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OPENING TALK

Life and Activities of David Gordeziani

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David Gordeziani was born in Tbilisi on December 9, 1937. Since 1945 he had been studying at Male Gymnasium No 1 (later known as Secondary School No 1 and currently Tbilisi Classical Gymnasium), which he graduated with the gold medal in 1956. The same year he became a student of the faculty of Mechanics and Mathematics at Iv. Javakhishvili Tbilisi State University (TSU).

David Gordeziani

In his student years he actively participated in student activities. At the student conference in 1961 he gave the talk [1]. After graduating TSU with honors in 1961 he continued his education as a post graduate student at the Chair of Approximate Analysis and Computational Technics of the same university where he had been working under the supervision of the outstanding Georgian scientist, Academician Shalva Mikeladze. In 1966 he defended PhD thesis in Computational mathematics at the Academic Council of A. Razmadze Institute of Mathematics of the Georgian Academy of Sciences (Head of the Council - Academiciam N. I. Muskhelishvili). The title of the thesis was: "About Numerical solution of some Parabolic Differential Equations of the Second and Higher Order" (under supervision of Academician Sh. Mikeladze).

Parents: Natalia Kireulishvili
and George Gordeziani

David Gordeziani began his scientific work at the Institute of Mathematics in 1964.

In 1968 he moved to TSU Scientific-Research Laboratory, which later became the Institute of Applied Mathematics (VIAM). Since 1970 he had been heading the department of Numerical Analysis of this institute. From 1979 till 1985 he was Deputy Director for science. By that time the institute was already named after Academician Ilia Vekua. From 1986 till the end of 2006 he was a Director of Ilia Vekua Institute of Applied Mathematics. From 1984 till 2006 David Gordeziani was also head of the Chair of Computational Mathematics and Informatics at the faculty of Mechanics and Mathematics of TSU. From 2006 till September 2009 he was Full Professor, and from 2009 till the end of his life he was Professor Emeritus at TSU and the head of the scientific direction "Mathematical Modeling and Numerical Analysis" of VIAM. He was Invited Professor at Sokhumi State University and St. Andrew the First Called Georgian University of Patriarchate of Georgia correspondingly in 1996-2006 and in 2008-2015.

Academician Sh. Mikeladze

David Gordeziani is a famous specialist in Computational and Applied Mathematics, who has made a considerable contribution to the development of science. His works are dedicated to such problems of modern mathematics as development and justification of numerical methods for resolving linear and nonlinear problems of Mathematical Physics, namely, by investigating mathematical models for I.Vekua plates and shells, studying correctness of nonlocal boundary value problems and developing methods of their discretization research of problems for computer realization of mathematical models of physics, chemistry, engineering mechanics, gasification, landslide, etc.

Academician N. Muskhelishvili,
Academician I.Vekua.

Scientific topics of Professor David Gordeziani mainly deal with:

With his wife Nanuli Turdzeladze

- Development and investigation of economic finite-difference algorithms;
- Justification and development of the theory of plates and shells of I. Vekua;
- Investigation of some problems of mathematical physics;
- Research of nonlocal initial-boundary and boundary value problems for partial differential equations.

In his earlier works David Gordeziani has constructed and studied the finite-difference schemes (having high precision of approximations) in various types of grids (right-angled, diamond type) for some non-stationary linear and nonlinear parabolic equations [2-7].

In those works Professor Gordeziani has shown convergence of constructed finite-difference schemes, studied stability and precision issues. Those researches present enhancement of results obtained by Academician Shalva Mikeladze for heat-conductivity equations. During the same period David Gordeziani has begun his studies in economic difference schemes (locally one-dimensional methods).

With Prof. G. Jaiani

His paper published in 1965, which was devoted to the investigation of locally one-dimensional schemes for 2m-order parabolic equations, was practically first among this type of works. In this paper the author has shown that for rather general high order equations the solution of a special one-dimensional system (additive model) in grid-points coincide with the solution of a multi-dimensional problem. The same paper studies stability issues of locally one-dimensional schemes for 2m-order equations [2].

The mentioned paper has attracted attention of many researchers, who have qualified it as of high importance (A.A. Samarskii, N.N. Janenko). The work has got large amount of citations in other scientific works and monographs.

With Academician A.N. Tikhonov

It should be noted that the first works of David Gordeziani became the basis for the development of such an important modern direction of Computational Mathematics in Georgia as investigation of economic algorithms for the problems of Mathematical Physics (locally one-dimensional method, fractional step method, decomposition method, split methods, method of variable directions, etc.) [2-7]. Theoretical basis of these methods was given in the papers of scientists from USA and USSR (J. Douglas, G. Reckford, D. Peacman, N. N. Janenko, A. A. Samarskii, G.I. Marchuk, etc.) at the end of 50th and beginning of 60th of the last century.

In 1968-72 David Gordeziani constructed new economic algorithms for the resolution of non-stationary problems of mathematical physics [5-7, 9, 10]. He called those algorithms “averaged” models. Applying those algorithms the parallel calculation were built and studied. The mentioned results were partially presented at the International Congress of Mathematicians held in Nice in 1970 (talk by A.A. Samarskii “On Works about Solution of Finite-Difference Schemes”). Part of the results was published during the period when David Gordeziani worked under the supervision of J.-L. Lions at the Laboratory of Numerical Analysis of the University Paris VI, France, in 1971-72. Those works have many citations (J.-L. Lions, R. Temam, V.L. Makarov, etc.). Averaged models (algorithms of parallel calculation) gained special attention after creation of computers with parallel processors. They are widely used for solution of certain applied problems of the theory of elasticity, plates and shell theory, magnetic hydrodynamics, etc.

In the next works of that cycle, David Gordeziani has constructed and investigated various types of locally one-dimensional and split schemes for non-stationary linear partial differential equations with variable coefficients in multi-dimensional cases as well, moreover, for nonlinear parabolic and hyperbolic equations. One important fact, characterizing David Gordeziani's works, should be noted here: investigation of algorithms is carried out on an abstract level, applying modern methods of functional analysis. In addition, results obtained are of a specific as well as applicable character. Later on, David Gordeziani has involved young scientists in studying the above mentioned problems. On their part, the young scientists have managed to develop some modern problems of numerical analysis and mathematical modeling.

Academician Jacques-Louis Lions

He continued to study these problems intensively with his followers [11-14, 16-19, 23, 25-29, 32, 35, 36, 46, 68, 71, 75, 77, 83, 113, 120, 123, 133, 136].

In 1971-1972 David Gordeziani took internship course at the laboratory of Numerical Analysis in University Paris VI and in Grenoble Institute of Applied Mathematics in France. His internship was supervised by the greatest contemporary mathematician, engineer and informatician, academician Jacque-Luis Lions.

During his stay in Paris, he devoted two interesting works to the construction and investigation of the difference schemes [13, 14].

One of the methods of constructing two-dimensional models of linearly elastic homogeneous plates with variable thickness, which are also called prismatic shells, was suggested by Ilia Vekua. Two-dimensional mathematical models for elastic plates and shells were constructed and studied by I. Vekua only in the spaces of classical smooth functions and the relationships between the original three-dimensional model and the obtained hierarchies of two-dimensional models were not investigated. The static two-dimensional models constructed by I. Vekua for general thin shallow shells in Sobolev spaces first were investigated by David Gordeziani in [20] and the existence and uniqueness of solution was proved. In the case of homogeneous isotropic linearly elastic plate with constant thickness the rate of approximation of the exact solution of the three-dimensional static problem by vector-functions of three space variables restored from the solutions of the constructed two-dimensional problems corresponding to I. Vekua's model in the spaces of classical smooth enough functions was estimated in [21].

The next stage of the investigation of dimensional reduction method suggested by I. Vekua and corresponding mathematical models of elastic structures begins with the works of Mariam Avalishvili and David Gordeziani, who noticed that Vekua's dimensional reduction method is a particular case of general methodology, which is based on approximation of the original spaces corresponding to three-dimensional problem by sequences of subspaces with special structure, which define a hierarchy of lower-dimensional models. Applying Vekua's dimensional reduction method two-dimensional dynamical [88, 96] as well as static [89] models were constructed and investigated for homogeneous isotropic linearly elastic prismatic shells. In papers [88, 96], variational formulation of three-dimensional initial-boundary value problem corresponding to prismatic shell with surface forces given along the upper and the power face surfaces and one part of the lateral boundary is considered, and the remaining part of the boundary is clamped. In order to construct a hierarchy of dynamical two-dimensional models of prismatic shell a sequences of subspaces of the spaces corresponding to the original three-dimensional problem is constructed, whose vector-functions are polynomials with respect to the variable x_3 of plate thickness. Considering the original problem on the introduced subspaces a hierarchy of dynamical two-dimensional models is obtained, which in the spaces of smooth enough functions is equivalent to Vekua's dynamical two-dimensional models. The two-dimensional initial-boundary value problems are investigated in corresponding spaces of vector-valued distributions with values in Sobolev spaces defined on two-dimensional space domain and the existence, uniqueness of solution and energy equality is proved, which implies continuous dependence on the given data. Moreover, in this paper convergence result on relationship of the original three-dimensional problem were first obtained and reduced to two-dimensional ones in the case of dynamical problem. It was proved that the sequence of vector-functions restored from the solutions of the two-dimensional models converges in suitable spaces pointwise with respect to the time variable and under additional regularity conditions the rate of convergence is estimated.

The results obtained in [88, 89, 96] and approach presented therein were developed and extended for various models of continuum mechanics. In [98, 99] dynamical and statical two-dimensional hierarchical models of multilayer elastic prismatic shells were constructed and investigated. In the latter papers the convergence of the sequence of vector-functions restored from the solutions of the reduced two-dimensional problems to the solution of the original three-dimensional problem is proved as the order of the model tends to the infinity or the minimum of the thickness of layers tends to zero and under additional conditions the rate of convergence is estimated by the order of the model and the minimum of the thickness of layers. The rate of convergence with respect to the order of the model obtained in [98, 99] improve the corresponding results of the previous papers. In [105] the extension of Vekua's method was used to construct dimensional reduction algorithm for general second order linear elliptic system, coinciding, in particular, with the linear system of the theory of elasticity, given on n-dimensional Lipschitz domain of cylindrical shape with thickness, which may vanish on a part of the boundary. The constructed $n-1$ dimensional problem is investigated in suitable weighted Sobolev spaces, that is caused by thickness degeneration, convergence of the approximate solutions restored from the reduced problems to the solution of the original problem is proved and under suitable conditions the rate of convergence is estimated.

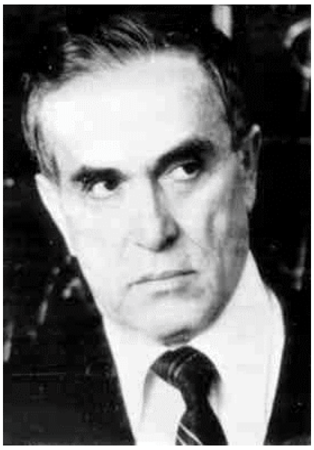
In [111, 119] static and dynamical linear three-dimensional models for homogeneous isotropic elastic general shells were considered, with thickness vanishing on a part of the lateral boundary. Applying variational formulation and approximation of corresponding spaces by suitable subspaces, consisting of polynomials with respect to the variable of shell thickness, hierarchies of two-dimensional static and dynamical models were constructed. Note that the two-dimensional models were obtained for the shell in curvilinear defined by arbitrary C^2 diffeomorphism, that is more general than linear with respect to shell thickness variable x_3 diffeomorphisms, which are usually used for general shells. The two-dimensional models of shells are obtained without any assumption of thinness or shallowness and hence are more exact than two-dimensional models of the shell obtained by I.Vekua. The constructed two-dimensional models were investigated in suitable weighted Sobolev spaces. The convergence of the sequences of vector-functions of three space variables restored from the solutions of the reduced two-dimensional problems to the exact solutions of the original problems is proved and under additional conditions the rates of convergence are estimated. Applying the above mentioned approach and the obtained results dynamical hierarchical two-dimensional models were constructed and investigated for prismatic shells consisting of elastic mixtures in [108, 112], for prismatic shells consisting of solid and fluid parts in [135], and static two-dimensional models for thermoelastic prismatic shells with microtemperatures were obtained and studied in [153].

