresponding uniqueness and existence theorems of solutions to the boundary value problems under consideration are proved. The smoothness properties and singularities of thermo-mechanical and electro-magnetic fields are investigated near the crack edges and the curves where the different types of boundary conditions collide. It is shown that the smoothness and stress singularity exponents essentially depend on the material parameters and an efficient method for their computation is described. By the inverse Laplace transform, the solutions of the original dynamical initial-boundary value problems are constructed and their smoothness and asymptotic properties are analysed in detail.

C O N T E N T S