It should be pointed out that until the papers [129, 138] in the papers devoted to hierarchical modeling of dynamical problems in the theory of elasticity the temperature field was mainly neglected. Dynamical two-dimensional hierarchical model of homogeneous isotropic thermoelastic plate with variable thickness was constructed and investigated in the first order Sobolev spaces in [129] applying the idea of Vekua's reduction method and variational approach when temperature vanishing on the whole boundary and with surface forces given along the upper and the lower faces of the plate. In [138] linear dynamical model for general homogeneous anisotropic thermoelastic plate with variable thickness and various boundary conditions for displacement and temperature fields along the upper and the lower faces of the plate were considered. Applying the idea of Vekua's reduction method and variational approach the hierarchies of two-dimensional dynamical problems in the second order Sobolev spaces were constructed and the relationship between the obtained hierarchical models and the original three-dimensional initial-boundary value problems was investigated. Recently, hierarchical two-dimensional models were obtained within the framework of Lord-Shulman model of the nonclassical theory of thermoelasticity, which depend on one relaxation time. The investigation of the latter hierarchical models and estimation of modeling error required further development and modification of the existing methods used in the previous papers.

In [104] hierarchy of linear one-dimensional models for homogeneous isotropic elastic rod with variable rectangular cross-sections the thickness and width of which may vanish at the ends of the rod were constructed, and the corresponding boundary value problems and the relationship between the original and the reduced problems were investigated in suitable weighted Sobolev spaces. One-dimensional hierarchical models for general linear elliptic problems were constructed and the corresponding dimensional reduction algorithm was investigated in [132].

On the basis of results of investigation of dimensional reduction algorithms for plates, shell and rods pluri-dimensional hierarchical models were constructed for elastic multistructures consisting of several parts with different geometric shapes. Statical hierarchical models of mulistructures consisting of three-dimensional part with general shape, plate clamped in it and rod clamped in the plate was constructed in [93]. Applying I. Vekua's dimensional reduction method and its generalization three-dimensional model for plate was reduced to two-dimensional one and for rod to one-dimensional model. The existence and uniqueness of solutions of boundary value problems corresponding to the constructed hierarchical models of multi-structure defined on the union of three-dimensional, two-dimensional and one-dimensional domain was proved in corresponding Sobolev spaces and the relationship between the constructed pluri-dimensional models and the original three-dimensional one was studied. In [155] hierarchical models of multistructures consisting of three-dimensional part with general shape and multilayer part composed of plates with variable thickness were constructed. A hierarchy of dynamical models defined on the union of three-dimensional and two-dimensional domains for dynamical three-

Academician, Prof. A. A. Samarskii



Academician, Prof. A. Bitsadze

dimensional model for the multi-structure is constructed. The pluri-dimensional initial-boundary value problems, defined on the union of three-dimensional and two-dimensional domains, corresponding to the constructed hierarchical models were investigated in suitable function spaces. The convergence of the sequence of vector-functions of three space variables, restored from the solutions of the constructed pluri-dimensional initial-boundary value problems to the solution of the original three-dimensional problem is proved and under additional regularity conditions the rate of convergence was estimated.

Talks around the mentioned topics were made in Paris at the Institute of Automatics and Informatics (INRIA) in 1977.

David Gordeziani had the honor of working with Shalva Milkeladze, Ilia Vekua, Andria Bitsadze, Alexsander Samarskii, Andrei Tikhonov, Jacque-Luis Lions, Phillip Ciarlet. They played an important role in shaping out his future field of research specialist of Applied Mathematics and Informatics. His very first scientific work was carried out under the supervision of Academician Shalva Mikeladze. Later, during his whole life he respected his supervisor and kept great memory of this outstanding mathematician. In general, Professor David Gordeziani always much appreciated and mentioned with love all his teachers, who greatly contributed to his scientific career.

It is necessary to note the work that he published together with Academician A. Samarskii [29].

It should be noted that the outstanding scientists A. Bitsadze and A. Samarskii in 1969 published a very important article on nonlocal boundary value problem. In 1970 D. Gordeziani devoted his note to this problem and

Academician, Prof. Jacques-Louis Lions and
Prof. D. Cioranescu

called it the Bitsadze-Samarskii problem. According to his approach this non-classical problem can be reduced to a sequence of classical problems and it can be simplified the algorithm for its approximate solution as well [8].

D. Gordeziani along with his disciples and followers devoted many works to the study of nonlocal problems for different types of equations [15, 31, 34, 44, 45, 49, 50, 52-54, 57-62, 64, 66, 67, 70, 72, 74, 78, 79, 81, 84, 86, 94, 114, 115, 140-142, 145, 148, 158, 159].

In 1981 David Gordeziani defended a thesis for a Doctor of Science degree in speciality "Computational mathematics" at the Moscow State University at the Academy Council of Computational Mathematics and Cybernetics (head of Council - Academician A. N. Tikhonov); The title of thesis was "Construction Methods of Approximate Solutions for Some Classes of Multidimensional Problems of Mathematical Physics".

He was given a title of Professor in 1985.

Professor David Gordeziani has published about 200 scientific works, including 4 inventions, 2 patents (USA, Denmark) and 3 monographs. He has received a lot of research grants. He has been supervisor of 7 scientific doctors and 17 candidates. He has supervised many scientific works for Master’s Degree. During many years he was heading various masters and doctoral programs. He was the author and co-author of many interesting syllabuses. Under- and Post-graduate and doctoral works carried out under his supervision have gained variety of diplomas, certificates and prizes on international as well as local conferences held within the framework of educational programs. 7 of his pupils have won presidential scholarship and 4 of them - scholarship of Soros Foundation. Among his students, being under his direct supervision are those, who successfully worked and have still been working abroad (Prof. E. Evseev – Israel, Dr. V. Iucys – USA, Dr. T. Jioev – Russia, Dr. I. Janashvili – Israel, etc.).

With Prof. G. Arsenishvili, Academician A. A. Samarskii, Prof. H. Meladze, Academician S. P. Kurdjumov, Academician A. Bitsadze

David Gordeziani participated in numerous international and other forums, among those are the International Congress of Mathematicians in

Warsaw (1983) and Zurich (1994), IUTAM Symposia, Athens Interdisciplinary Olympiad, etc. He was many times invited to famous Universities of Paris, Rome, Grenoble, Athens, Jena, Moscow, Kiev, Minsk and other cities to give lectures and carry out joint scientific researches.

In 1983 at the International Congress of Mathematicians in Warsaw David Gordeziani gave a talk on investigation of a new type nonlinear parabolic integro-differential equation [37].

It is well known that process of electromagnetic field penetration in the substance is described by the system of Maxwell equations. By initiative of David Gordeziani the Maxwell model with Joule heating were reduced to the following system of Volterra-type nonlinear integro-differential equations

$$\frac{\partial W}{\partial t} + rot\left[a\left(\int_0^t |rotW|^2 \, d\tau \right) rotW \right] = 0 ,$$

where $W = (W_1, W_2, W_3)$ and $a = a(S)$ is defined for $S \in [0, \infty)$.

The existence of global solution of the initial-boundary value problem for scalar and one-dimensional case of Volterra-type nonlinear integro-differential equation and uniqueness for more general cases at first are studied in [38,39].

Based on this system some generalizations were proposed and the following system of averaged type were obtained by Professor G.I. Laptev

$$\frac{\partial W}{\partial t} = a\left(\int_0^t \int_\Omega |rotW|^2 dx d\tau \right) \Delta W .$$

These integro-differential type models are complex and have been studied by many authors intensively since 80ies of the last century.

For a more thorough description of electromagnetic field propagation in the medium, it is necessary to take into consideration different physical effects. First of all, it should be heat conductivity of the medium. Professor D. Gordeziani published works in this direction too [41,42].

Results of these research were applied in practice at I.V. Kurchatov Institute of Atomic Energy and Sokhumi Physical-Technical Institute.

It is worth noting that the researches carried out by David Gordeziani together with his students and colleagues (landslide spread calculation, calculation of flow in city gas-network and its optimization, heat facilities) have found practical applications in technique and agriculture. Along with scientific activity, David Gordeziani had been participating in works dealing with creation of special building mechanisms.

One must note the role of D. Gordeziani in mathematical modeling and in the study of practical problems of high importance.

Dynamics of transfer of pollutants in river water is described by the diffusion equations

$$\frac{\partial \Phi}{\partial t} = \frac{\partial}{\partial x}\left(K_x \frac{\partial \Phi}{\partial x}\right) + \frac{\partial}{\partial y}\left(K_y \frac{\partial \Phi}{\partial y}\right) + \frac{\partial}{\partial z}\left(K_z \frac{\partial \Phi}{\partial z}\right) -$$

$$- V_x \frac{\partial \Phi}{\partial x} - V_y \frac{\partial \Phi}{\partial y} - V_z \frac{\partial \Phi}{\partial z} - u \frac{\partial \Phi}{\partial y} - K(\Phi) + f(x, y, z, t),$$

where Φ is averaged in time concentration of the non-conservative dissolved substance; t is time; x, y, z are spatial co-ordinates (the axis x is horizontal and its direction coincides with the direction of averaged current of all the streams, the axis y is perpendicular to the free surface and it is directed downwards, the axis z is directed across to the stream); K_x, K_y, K_z are coefficients of the turbulent diffusion in the direction of axes x, y, z; V_x, V_y and V_z are averaged in time components of speeds on axes x, y, z; u is hydraulic largest of particles; $K(\Phi)$ is the member characterizing the non-conservativeness of pollutant (one often uses simple approximation of this dependence $K(\Phi) \equiv K \cdot \Phi$, where K is the coefficient of non-conservativeness); $f(x, y, z, t)$ is total intensity of external sources of pollution. In general case, the coefficients K_x, K_y, K_z, V_x, V_y, V_z and $K(\Phi)$ are the functions of a point of space and time.

Boundary conditions in the lower end of the section can be classical or non-classical. Classical condition looks like

$$\frac{\partial}{\partial x} \Phi(t, r)\Big|_{x=\Im} = 0\,; \text{ (condition of full mixing)}$$

non-classical condition is

$$\Phi(t, r)\Big|_{x=\Im} = q \cdot \Phi(t, r)\Big|_{x=\Im-l}\,. \text{ (not local boundary condition)}$$

where q is the coefficient of self-purification of the river on the considered section; ω is the concentration of the pollutant dropped by pollution source in the point $x = \Im$.

For the first time the above nonclassical boundary condition was considered in [69] and was after investigated in many scientific works published together with his followers.

These equations are usually solved numerically, with the help of difference schemes. Here arise some questions, related to the choice of methods for solving various problems depending on the practical realizability, the accuracy and the duration of obtaining the solution on the computer. In particular: a) the analytical description of plane or spatial area for which the equations of diffusion and boundary conditions are investigated, i.e. the analytical description of coastal lines and a river bed; b) the analytical description of the dependence of coefficients of the equation from spatial co-ordinates; c) the analytical description of dependence on spatial co-ordinates and from time of non-uniform parts of the solved equations of diffusion, i.e. powers of pollution sources; d) the correct choice in difference scheme ratios between spatial steps of the grid, and also between them and the step of digitization of time.

These problems and simulation of real pollution process of rivers were solved in some international scientific projects:

1. The project of ISTC (International Science and Technology Center) G-047 “Identification of River Water Pollution Sources by Means of Automated Control Systems”, 1998-2000. (http://www.istc.ru).
2. Georgia-USA (CRDF/GRDF) joint project “Development and research of deterministic and stochastic mathematical models for control and management of pollution level of fluvial waters and their realization by application package”, 2002-2004. (http://www.grdf.ge).

and obtained results were published in [80, 85, 95, 109, 125-128].

The developed methods and algorithms were realized in software packages with participation of Prof. D.Gordeziani:

1. Application package for realization of mathematical models of pollutants transfer in rivers (Version 2.0);
2. Automatic detection of river water emergency pollution sources (Version 2.0).

They were demonstrated at different national and international conferences and exhibitions. Developed packages for environmental monitoring automation and processing of experimental data have taken the first place and were awarded the main prize at the program products exhibition, which were developed and held in Georgia on June 1999 at the Georgian Technical University.

The development and application of numerical regional and local scales atmosphere chemistry transport models (CTMs) are discussed in [67, 73, 107, 131]. Motivation for the employment of these models in atmospheric science is a strong need for application of CTMs to environmental policy and planning and air quality forecasts as an important factor for the reduction of health risks. The problem of pollutants transfer through the troposphere is considered in [67, 73] based on the following mathematical model:

$$
\frac{\partial S}{\partial t} + u\frac{\partial S}{\partial x} + v\frac{\partial S}{\partial y} + w\frac{\partial S}{\partial z} = K_x \frac{\partial^2 S}{\partial x^2} + K_y \frac{\partial^2}{\partial y^2} + \frac{\partial}{\partial z} K_z \frac{\partial S}{\partial z} - \alpha S + F, \tag{1}
$$

where S is concentration of aerosol substance, u, v, w are the axial components of wind velocity (which are defined by integration of full system of the hydro-thermodynamic equations), $K_{x,} K_y$ and K_z are the coefficients of turbulent diffusion; α is the velocity of substance concentration decomposition and transformation, $F(x, y, z, t)$ is the internal source. Equation (1) is solved with the following nonlocal boundary condition

$$
S(x,y,z,t) = q(x,y,z,t)S(x',y',z',t) + \psi\ (\mathrm{x}, \mathrm{y}, \mathrm{z}, \mathrm{t}), \tag{2}
$$

where ψ is a given concentration, $q(x,y,z,t) = [1-\varepsilon(x,y,z,t)]'^{\,\mathrm{P}(xx^1)}$, $0 < \varepsilon < 1$.

In case of two-dimensional model the existence and uniqueness of the regular solution of the problem is proved in [67]. For numerical solution of two-dimensional model decomposition method was used on the basis of which averaged additive models are built up [73]. In the zonal averaged mathematical model predesigned for the Georgian transport corridor pollution influence of orography was taken into account in [67]. Results of numerical calculations have shown that nonlocal boundary conditions should be used [67].

The chemistry transport model [107] consists of a set of continuity equations for the sample of minor constituents complimented by boundary conditions for each transported species which are usually formulated in flux form. In [107] practical aspects of the treatment of boundary and initial conditions and zooming or nesting methods are discussed, i.e. the use of variable resolution and grids are discussed. The European Air Pollution Dispersion model system (EURAD) was planning to chain with the Tbilisi Air Pollution model (TAP) [131]. Using mathematical model (1) based on integration full system of hydrothermal dynamics, distribution of concentration of harmful substances *NOx* at the crossroad of Agmashenebeli and King Tamar Avenues and for the whole territory adjoined to the crossroad have been studied in [131]. Taking as a basis these experimental calculations, it was concluded that the growth of harmful substances wholly depends on the traffic intensity as well as on the light-signal's placement and working cycle [131].

Migration and distribution of the oil spilled in the sea is studied by the numerical model based on the nonstationary 2D advection-diffusion equation for nonconservative substance [149]. The change of oil concentrations owing to different factors (evaporation, dispersion, emulsification, sedimentation, microbiological decomposition, etc.) is taken into account parametrically. In this paper the iterative procedure for reducing non-classical problems to a sequence of classical initial-boundary value problems of Cauchy-Dirichlet is proposed. The convergence of iterative process is proved and estimation of the speed of convergence in certain cases is given. By the operator decomposition method of the original problem the absolutely monotone averaged additive difference scheme of parallel calculation is constructed and investigated. The questions of stability, convergence and accuracy of such schemes are discussed in [149]. Numerical model describing mineral oil infiltration into the soils was elaborated in [147]. Results, obtained by numerical calculations have both as theoretical as practical interest for the territory of Georgia [147]. To the analysis of some mathematical models describing a movement of subsoil waters (liquids) into the soil having the non-homogeneous multilayer structure in the vertical direction was devoted the paper [160]. The corresponding systems of two-dimensional differential equations in stationary and non-stationary cases are considered in [160]. For the first one the problem with classical and non-classical boundary conditions is stated. For numerical solution of the problem with nonlocal boundary conditions the iteration process is constructed, which allows one to reduce the solution of the initial problem to the solution of a sequence of the classical Dirichlet problems. Some results of numerical calculations for the soil having two-layer structure are presented. These problems and simulation going on in environment were studied in some international scientific projects (the list of these projects is given below) and obtained results were published in [67,73,107,131,143,147,149-151,154,160].

Scientific grants:

1. Grant of Shota Rustaveli National Scientific Foundation # 09-614-5-210 on "Assessment of Risk Factors of Emergency Cases at Oil and Gas Pipelines and Possible Pollution of Environment by Means of Mathematical Modelling" 2010-2011.
2. Grant No. 1.01.81 of Education and Scientific Ministry of Georgia for project proposal "Non-ordinary Hydro-meteorological Processes and Environmental Pollution Mathematical and Numerical Modeling " 2005.
3. Grant of Scientific and Tecnology Department of Georgia for project proposal "Mathematical Modeling of Georgian Transport Corridor Pollution for Risk Assessment" 2001.
4. Grant of Scientific and Tecnology Department of Georgia for project proposal "Mathematical Modeling of Ecological Regime of Georgian Territory" 1997- 1998.

Natural phenomena, such as earthquakes and floods, may cause occurrence of cracks in the pipes of great diameter, and in gas and petrol tanks. Increase of the number and dimensions of cracks, from its part, causes to increase the volume of leakage that will result in great ecocatastrophe. The study of boundary value problems for the composite bodies weakened by cracks has a great practical significance. Equations of the antiplane elasticity theory for composite bodies weakened by cracks can be used as an initial approximation of the mathematical model of problems provided by the project Grant #GNSF 09-614_ 5-210. In the articles [144,150,151,154] two methods of solution of elasticity theory antiplane problems for piece-wise homogeneous orthotropic plane weakened by cracks are investigated by integral equations ([143, 150]) and finite-difference methods ([151], [154]).

In Paris

Professor Gordeziani has taken part in organizing and carrying out many international and local congresses, symposia, conferences, schools in the area of computational mathematics, mechanics, shell theory, hydrodynamics, magnetic hydrodynamics, informatics.

Professor Gordeziani was invited as an official opponent of a Ph.D. theses in scientific-educational centers of France, Germany, Italy, Russia, Denmark, Poland, Greece, Ukraine, Belorussia, Uzbekistan, Azerbaijan and Moldova. Often he was an opponent and expert in Georgia in the areas of Mathematics, Informatics and Mechanics.

In 1993 Professor Gordeziani was elected a member of International Academy of Computer Sciences and systems; he was a member, president, and honorary president of Georgian Academy of Natural Sciences, the member of Georgian Engineering Academy, one of the founders of the Georgian National Committee of the Theoretical and Applied Mechanics. He was also a member of Coordination Council of the USSR Academy of Sciences in Mathematical Modeling, deputy head of Coordination Council of Georgian Academy of Sciences in Mathematical Modelling, member of Iv. Javakhishvili Tbilisi State University Scientific Council (1985-2006), etc.

D. Gordeziani's work as a professor at Iv. Javakhishvili Tbilisi State University was productive. He had been working there for decades of years: he was a head of Chair of Informatics and Computational Mathematics and was delivering lectures, sharing his knowledge and experience with his students; he was carrying out joint scientific works with most successful students. The contents of his lectures was very diverse: Programming on Computer, Computational Mathematics, Mathematical Modelling, Functional Analysis and Computational Mathematics, Vekua Plates and Shell Theory, Finite-Difference Methods for Solution of Partial Differential Equations, Decomposition Methods, Numerical Analysis, Scientific Compating, Mathematical Modeling and Computational Mathematics, Numerical Methods of Linear Algebra, Computer Algebra, etc. He was a supervisor of several bachelor, master, and doctoral programs.

Particular note should be made pertinent to David Gordeziani's activities at I. Vekua Institute of Applied Mathematics of Iv. Javakhishvili Tbilisi State University. There he passed the way starting from the position of a senior researcher up to an Director of the Institute. During the last ten year he was leading one of the main scientific directions (Mathematical Modeling and Computational Mathematics) of the Institute.

During different time periods Professor David Gordeziani had been a board member of Scientific Quality and Title Council; Member of the Council on Problematic Issues at Russian Academy of Sciences in Mathematical Modeling; Member of German Council of Informaticians; Member of Editorial Boards of several authoritative scientific journals. He had been one of the Founders and Chief Editor of Georgian-Spain joint scientific journal "Applied Mathematics, Informatics and Mechanics".

Such outstanding scientists as Acad. Ph. G. Ciarlet (France), Acad. A.A. Samarskii (Russia), Acad. V.A. Ilyin (Russia), Acad. A.V. Bitsadze (Russia), Acad. E. Moiseev (Russia), Professors: A.L. Skubatchevskii (Russia), B.P. Paneyakh (Russia), M. Bernadou (France); I. Babushka (USA), M.Vogelius (USA), S. Iensen (USA); V.L. Makarov (Ukraine), V.I. Gulyaev (Ukraine), V.A. Bagenov (Ukraine), P.P. Lizunov (Ukraine), P.N. Vabishevich (Russian), P.P. Matus (Belarus), S. Abrachin (Belarus), V. Asmokil (Belarus), N. Ionkin (Russia), S. Lapko (Ukraine), N. Chadaev (Ukraine), M. Dauge (France), M. Lods (France), N. Long (USA), M. Sapagovas (Lithonia), A. Guchin (Russia), G. Laptev (Russia), and many other Georgian and foreign scientists have made references and quoted the works of D. Gordeziani in their Monographies and articles.

Below one can find the list of some monographs, handbooks, reviews and historical works, encyclopedias of well-known researchers of Computational, Applied Mathematics, and Mechanics, where papers of Professor David Gordeziani are cited (see also http://www.books.google.com/books?q=gordeziani d):

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3. Ph. G. Ciarlet. “Mathematical Elasticity”. Vol. II, 1997, Elsevier, 1997 (monograph);
4. Ph. G. Ciarlet. “Mathematical Elasticity”. Vol. III, 1998, Elsevier, 1998 (monograph);
5. M. Vogelius and I. Babushka. “On a Dimensional Reduction Method”. I, II, III port., Math. of Comput.., 1981, vol. 37, # 156 (Review);
6. А.А. Самарский. „Теория разностных схем“. М., Наука, 1983 (textbook);
7. П.И. Вабищевич, А.А. Самарский. „Вычислительная теплопередача“. Изд. УРСС, Москва, 2003 (monograph);
8. Международная конференция математиков в Нице, Доклады Советских Математиков, Москва 1972 (Review of important results of the scientific area);
9. А.А. Самарский, П.Н. Вабищевич, П.П. Матус. „Разностные схемы с операторными множителями“. Минск, 1998 (monograph);
10. Т.Ю. Хома. „Обобщенная теория оболочек“. Киев, Наукова-Дутка, 1986 (monograph);
11. Monique Dauge, Erwan Faou, Zohaz Yosibash. “Plates and Shells Asymptotic Expansions and Hierarchical models”/ Encyclopedia for Computational Mechanics, 2004, Edited by Ervin Steinm Rene de Borst Thomas J.R. Highes (encyclopedia);
12. А.В. Бицадзе. „Некоторые классы уравнений в частных производных“. М. Наука, 1981 (monograph);
13. E. Obolashvili. “Higher Order Pratial Differential Equations in Clifford Analysis. Effective Solutions to Problems”. Progress in Mathematical Physics, 28, Birkhauser Boston, Inc., Boston, MA, 2003 (monograph);
14. Jensen S. “Adaptive Dimensional Reduction and Divergence Stability”. Math. Mod., v. 8, #9 (review);
15. „История отечественной математики“. т. 4, кн. 2, СССР, 1917-1967;

16. Dikmen M. „Theory of thin elastic shells“(monograph type textbook).

He spoke fluent Russian, French and English and the knowledge of languages facilitated his relations with foreign colleagues.

With his wife Nanuli Turdzeladze

His hobby was sculpture, music and history. Everyday life was a complete sporting life, especially he liked Yoga.

A faithful companion and spouse of Professor David Gordesiani, educated in Mathematical Linguistics, Ms. Nanuli and two daughters Candidates of Physics and Mathematics Nato and Eka, tried their best to support husband and father who was in love with mathematics, his disciples, colleagues and family.

D. Gordeziani has published several works together with his daughters as well [47, 48, 51, 54-56, 58, 59, 62, 64, 68, 69, 71, 75, 120].

All those who had been even slightly acquainted with him, were saddened by his death and sincerely sympathize with his family.

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Plenary Talks

Modeling Symmetric Eigenvalue Problem with Time Dependent Equations

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Symmetric eigenvalue problem is formulated as equivalent problem for corresponding time dependent equation. Examples of application of the developed approach are given.

Nonlinear Estimates for Surfaces in Terms of Their Fundamental Forms, and Applications

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It is well known that a surface can be recovered from its two fundamental forms if they satisfy the Gauss and Codazzi-Mainardi compatibility equations on a simply-connected domain, in which case the surface is uniquely determined only up to isometric equivalence. It is less known that in this case the surface becomes a continuous function of its fundamental forms, again up to isometric equivalence, for various topologies, such as the Frechet topology of continuously differentiable functions, or those corresponding to various Sobolev norms. In this talk, we will review such continuity results obtained during the past fifteen years, with special emphasis on those that can be derived by means of nonlinear Korn inequalities on a surface. We will also mention potential applications of such results, such as the intrinsic approach to nonlinear shell theory, where the unknowns are the fundamental forms of the deformed middle surface of a shell, or the numerical reconstruction of the Earth surface by means of the knowledge of its fundamental forms on a discrete grid.

Few Results Concerning Non Destructive Testing of Structures

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The non destructive testing (NDT) consists in detecting the existence, the shape and the position of a defect in a structure. In this talk the ultrasonic method is discussed in the framework of NDT. The use of a mathematical model is a necessity for solving this challenge . In fact, several problems should be tackled. For instance for a given frequency, a defect can be hidden by the ultrasonic waves used. The goal of the mathematician is to give informations on these invisible defects in order to adjust the frequencies used. Furthermore, only a part of a structure is equipped with sensors and therefore a lot of the signal reflected by the defect is lost (or can be lost). The use of a so-called harvester (based on a control method) seems to be a promising improvement in such a technology. In addition the problem set is an inverse one with many controllability difficulties. Finally a fast simulator would be required in order to operate in real time. This talk is mainly oriented towards a definition of the problems to be handled and partial results are given.

Laplace-Beltrami Equation on Hypersurfaces and Γ-Convergence

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Let us consider heat conduction by an "isotropic" media, governed by the Laplace equation with the classical Dirichlet-Neumann mixed boundary conditions on the boundary in the layer domain $\Omega^\varepsilon := \mathcal{C} \times (-\varepsilon, \varepsilon)$ of a thickness 2ε. More precisely we impose zero Dirichlet and non-zero Neumann data on the corresponding parts of the boundary

$$\begin{aligned}
&\Delta_{\Omega^\varepsilon} T(x,t) = f(x,t), && (x,t) \in \mathcal{C} \times (-\varepsilon, \varepsilon),\\
&T^+(x,t) = 0, && (x,t) \in \partial\mathcal{C} \times (-\varepsilon, \varepsilon),\\
&\pm(\partial_t T)^+(x, \pm\varepsilon) = q(x, \pm\varepsilon), && x \in \mathcal{C},
\end{aligned}$$

where $\pm\partial_t = \partial_\nu$ represents the normal derivative on the surfaces $\mathcal{C} \times \{\pm\varepsilon\}$. Here $\mathcal{C} \subset \mathcal{S}$ is a smooth subsurface of a closed hypersurface $\mathcal{S}$ with smooth nonempty boundary $\partial\mathcal{C}$.

The suggested approach is based on the fact that the Laplace operator $\Delta_{\Omega^\varepsilon} = \partial_1^2 + \partial_2^2 + \partial_3^2$ is represented as the sum of the Laplace-Beltrami operator on the mid-surface and the square of the transversal derivative: $\Delta_{\Omega^\varepsilon} T = \sum\limits_{j=1}^{4} \mathcal{D}_j^2 T = \Delta_{\mathcal{C}} T + \partial_t^2 T$.

In the report we will review what happens with the above mentioned mixed boundary value problem when the thickness of the layer converges to zero in the sense of Γ-convergence. It is proved that the limit coincides with the Dirichlet BVP for the Laplace-Beltrami equation, which is described explicitly. It is shown how the Neumann boundary conditions from the initial BVP transform during the Γ-limit and wanders to the right hand side of the limit BVP. For this we apply the variational formulation and the calculus of Günter's tangential differential operators on a hypersurface and layers, which allow global representation of basic differential operators and of corresponding boundary value problems in terms of the standard Euclidean coordinates of the ambient space $\mathbb{R}^n$.

This part of the investigation has appeared in the paper [1]. The finite element method for the solution appeared in [2].

The final goal of the research is to derive a shell equation with the Γ-convergence, write the equations of the shell surface in Günter's derivatives and work out efficient numerical solution to such equations.

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Geometry of the Space of Curves. Application to a Problem in Endodontics

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In the first part of the talk we will review the geometry of spaces of maps with special emphasis in the spaces of curves. The second part will be devoted to describe an application to a problem proposed by the stomatology researchers B. Buenrostro and L. Forner concerning the root canal curvature of natural human teeth. We will report the results of our joint work in progress.

On the First Eigenvalue of Free Vibrating Membrane (of Neumann-Laplace Operator)

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This talk is devoted to connections between geometry of bounded simply connected domains and the Neumann eigenvalues of the Laplace and p-Laplace operator. Main results concern to a large class of bounded non convex domains under some additional restrictions on its geometry. In the plane case this class includes quasi discs (images of the unit disc under quasi conformal homeomorphism of the plane). The talk is focused on lower estimates or the first eigenvalue into non convex domains and its dependence on (quasi) conformal geometry. Recall that quasi discs can have fractal boundaries.

Situation in space domains is more complicated and will be discussed also.

This is a joint work with A.Ukhlov and V.Pchelintsev

Lipschitz Continuity of the Fréchet Gradient in an Inverse Coefficient Problem for a Parabolic Equation with Dirichlet Measured Output

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Dedicated to Professor David Gordeziani on his 80th birthday

The inverse problem of identifying the leading coefficient $k(x)$ in

$$\begin{cases} u_t(x,t) = (k(x)u_x(x,t))_x, \quad (x,t) \in \Omega_T := (0,l) \times (0,T], \\ u(x,0) = 0, \quad 0 < x < l, \\ -k(0)u_x(0,t) = g(t), \quad u_x(l,t) = 0, \quad 0 < t < T, \end{cases} \tag{1}$$

from the measured temperature $f(t)$ at the left boundary $x = 0$ of a nonhomogeneous rod:

$$f(t) := u(0,t), \quad t \in [0,T] \tag{2}$$

is studied.

The main result here is the Lipschitz continuity of the Fréchet gradient of the Tikhonov functional

$$J(k) = \frac{1}{2} \|u(0,\cdot;k) - f\|^2_{L^2(0,T)}, \; k \in \mathcal{K}$$

corresponding to inverse coefficient problem (1)-(2). In addition, the compactness and Lipschitz continuity of the input-output operator $\Phi[k] := u(x,t;k)|_{x=0^+}$, $\Phi[\cdot] : \mathcal{K} \subset H^1(0,l) \mapsto H^1(0,T)$, as well as solvability of the regularized inverse problem, the Lipschitz continuity of the Fréchet gradient of the Tikhonov functional are proved. Furthermore, relationships between the sufficient conditions for the Lipschitz continuity of the Fréchet gradient and the regularity of the weak solution of the direct problem, as well as the measured output $f(t) := u(0,t;k)$ are established. In particular, it follows from the obtained results that the Lipschitz continuity of the Fréchet gradient of the Tikhonov functional requires more regular coefficient $k(x)$ and the measured output $f(t)$.

One of the derived lemmas also introduces a useful application of the Lipschitz continuity of the Fréchet gradient. This lemma shows that an important advantage of gradient methods comes when dealing with the functionals of class $C^{1,1}(\mathcal{K})$. Specifically, this lemma asserts that if $J \in C^{1,1}(\mathcal{K})$

and $\{k^{(n)}\} \subset \mathcal{K}$ is the sequence of iterations obtained by the Landweber iteration algorithm $k^{(n+1)} = k^{(n)} + \omega_n J'(k^{(n)})$, then for $\omega_n \in (0, 2/L_g)$, where $L_g > 0$ is the Lipschitz constant, the sequence $\{J(k^{(n)})\}$ is monotonically decreasing and $\lim_{n\to\infty} \|J'(k^{(n)})\| = 0$.

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New Trends in Function Spaces Theory and Applications to the BVPs

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The goal of our talk is to introduce and explore new function spaces having an ideological link with grand Lebesgue spaces), introduced by T. Iwaniec and C. Sbordone. Our aim is to define new scale of "grand" spaces which are more finaly turned to the application in non-linear PDEs and various BVPs. These spaces serve the integral operators of linear and nonlinear Harmonic Analysis. We intend to present the boundedness criteria for basic integral and differential operators in new function spaces and to give some applications to the BVPs for analytic functions.

Anti-Unification in Description Logic EL

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In unification theory, anti-unification is a process for computing generalizations, i.e., to obtain more general objects from concrete ones. The objects are represented as logical expressions (terms, term sequences, clauses, etc.) and the generalization relation is specified (e.g., with respect to syntactic equality, equality modulo given equations, subsumption, etc.). The name anti-unification underlines its duality with unification that essentially computes common instances of the given general objects. Current research on anti-unification is dominated by practically oriented topics. It is not surprising, because generalization computation, in one form or another, is a very important ingredient of various applications in reasoning, learning, information extraction, data compression, software development and analysis, etc.

In this talk, we present anti-unification for the description logic EL. Description logics are a family of knowledge representation languages, which provide logical formalism for ontologies and Semantic Web. Compared to the other description logics, EL has a limited expressive power, but it attracts significant interest due to its efficient inference (subsumption problem is polynomial) and successful application in biomedical informatics (large biomedical ontologies are defined in EL).

We introduced the notion of least general generalization in EL, which generalizes simultaneously the notions of least common subsumer and concept matching. The idea of generalization of two concepts is to detect maximal similarities between them, and to abstract over their differences uniformly. We show that generalization for EL is finitary, present an anti-unification algorithm, discuss its properties, and report on preliminary experimental evaluation.

This is a joint work with Boris Konev (University of Liverpool).

Exact and Approximate Solutions of the Spectral Problems for the Differential Schrödinger Operator with a Polynomial Potential in R^k, $k \geq 2$

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Spectral problems for the Schrödinger operator with polynomial potentials in R^k, $k \geq 2$ are considered in this paper. For the set of specifically given potentials we found in the exact form a few first eigenvalues of the given problem using a technique based on the combination of functional-discrete (FD-) method and Maple computer algebra system. In the case when the application of traditional FD-method to the given problem leads to a divergent scheme (the power of the polynomial potential of at least one of the independent variables exceeds 2) we propose a modification of FD-method, which is proved to be effective for the class of the problems under consideration. The obtained theoretical results are illustrated by numerical examples.

Rule-Based Programming: The Mathematica Experience

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Rule-based programming provides a common framework to model the two main processes of concern to computer science: Computation and Deduction. Both of them can be expressed as sequences of transformations that lead to a desired result or proof. Various kinds of systems of conditional rewrite rules emerged as suitable formalisms for the specification of stepwise transformations, whereas the sequences of transformations of interest are controlled by strategies. The last two decades witnessed the advent of powerful formalisms, such as rewriting logic and rewriting calculi, that led to the emergence of rule-based languages and systems, like Elan, Maude, and Stratego. Their practical applications span from computations in various systems to deduction in theorem provers, program transformation, implementation of constraint solvers, access control policies. In this talk I will describe our experience with the design of a calculus for rule-based

programming called RhoLog, its implementation in Mathematica, and some practical applications.

On Some Parallel Algorithms for Approximate Solution of Problems of Mathematical Physics

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The present talk is devoted to the investigation of special decomposition methods for stationary and nonstationary problems of partial differential equations: the decomposition of the basic area or the basic operator of the initial problem. These methods are based on the reduction of the solution of initial problem to the solution of some more "simple" sub-problems and open the great possibilities in designing algorithms of parallel implementation and creation the program products for computers. We consider also the parallel version of the Schwarz alternating method, based on area decomposition. The independent problem is the solution of difference problems representing itself the system of linear or nonlinear algebraic equations. The parallel iterative methods for the numerical solution of nonlinear equations and systems of equations will be considered as well. In the talk the primary attention will be inverted on the works, conducted in the Tbilisi State University and I.Vekua Institute of Applied Mathematics.

On I. Vekua's Method of the Consistent Theory of Elastic Shells

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The methods of tensor calculus, as well as those of surface theory in tensor notation [1], are systematically employed in the talk.

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Application of Bdie Method in the Theory of Acoustic Scattering by Inhomogeneous Anisotropic Obstacles

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We consider the time-harmonic acoustic wave scattering by a bounded layered anisotropic inhomogeneity embedded in an unbounded anisotropic homogeneous medium. The material parameters and the refractive index are assumed to be discontinuous across the interfaces between inhomogeneous interior and homogeneous exterior regions. The corresponding mathematical problem is formulated as a boundary-transmission problem for a second order elliptic partial differential equation of Helmholtz type with discontinuous variable coefficients. We use the results obtained in [1,2] and show that with the help of localized potentials constructed by a harmonic parametrix the boundary-transmission problem can be reformulated as a system of localized boundary-domain pseudodifferential equations (LBDIE) and prove that the corresponding localized boundary-domain pseudodifferential operator is invertible in appropriate function spaces. This leads to the unique solvability result for the original acoustic wave scattering problem with arbitrary frequency parameter. The isotropic case has been analyzed by P.Werner [3] and P.Martin [4].

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On the Modeling of Transport Phenomena in Continuum and Statistical Mechanics

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The formulation of balance laws in continuum and statistical mechanics is expounded in forms that open the way to revise and review the correspondence instituted, in a manner proposed by Irving and Kirkwood and improved by Noll, between the basic balance laws of Cauchy continua and those of standard Hamiltonian systems of particles.

Computation of Spectral Characteristics for Charged Integral Equations

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The Rayleigh-Ritz and the inverse iteration methods are used in order to compute the eigenvalues of charged Fredholm-Stieltjes integral equations, i.e. Fredholm equations with respect to suitable Stieltjes-type measures. Some applications are shown, including approximation of the relevant eigenfunctions. Starting from the problem of a string charged by a finite number of cursors, a survey including the extensions to the 2D and 3D dimensional problems is presented.

Corner Calculus for Higher Singularities

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The results of our talk are partly obtained in joint work with Der-Chen Chang (Georgetown University, Washington, DC) and Mahdi Hedayat Mahmoudi (University of Potsdam). We outline recent achievements of the program for establisheing parametrices of elliptic partial differential equations on configurations with corner singularities in terms of pseudo-differential algebras. Those are based on operator-valued Mellin symbols and extend the well-established edge pseudo-differential calculus with its symbolic hierarchies, see also a joint monograph with D. Kapanadze, Tbilisi, and other monographs of the speaker. The operators act in weighted corner Sobolev spaces with multiple weights, formed by an iterative approach, starting with conical and edge singularities. The spaces are modeled on Hilbert spaces, equipped with strongly continuous group actions. We illustrate the approach for singularities of second order, where the model cone of wedges itself is based on a compact space with edge singularities.

On Investigation of Two Problems

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In this report we will be discussing some results of investigations connected to the problems of Informatics and Numerical Methods of Linear Algebra: (1) The new method of the multiplication of polynomials from one variable is created by which we estimate the order of a number of multiply of n-digital integers. In this way corresponding results of A.Toom-S.A.Cook [1, ch.4, point 4.3.3] are refined; (2) The full problems of finding the eigenvalues with high order of accuracy for sufficient big matrices of special classes by stability method are solved using methodologies of U.J.J. Le Verrier-D.Faddeev, D.H.Lemer.

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20 Minutes Talks

Transfer of Generalized Groupoid Action Along a Morita Equivalence

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The authors define and study fibrations of topological groupoids in [2]. They interpret a groupoid fibration $L \to H$ with fibre G as an action of H on G by groupoid equivalences. The idea is the following. An action of H on G should give a transformation groupoid $L := G \rtimes H$ that contains G and comes with a continuous functor $L \to H$. Thus defining actions of topological groupoids on topological groupoids amounts to characterising which chains of continuous functors $G \hookrightarrow L \to H$ correspond to actions. We require $L \to H$ to be a "groupoid fibration" with "fibre" $G \subseteq L$. This gives the same notion of action if H is étale. So, all this give the authors of [2] a possibility to generalise a topological groupoid action.

My result shows how can we transfer this generalized action along a Morita equivalence. In more details, if I have two Morita equivalent topological groipoids G and K and third topological groupoid H which acts on G, (this means that there is a topological groupoid L and a groupoid fibration $L \to H$ with fibre G) then I can construct a topological groupoid R and groupoid fibration $R \to H$, such that R and L are Morita equivalent and K is a fibre of fibration $R \to H$. This all gives us a generalized action of H on L.

Acknowledgment. This research is a part of my doctoral dissertation under supervising professors Ralf Meyer and Chenchang Zhu. My doctoral program is funded by "LEPL International Education Centre".

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Adaptive Multischeme Refinement in the Spatially Two Dimensional Case

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Multischeme is smart combination of several different numerical schemes and space time meshes for hyperbolic conservation laws. It enjoys important qualities such as approximation, stability, mass conservation and convergence. Multischemes are implemented as C++ code. Several adaptation strategies are discussed. Numerical tests demonstrate efficiency of the developed approach.

Investigation of a Nonlinear Dynamical System Describing the Process of Linguistic Globalization

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Recently, mathematical modeling of social processes is of special interest. Earlier we have offered and investigated by means of computer modeling, mathematical models of bilateral and two-level assimilation (micromodels on the example of two powerful states with various state languages assimilating the population of the third state formation (autonomy) with less widespread language; the first powerful state assimilating the population of the second less powerful state and autonomy which population, in turn, assimilates as well the second state).

In this work the new nonlinear continuous mathematical model of linguistic globalization is considered (macromodel). Two categories of the population of Earth are considered: interfering and promoting an ascendent position of English. At positive demographic factor of the population interfering globalization and negative demographic factor of the population promoting globalization it is shown that the dynamic system describing this process allows existence of two topological not equivalent phase portraits (steady node, a limit cycle).

At some restrictions for model parameters, the theorem of absence of periodic trajectories of dynamic system is proved and the asymptotic stable position of balance (a limit cycle) is found, i.e. the equilibrium coexistence of both categories.

It is proved that linguistic globalization at positive demographic factor of the population interfering globalization and negative demographic factor of the population promoting globalization is impossible.

In case of positive demographic factors of the population interfering and promoting globalization it is shown that the dynamic system describing this process allows existence of two topological not equivalent phase portraits (a global attractor, steady node (full linguistic globalization), a limit cycle).

Thus, it is established that full linguistic globalization is impossible if the demographic factor of category of the population of Earth promoting domination of English isn't positive. Full linguistic globalization is possible only in case of positivity of demographic factor of category of the population of Earth promoting domination of English and some restriction for the model parameters connected with assimilation coefficient.

Parallel Type of Decomposition Scheme for Quasi-Linear Abstract Hyperbolic Equation

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We consider an abstract hyperbolic equation with a Lipschitz continuous operator, where the main operator is a sum of finite number self-adjoint and positive definite operators. Semi-discrete implicit difference schemes corresponding to the summand operators are solved independently (parallelly) on each local interval. It is proved that the weighted sum of solutions of the semi-discrete implicit difference schemes converges to the exact solution of the given abstract hyperbolic equation.

Optimal Observation Network Assessment for Controlling High Dimensional Advection-Diffusion-Reaction Equations

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Geophysical reactive flow as simulated by solvers of advection-diffusion-reaction equations are mostly formulated as an initial value problem. However, in practice the initial values (often of dimension of 10^7-10^8, are poorly known and and estimated by a data assimilation procedure, typically a spatio-temporal Best Linear Unbiased Estimator (BLUE). In case of boundary values being poorly known as well, the data assimilation procedure must be generalized to a joint optimisation problem to estmate both parameter

families in tandem, rendering the optimisation problem more ill-posed. The presentation addresses the question, which parameters can be optimised, given an observation network, in the context of atmospheric chemistry simulations, with emission rates as insufficiently known boundary values. The method proposed is based on a Kalman smoother framework, identifying singular vectors of sensitivity to observations, separately for initial values and emission rates as boundary flow parameters. For the nonlinear reactive flow maximal sensitivities are calculated by the Raleigh-quotient with Oseledec-operator.

The One-Dimensional Modified Weyl-Berry Conjecture: an Elementary Approach

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Let Ω be a bounded domain in $\mathbb{R}^n$ with boundary $\delta\Omega$ and consider the eigenvalue problem:

$$-\Delta u = \lambda u \text{ in } \Omega \text{ and } u\,|_{\delta\Omega} = 0.$$

Its set of eigenvalues, $0 < \lambda_1 \leq \lambda_2 \leq \ldots \leq \lambda_k \leq \ldots$ - each eigenvalue being repeated according to (algebraic) multiplicity - is countable and we can define the eigenvalue counting function as:

$$N(\lambda) := \#\{0 < \lambda_k < \lambda\},$$

for a given positive λ. The modified Weyl-Berry conjecture [1] for the asymptotics of the eigenvalues of the Laplacian on a bounded open subset of the line then states that:

$$N(\lambda) = \pi^{-1}\,|\Omega|_1\,\lambda^{\frac{1}{2}} + \mathcal{O}(\lambda^{\frac{d}{2}}),$$

with $|\Omega|_1$ being the one-dimensional Lebesgue measure of Ω and $d \in [0,1]$ the Minkowski dimension of the boundary. Proved in 1993 by M.L. Lapidus and C. Pomerance [2], they incidentally discovered an interesting connection with the Riemann hypothesis. A simplified proof may be found in [3].

Based upon techniques developed previously by the author [4, 5], it will be shown how to obtain some of the key results on the one-dimensional modified Weyl-Berry conjecture through elementary methods.

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Adjoint Development for Linear Advection Schemes in ICON Model of the German Weather Service

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Among the most advanced and sophisticated methods for state analysis of an atmospheric system, is 4-dimensional variational data assimilation method (4D-var). The numerically challenging task of this approach is the development and application of the adjoint model components. We aim at developing adjoint solver for the linear advection equation in the German Weather Servise's ICON model, as a part of the variational data assimilation. For the tracer accuracy of numerical schemes is vital. It is even more important for applications in space-time variational data assimilation with adjoint model version.
New approach for adjoint development by means of adding source term is introduced. It has several benefits compared to the traditional adjoint model building technique. The stability, convergence and accuracy of the adjoint scheme is investigated. The method is successfully implemented in the ICON model in the framework of 4d-var data assimilation. Numerical test using both approaches are given and compared against each other.

Asymptotic Properties of Solution and Difference Scheme for One Nonlinear Integro-Differential Model

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The nonlinear integro-differential equations and their systems describe various processes in physics, economics, chemistry, technology and so on. It is doubtless that the study of qualitative and structural properties of the solutions of initial-boundary value problems for those equations and systems, construction and investigation of discrete analogues and the study of numerical algorithms are very important. One type of integro-differential systems arise, for instance, in mathematical modeling of the process of penetrating of magnetic field into a substance. It is known that the mentioned process is described by the system of Maxwell equations.

One nonlinear partial integro-differential model is considered in the present study. The model is obtained by reducing of the above-mentioned Maxwell equations to the integro-differential form. Initial-boundary value problem with Dirichlet boundary conditions is considered. Asymptotic behavior as $t \to \infty$ of solutions is studied. Rates of stabilization are given. Stabilization and convergence of discrete analogs are proven. Wider classes of nonlinearity are investigated than the ones studied earlier. Various numerical experiments are carried out. Results of numerical experiments with the corresponding graphical illustrations are given and compared to the theoretical ones.

An Alternative Transient Analysis for Closed Semi-Markov Queuing Systems

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In Mathematical Theory of Reliability and Queuing Theory (QT) it is generally accepted, that the transient (time dependent) investigation of closed Semi-Markov Queuing systems is the problem of first rate importance [1 - 2].

The point is that time dependent investigation of Semi Markov systems traditionally is considered to be a very difficult theoretical problem. Many prominent experts have been involved in this matter although the results are quite scarce [4].

At the same time, such investigation of the models often is very important also for practical applications, particularly, in the problems of reliability and dependability analysis [4, 5].

In this paper, unlike classical QT we offer a novel method for investigation of closed Semi-Markov queuing systems. To illustrate the method we consider M/G/1/N model.

Using pure probabilistic argumentation, we prove the theorems here, which, on one hand simplifies M/G/1/N queuing systems analysis using supplementary variables method. On the other hand, it allows not using partial differential equations system at all in non classical boundary value problem of mathematical physics, and directly yields the systems solution in terms of operational calculus.

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Calculation of Plates and Shells with Ribs Rigidity by the Finite Element Method

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Seit Bliadze

In this talk, the development of the stages of calculating plates and shells with stiffeners is described.It is noted that the theory of ribbed shells is one of the most controversial and incomplete sections of the general theory of shells.We consider shell calculations with longitudinal and transverse stiffeners by the finite element method. In the first embodiment, the stiffener is considered as a beam end element. In the second case, the edges are approximated as thin-walled plates and, finally, in the third case, the edge is represented as a three-dimensional finite element.A comparative analysis of the theoretical and experimental results is given.

Examined is the ribbed shell of double curvature, whose dimension in the plan is 565× 370mm, the distance between the longitudinal edges is 16 mm, the distance from the edge to the transverse rib is 120 mm, and the distance between the transverse ribs is 340 mm. The thickness of the longitudinal edges is 7 mm, the transverse ribs are 3.5 mm, the ribs height is 26 mm, the thickness of the shell itself is 4.5 mm. Material: aluminum alloy AK-4. Modulus of elasticity of material E = 7100 kgf / mm^2, Poisson's ratio$\nu = 0.3$, the pressure on the surface of the shell is p = 0.0098 Mpa

Boundary conditions: two opposite narrow ends are fixed rigidly, and the remaining edges are freely supported.

The shell modelFor the first case, the entire shell is modeled by six nodal three-dimensional finite elements, in this case the maximum displacement is$w = 0.077$mm, Fig.1 (In the center of the shell), and the equivalent stress of von Mises equals1.035 kgf/mm^2, Fig.2.

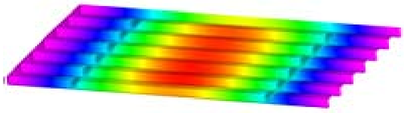

Figure 1: Displacement

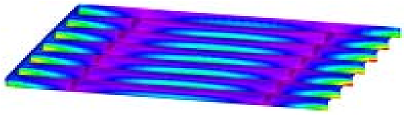

Figure 2: Von Mises stress

In the second case, the base and ribs of the shells were modeled by 4-node plate-like elements;Analysis showed that the maximum displacement is in the center of the shell and is equal to $w = 0.080$mm (Fig.3), and the equivalent stress of von Mises equals1.04kgf/mm^2(Fig.4).

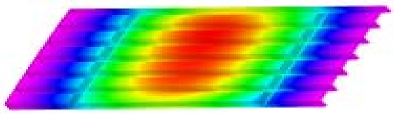

Figure 3: Displacement

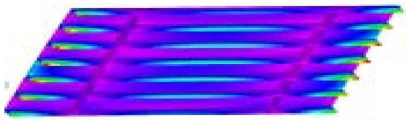

Figure 4: Von Mises stress

In the third case, the base of the shell was modeled by 4-node plate elements, and the edges by beam end elements. The analysis showed that the maximum deflection is in the center of the shell and is equal to $w = 0.0834$mm(Fig.5),and the equivalent stress of von Mises equals1.271kgf/mm^2, (Fig.6).

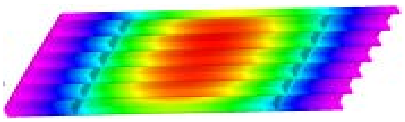

Figure 5: Displacement

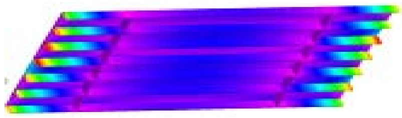

Figure 6: Von Mises stress

For the fourth case, the shell is modeled by eight-node three-dimensional finite elements, and the edges were modeled by beam end elements; In this case the maximum deflection is $w = 0.081$mm, (In the center of the shell),and the equivalent stress of von Mises equals1.06kgf/mm^2.

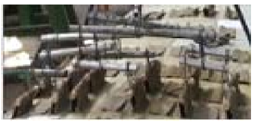

Figure 7: Experiment

The experiments of these shells were carried out on the basis of a certified BS EN ISO 9001 and EN 9100 of JSC "TAM" Tbilisi Aircraft Manufacturing on the surface of the shell, the pressure p=0,0098Mpa.

With the use of strain gauges, the movements in the center of the shell and the stress at the fastening points were checked. The maximum displacement was 0.75 mm, and the equivalent voltage was equal to 1.03 kgf / mm^2.

On the Mathematical Model of Drug Treatment of Rheumatoid Arthritis

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The model-based investigation of diseases is a complex and evolving field. We herein report about further development of the mathematical model of immune mediated disorders. We improved our previously reported model [3] by providing the treatment components. Model is implemented for rheumatoid arthritis and toclizumab as a drug for its treatment.

Mathematical models of autoimmune diseases provide an analytic framework in which we can address specific questions concerning disease immune dynamics and the choice of treatment. Such models are actively reported in the field of tumor immunology and immunotherapy by de Phillis et al [1, 2]. Improvement includes also estimation of model coefficients and the software that solves the Cauchy problem for this system and visualizes the obtained solution. Solutions can be obtained for the case of each individual patient to decipher the disease progress and treatment effect. In our implementation we used real data of disease progression and treatment of patients [4].

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On a Cyclically Symmetric Problem of Plate Bending with Partially Unknown Boundary

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The paper considers a problem of bending of an isotropic elastic plate, bounded by a regular hexagon weakened with a required full-strength hole whose symmetric axes are the regular hexagon diagonals. Rigid bars are attached to each component of the broken line of the outer boundary of the plate. This plate bends under the action of concentrated moments applied to the middle points of the bars. The unknown part of the boundary is free from external forces.

The formulas of Kolosov-Muskhelishvili [1] are used to investigate this problem. Using the conformal mapping, the investigation of the problem is reduced to a Riemann-Hilbert problem. The solution is written in quadratures. Using the methods of complex analysis [3], the plate deflection and required full-strength contours are determined [2]

$$t = w(\xi_0),\ \xi_0 \epsilon (-1, 1) \tag{1}$$

The conformal mapping function (1) is a generalization of Christoffel-Schwarz This problem is both of mechanics and of geometry since the shape of a hole of a plate is required and the conformal mapping function is used to define it.

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On Approximate Solution of Anti-Plane Problem of Composite Bodies Weakened by Crack

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In the present article an anti–plane problem of the elasticity theory for the composite (piece–wise homogeneous) orthotropic body weakened by cracks, intersecting the interface (problem 1) or reaching it (problem 2) at the right angle is studied. The studied problem is reduced to the singular integral equation (when the crack reaches the interface) and system (pair) of singular integral equations (when the crack intersects the interface) containing an immovable singularity with respect to the unknown characteristic functions of the cracks disclosure. Behavior of solutions in the neighborhood of the crack endpoints is studied by the method of discrete singularity with uniform division of an interval by knots. In both cases (crack intersects and crack reaches to the interface) the question of behavior of approximate solutions are investigated. The corresponding algorithms are composed and realized. The results of numerical investigations are presented.

A Nonlinear Equation for the Rectangular Dynamic Shell

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From an initial boundary value problem for the system of nonlinear differential equations, which describes the large deflection of a rectangular shell, we obtain a nonlinear integro-differential equation for the transverse displacement. This equation is analogous by its structure to the Kirchhoff equation for a string [2], the Woinowsky-Krieger equation for a beam [4] and the Berger equation for a plate [1], [3]. It is shown that after getting the transverse displacement for finding the longitudinal displacements we should solve the linear plane problem of elasticity.

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The Trial Version of the Georgian Universal Smart Corpus, the Aims of Defence of Georgian and Abkhazian Languages from the Danger of Digital Extinction, and the Law of Georgia "On Official Language"

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At the presentation we will overview Georgian Intellectual web-corpus (http://corpus.ge), which is elaborated on the basis of Pkhakadze's Logical Grammar of Georgian Language in the Center for the Georgian Language Technology of the Georgian Technical University and, also, we will overview innovative Georgian language technology systems, which are already inbuilt in the corpus and already functioned in trial mode (see [2] and, also: AR/122/4-105/14-17, geoanbani.com)

It must be also, underlined, that this already elaborated corpus, inspite of the fact that it is only a trial version of the Georgian universal smart corpus, is the only Georgian selfdeveloping, multimodal and multilingual corpus, which as sub-corpus contains:

1. The largest and technologically most of all supported corpus of the modern Georgian written language, by the existence of which the Georgian language acquired much more safety from danger of digital extinction than it was before when it was constructed.

2. The only was made abkhazian corpus, by the existence of which the first important step toward defense of Abkhazian language from Danger of digital extinction.

In addition to this, at the presentation, on the basis of the currently existed low technological support of the Georgian Language and, also, on the basis of the results of the research European languages in the Digital Age done by META-NET (meta-net.eu), we will show a reality of the danger of the digital extinction which the Georgian language is facing [1], [2]. We will Also show that another state language of Georgia - Abkhazian language, which is much more less technologically supported than Georgian, is under the much more higher-level danger of the digital extinction.

Thus, the above mentioned makes clear the very high national value and actuality of the law of Georgia "On Official Language" and, also, the obligatory necessity of its urgent activation. The thing is in the next: According to the 37th Article of the Chapter X of this law, date of using of which is 22/07/2015, we have a state and national duty of the elaboration of the unified program of the official language, which, together with other also very important topics, will contain also the topics, which will be related with the problems of defend Georgian and Abkhazian languages from danger of digital extinction in rapidly forthcoming digital age.

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Multi-Orientation Diffusion Schemes for Image Reconstruction

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Tensor-based anisotropic diffusion has been widely used in computer vision and image processing (see [2, 8] for overviews). It is a scale-space, regularization, and image reconstruction technique and has been used for many different applications including e.g. image inpainting, superresolution, noise removal, or reduction of JPEG artifacts (see e.g. [6] and many others).

Here we construct and investigate similar tensor-based diffusion schemes allowing to smooth along multiple orientations simultaneously. To do so, we first look into how diffusion schemes can be derived from suitable energies or cost functionals via calculus of variations. For anisotropic diffusion with a diffusion tensor as introduced by Weickert [7] no such energy is known. However, we propose an energy formulation based on a single-orientation data model allowing to derive this type of anisotropic diffusion as a reasonable approximation when minimizing the energy by gradient descent.

The energy term is a nice result by its own, and allows to transfer many results derived for energy minimization methods to anisotropic diffusion. E.g. one can use a richer set of learning methods to optimally parameterize anisotropic diffusion, when an energy is available [1]. The energy can also be used to give a theoretical justification for the combined optical flow and denoising algorithm from [3]. Or one may use different solving strategies based on the energy instead of simulating time evolutions due to diffusion.

Here, we use this energy formulation to derive novel diffusion-like schemes. Construction of the energy is not restricted to the single orientation data model leading to standard anisotropic diffusion, but other linear data models may be plugged in instead. We demonstrate this using double-, and triple-orientation models which we constructed from a single orientation model in the same way as transparent motion models [5, 4] are constructed from single motion or optical flow models. Exchanging the single orientation model in our energy formulation by multi-orientation models results in diffusion-like schemes suitable for reconstruction of multiply oriented structures.

We give examples for double- and triple-orientation diffusion including discretizations and update schemes. Indeed, these schemes can reconstruct richer image structures than standard anisotropic diffusion, e.g. crossing lines or superimposed oriented patterns (cmp. the demonstration experiment in Figure 1).

Experiments investigate filter kernels applied when using these schemes, and their performance in image reconstruction applications like denoising and image inpainting.

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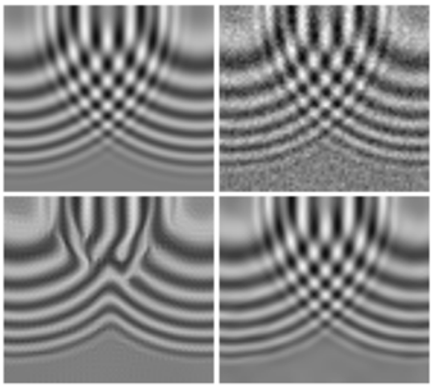

Figure 8: Denoising of transparent rings. Top left: original, top right: noise added, bottom left: reconstructed by anisotropic diffusion, bottom right: reconstructed by double-orientation diffusion.

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Application of Symbolic Sequence Analysis to Characterize Cyclic Variations in a Gasoline Engine

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Many systems show complex behavior like the presence of bifurcation and deterministic chaos in their dynamics. The nonlinear dynamics of such system may have patterns that are predictable for short duration but not for long duration. However, the patterns are often obscured by the noise. The characterization of such dynamics is not possible using conventional techniques like Fourier transformation. The present work discusses a technique which uses symbolic codes to extract information about such dynamic system. It involves the conversion of a time series data into a series of discretized symbols, i.e., a high-resolution signal to low-resolution signal. The raw data is partitioned into many bins, and each bin is allocated a symbol. The short string of consecutive symbols are grouped to form a sequence code. The length of the string in a sequence code is known as sequence length. The sensitivity of this method to characterize experimental time series is dependent upon the length of sequence code and a number of bins into which data is partitioned. Since the method involves coarse-graining technique, it is not affected by noise. On a symbol sequence histogram (SSH), a stochastic signal or a signal having no temporal correlation are identified with relative frequencies of sequence code lying close to $1/n^L$ where n is number of bins and L is the sequence length. The deterministic patterns result in structured SSH with frequencies of such patterns showing spikes on SSH.

The application of symbol sequence analysis is shown to be helpful in investigating the cyclic variability of a single cylinder gasoline engine. Cycle dynamics is found to be stochastic at high engine loads with low cyclic variability. The frequency of deterministic patterns with close coupling between consecutive cycles is determined to be high at low loads and lean mixtures with high cyclic variability. The work suggests that nature of cyclic variability ranges from completely stochastic to deterministic process superimposed on the stochastic process; with prior-cycle or deterministic effects more noticeable at low load and lean mixtures. The deterministic information in cycle dynamics can be used to design a predictive controller, which uses the knowledge of the current event, to predict the state of subsequent events

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On Nonlinear MEMS Statics and Dynamics

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The recent results on mathematical stability and pull-in analysis of MEMS parallel-plate capacitors will be presented [1, 2]. Lumped-parameter modeling with nonlinear restoring forces is used to establish some nonlinear mass-spring systems for various cantilever beams of some nonlinear materials subject to the standard constraints in MEMS applications. The nonlinear materials considered are some common annealed metals and other materials which can be modeled by the Hollomon's nonlinear stress-strain constitutive equations. The parameters in the corresponding nonlinear restoring forces and the effective masses are presented based on extension of the Euler-Bernoulli's beam assumptions and the nonlinear stress-strain model for the constraints.

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Facility Location for new Stone Crusher in Sarcheshmeh Copper Mine with AHP and TOPSIS methods

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This study offers an approach for dealing with prediction of the new stone crusher in Sarcheshmeh copper mine. The most important property of In-pit crusher is reducing the extraction costs. So it must be situated in a place that operating costs of conveyor, trucks and the number of trucks be at least. It is possible the various positions be better than others and the factors of site selection choice are not in a direction, because the organized and comprehensive study of possible places of In-pit crusher is necessary. Multi-criteria decision method is used for ranking of waste in-pit crusher site selection in Sarcheshmeh copper mine. The importance and sensitivity of waste rock is less than ores but size distribution and the ability of load and transport are the most important variables The main criteria in four main groups of technical factors, costing factors, operating factors and environmental factors. Affecting parameters on In-pit crushing site selection choice include. TOPSIS and AHP are used in this study as multi-criteria decision making method. In ranking of crusher site selection, the west case by 0.702 score in first rank, the south case by 0.613 as second rank, east case as third by 0.508 score and the east as fourth rank by 0.383 score was suggested.

AStudy of Algebraic Structure Involving Banach Space through q-Analogue

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The aim of the present presentation is to study the Banach Space and Combinatorial Algebraic Structure of R.It is further aimed to study algebraic structure of set of all q-extension of classical formula and function for $0 < q < 1$.

Satellite Workshop on Practical and Theoretical Aspects of Cryptography and Information Security

Reducing Key Sizes in Rainbow: Partially Hadamard-Rainbow

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In this talk, we propose to use partially Hadamard MDS matrices in the invertible affine maps. Then, the key sizes of these affine maps can be reduced significantly since we only need to store the first row of the matrix. We call this a partially Hadamard-Rainbow.

Computing Square Roots in Prime Fields via Singular Elliptic Curves

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There are many papers proposed for computing square roots in finite fields having some applications in cryptography [1, 2, 3, 4, 5, 6, 7, 8, 9]. While the algorithms given in [6] and [8] are deterministic, the remaining algorithms are probabilistic. In order to compute square roots in the finite field $\mathbb{F}_p$, where p is a prime number, these algorithms use the properties of finite fields, elliptic curves and second order linear recurrence sequence over $\mathbb{F}_p$. Compared to the most used algorithms Cipolla [2] and Tonelli-Shanks [7], the algorithms proposed by Ozdemir [4] have higher probability of success and efficient group operations on elliptic curves over $\mathbb{F}_p$.

In this study, inspired by the work of Ozdemir [4], we give an example of singular elliptic curves in the form $E\ :\ y^2 = x(x-5a)^2$ over $\mathbb{F}_p$ with quadratic residue a to compute square roots in $\mathbb{F}_p$. This is carried out by computing $2P = (x_3, y_3) = (a, -4a\sqrt{a})$ for the non-singular point $P = (a, 4a\sqrt{a})$ of order 3 in $E(\mathbb{F}_p)$. This stands for $2P = -P$, and therefore P and $-P$ are the only points of order 3.

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On the Generalization of Linear-In-One-Argument Form of Multivariate Polynomials for Post-Quantum Cryptography

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In this paper, we propose efficient constructions based on multivariate polynomials of degree $d \geq 4$ given in [1] as an open problem. The hardness of this problem comes from the nonlinear terms in $f(x+y) - f(x) - f(y)$.

Multivariate cryptography depends on solving systems of multivariate quadratic (MQ) polynomials over a finite field. The system of MQ polynomials is given as follows:

$$
\begin{aligned}
f^{(1)}(x_1,\ldots,x_n) &= \sum_{i=1}^{n}\sum_{j=i}^{n} f_{ij}^{(1)} \cdot x_i x_j \quad + \quad \sum_{i=1}^{n} f_i^{(1)} \cdot x_i + f_0^{(1)} \\
f^{(2)}(x_1,\ldots,x_n) &= \sum_{i=1}^{n}\sum_{j=i}^{n} f_{ij}^{(2)} \cdot x_i x_j \quad + \quad \sum_{i=1}^{n} f_i^{(2)} \cdot x_i + f_0^{(2)} \\
&\vdots \\
f^{(m)}(x_1,\ldots,x_n) &= \sum_{i=1}^{n}\sum_{j=i}^{n} f_{ij}^{(m)} \cdot x_i x_j \quad + \quad \sum_{i=1}^{n} f_i^{(m)} \cdot x_i + f_0^{(m)}.
\end{aligned}
$$

where the coefficients ($f_{ij}^{(k)}$ and $f_i^{(k)}$ for $1 \leq k \leq m$) are in a finite field. Then, the MQ problem, which is NP-hard, can be defined as follows:

Definition 1. Given m multivariate quadratic polynomials $f^{(1)}(\mathbf{x}), \ldots,$ $f^{(m)}(\mathbf{x})$ as shown in equation (1), find a vector $\bar{\mathbf{x}} = (\bar{x}_1, \ldots, \bar{x}_n)$ such that $f^{(1)}(\bar{\mathbf{x}}) = \ldots = f^{(m)}(\bar{\mathbf{x}}) = 0$.

Its main security is relied on the hardness of solving nonlinear equations over finite field [2]. The cryptosystems whose security relies on the MQ problem are considered as quantum secure since there is no polynomial-time algorithm that can be solved by the powerful quantum computers of the future [3].

In the literature, identification scheme based on multivariate quadratic polynomials as well as cubic ones have received interest. In [4], 3 and 5-pass zero knowledge identification protocols based on multivariate quadratic polynomials over a finite field were proposed. Then, in [5], these identification schemes were improved in view of communication complexity by using new dividing techniques with the help of bilinearity of a polar form of the MQ function.

Let $F(x) = (f_1(x), f_2(x), ..., f_m(x)) \in MQ(n, m, \mathbb{F}_q)$ where n is the size of the x and m is the number of polynomials. Then polar form G of the MQ function is defined as $G(x, y) = F(x + y) - F(x) - F(y)$. The function G is bilinear since $G(x, y) = \sum_{ij} a_{l,i,j} x_i y_j + y_i x_j$, where x, y are $n-$dimension vectors. Let $G : AxA \to B$, then, $G(x + y, z) = G(x, z) + G(y, z)$, where $x, y \in A$ and $z \in B$.

Note that this definition of polar form is defined for only MQ systems. There is also a polar form for multivariate cubic polynomial systems. It is difficult to control the terms in the opening because the number of terms increases when higher-order functions are used. In this paper, we focus on the generalization polar form for multivariate ℓ-degree polynomials.

In Theorem 1, we introduce the function of degree $d \geq 4$ which is a modified construction derived from the MQ.

Theorem 1. *The generalization of linear-in-one-argument form for the function of degree $d \geq 4$ is defined as follows: $F(x + y) = F(x) + G(x, y) + G(y, x) + F(y)$, where x, y are $n-$dimension vectors, P is the permutation,*

$$G(x,y) = xy + \sum_{z=3}^{d} \begin{cases} \sum_{i=1}^{\frac{(z-1)}{2}} \begin{pmatrix} z \\ i \end{pmatrix} P(x^{z-i}, y^i) \quad \textit{if } z \textit{ is odd} ; \\ \\ \sum_{i=1}^{(\frac{z}{2}-1)} \begin{pmatrix} z \\ i \end{pmatrix} P(x^{z-i}, y^i) + \sum_{i=\frac{z}{2}} \frac{\begin{pmatrix} z \\ i \end{pmatrix}}{2} P(x^{z-i}, y^i) \\ \qquad \textit{if } z \textit{ is even} . \end{cases}$$

and

$$G(y,x) = yx + \sum_{z=3}^{d} \begin{cases} \sum_{i=1}^{\frac{(z-1)}{2}} \begin{pmatrix} z \\ i \end{pmatrix} P(x^i, y^{z-i}) \quad if\ z\ is\ odd\ ; \\ \sum_{i=1}^{(\frac{z}{2}-1)} \begin{pmatrix} z \\ i \end{pmatrix} P(x^i, y^{z-i}) + \sum_{i=\frac{z}{2}} \frac{\begin{pmatrix} z \\ i \end{pmatrix}}{2} P(x^i, y^{z-i}) \\ \qquad if\ z\ is\ even\ . \end{cases}$$

Example. Let $d = 4$. Then, multivariate quartic polynomials system F is in the following form:
$F(x+y) = \sum_{i,j,k,t} a_{l,i,j,k,t}(x_ix_jx_kx_t + y_iy_jy_ky_t + x_ix_jx_ky_t + x_ix_jy_kx_t + x_iy_jx_kx_t + y_ix_jx_kx_t + x_ix_jy_ky_t + x_iy_jx_ky_t + x_iy_jy_kx_t + y_iy_jy_kx_t + y_iy_jx_ky_t + y_ix_jy_ky_t + x_iy_jy_ky_t + y_iy_jx_kx_t + y_ix_jy_kx_t + y_ix_jx_ky_t) + \sum_{i,j,k,t} b_{l,i,j,k}(x_iy_jx_k + y_ix_jx_k + x_ix_jy_k + y_ix_jy_k + x_iy_jy_k + y_iy_jx_k + x_ix_jx_k + y_iy_jy_k) + \sum_{i,j} c_{l,i,j}(x_ix_j + y_iy_j + x_iy_j + x_jy_i) + \sum_i d_{l,i}(x_i + y_i)$.
By using Theorem 1, the linear-in-one-argument form of F is:

$$G(x,y) = xy + \begin{pmatrix} 3 \\ 1 \end{pmatrix} P(x^2,y) + \begin{pmatrix} 4 \\ 1 \end{pmatrix} P(x^3,y) + \frac{\begin{pmatrix} 4 \\ 2 \end{pmatrix}}{2} P(x^2,y^2). \quad (1)$$

$$G(y,x) = yx + \begin{pmatrix} 3 \\ 1 \end{pmatrix} P(x,y^2) + \begin{pmatrix} 4 \\ 1 \end{pmatrix} P(x,y^3) + \frac{\begin{pmatrix} 4 \\ 2 \end{pmatrix}}{2} P(x^2,y^2). \quad (2)$$

$P(x^2, y)$ computed in Equation 1 is the complement of $P(x, y^2)$ in view of variables given in Equation 2, i.e., $P(x^2, y) = \overline{P(x, y^2)}$. In a similar manner, all permutations used in $G(x, y)$ are selected as a complement of those used in $G(y, x)$.

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Human–Internet of Things (IoT) Communication: Security issues

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Internet of Things (IoT) offers a common communication paradigm for all objects via the Internet and its protocols. It is being applied in all areas of life, such as environmental monitoring, health-care, cities management, industry and education. In addition, the growth of IoT and embedded and wearable devices have widespread and beneficial effects on the way of improving life quality and health-care. Powerful devices capable of sensing, processing and exchanging data, new low-energy communication protocols, big data and cloud systems it is obviously very important to use existing security technologies for dealing with security issues.

On the other hand, the applications of Brainwave Computer Interface (BCI) have the potential to improve the quality of life for disabled patients and improvement of human thought concentration. Technology that allows paralyzed people to use thought commands to manipulate computers already exists. However, it is not practical for everyday use: the design is complicated and messy, involving many wires and laboratory environments. The Bio-Brain Signals designed for Internet of Thing is aiming to share the challenges resulting from the connectivity of both IoT and Human Bio-Signals wirelessly, which will influence the future of everything – everywhere.

The security issue of the embedded devices with IoT are never easily provided, but with the increasing importance of security at development stages and the use of analytics to monitor the devices and protect the network in which the devices are located, it is necessary to prevent the attackers from infecting low-power devices. Goals are more vital and important. The paper will address these issues further by moving to a more systematic approach to IoT security with more focusing on Human Interaction with IoT through BCI.

Modern Methods of Cryptanalysis

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Cryptanalysis is the science of cracking codes and decoding secrets. It is used to violate authentication schemes, to break cryptographic protocols, and to find and correct weaknesses in encryption algorithms. This article discusses modern cryptanalysis methods such as: Linear Differential Cryptanalysis, the two most significant attacks applicable to symmetric-key block ciphers.

On the Protocol Analysis in Pρlog

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In this talk we consider the problem of cryptographic protocol verification and analysis. Cryptographic protocol is used for secure communication over the network by two or more agents. Cryptographic protocol verification is a task, that determines whether the protocol is secure and can be broken by different kinds of attacks, like "men in the middle", etc. We try to model a cryptographic protocol in the Pρlog system and show that it is not vulnerable for attacks. We would like to mention, that it is an easier task (decidable) to find out whether a protocol is vulnerable for attacks, than to find an attack that breaks the protocol (not decidable in general).

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Nearly Perfect Sequences and Cryptographic Functions

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Let ζ_m be a primitive m-th root of unity. A v-periodic sequence $\underline{a} = (a_0, a_1, \ldots, a_{v-1}, \ldots)$ is an *m-ary sequence* if $a_0, a_1, \ldots, a_{v-1} \in \mathcal{E}_m = \{1, \zeta_m, \zeta_m^2, \ldots, \zeta_m^{m-1}\}$ and an *almost m-ary sequence* if $a_0 = 0$ and $a_1, \ldots, a_{v-1} \in \mathcal{E}_m$. For $0 \leq t \leq v-1$, the *autocorrelation function* $C_{\underline{a}}(t)$ is defined by $C_{\underline{a}}(t) = \sum_{i=0}^{v-1} a_i \overline{a_{i+t}}$, where $\overline{a}$ is the complex conjugate of $a \in \mathbb{C}$. An m-ary or almost m-ary sequence $\underline{a}$ of period v is called a *perfect sequence* (PS) if

$C_{\underline{a}}(t) = 0$ for all $1 \leq t \leq v-1$. Similarly, an almost m-ary sequence $\underline{a}$ of period v is called a *nearly perfect sequence* (NPS) of type γ if $C_{\underline{a}}(t) = \gamma$ for all $1 \leq t \leq v-1$. The existence cases of nearly perfect sequence of type γ for small $|\gamma|$ is considered in this study. Perfect sequences with small integer γ values are used in many applications. Hence, we look for v, m and γ, for which a v periodic m ary nearly perfect sequence of type γ exists. We obtain many new nearly perfect sequences of type γ, indeed we have some sequences with very small $|\gamma|$. Next, the relationship between a nearly perfect sequence and a cryptographic function is investigated. In cryptography, secrecy is generally satisfied by using block ciphers which confuses a message into a ciphertext via a nonlinear *Boolean function.* A nonlinear Boolean function attaining the maximum nonlinearity is called a *bent function.* It is known that a perfect sequence is equivalent to a cryptographic bent function. By using this equivalence, we convert a nearly perfect sequence into a Boolean function. It is seen that one can find a highly nonlinear Boolean function via a nearly perfect sequence of type γ having very small $|\gamma|$ values.

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Analysis of Trust Models in Wireless Networks

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In general, there are different forms of trust to address different types of network security problems and reduce risk in certain conditions. There are a number of trust models applied by various cryptographic schemes, respectively. This talk explores three of them: a) the web of trust employed by Pretty Good Privacy (PGP) users using their own set of trusted public keys, b) Kerberos, a secret key distribution scheme using a trusted third party, c) certificates, which allow a set of trusted third parties to authenticate each other and, by implication, each other's users. Each of the above mentioned trust models differs in complexity, scope, scalability and general applicability. Which model of trust to apply in certain circumstances and types of wireless networks is discussed in the given talk. It describes the major security issues and their techniques of building trust model by monitoring network behavior.

On the Number of k-normal Elements over Finite Fields

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Normal bases are widely used in areas such as cryptography, coding theory and signal processing (see, for example, [1, 2]). They are practical in implementing finite field arithmetic, especially in multiplication and exponentiation due to the structure of the finite fields. For implementation purposes the chosen bases of the finite fields should have low complexities. For some finite fields there exist special normal bases, optimal normal bases and Gaussian normal bases, which have low complexities [3, 4]. But for many others we do not have such bases.

k-normal elements were introduced in [5] by generalizing normal elements. They arise implicitly in constructing quasi-normal bases [6], which is a class of $\mathbb{F}_q$-bases of $\mathbb{F}_{q^n}$ that offer efficient multiplication in finite fields. These bases are useful when there is no optimal normal basis for a given finite field and gaussian normal basis of this field have high complexity.

In this article, for some special values of n we give an explicit formula for the number of k-normal elements of $\mathbb{F}_{q^n}$ over $\mathbb{F}_q$ where q is a prime power. For this case we use the properties of cyclotomic polynomials and composed products of polynomials [1]. Note that previously this problem has been considered for the case $n = p^m$ for a positive integer m and a prime p [7].

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