SOME BOUNDARY VALUE PROBLEMS OF POROUS COSSERAT MEDIA WITH VOIDS

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Abstract

In this paper the body is an elastic Cosserat media with voids. The two-dimensional system of equations corresponding to a plane deformation case is written in a complex form and its general solution is presented with using of two analytic functions of a complex variable and two solutions of the Helmholtz equations. On the basis of the general representation, specific boundary value problems are solved for a circle and infinite plane with a circular hole.

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1 Introduction

The non-linear version of elastic materials with voids was proposed by Nunziato and Cowin [1] and the linear version was developed by Cowin and Nunziato [2] to study mathematically the mechanical behaviour of porous solids. The linear theory of thermoelastic porous materials with voids is the generalization of the classical theory of elasticity. Porous materials with voids have applications in many fields of engineering, such as the petroleum industry, material science and biology. This theory enables us to analyze the behaviour of elastic porous materials which can be found in engineering, such as rock and soil, bone, the manufactured porous materials. The voids are assumed to contain nothing of mechanical or energetic significance.

It should be noted that all the papers mentioned above dealt with a classical (symmetric) medium. We consider the problem of elasticity for solids with triple-porosity in the case of an elastic Cosserat medium [3].

Many problem are investigated by several researchers in the elastic materials with the microstructure . Some of these results are presented in [4-7] and in references therein.

2 Basic equations

Assume an elastic body with voids occupies the domain $\overline{\Omega} \in \mathbb{R}^3$. Denote by $x = (x_1,\ x_2,\ x_3)$ a point of the domain $\overline{\Omega}$ in the Cartesian coordinate system. Assume the domain $\overline{\Omega}$ is filled with an elastic Cosserat media having voids.

In this case, a system of static equilibrium equations is [2, 8]

$$\begin{array}{l} \partial_i\sigma_{ij}(x) + \rho F_j(x) = 0, \\ \partial_i\mu_{ij}(x) + \in_{jik}\sigma_{ik}(x) + \rho G_j(x) = 0, \quad i,j,k = 1,2,3 \\ \partial_i h_i(x) + g(x) + \rho l(x) = 0, \end{array} \tag{1}$$

where σ_{ij} are stress tensor components; ρ is material density; F_j are the components of the mass force vectors; μ_{ij} are moment stress tensor components; $\in_{ijk}$ is the Levi–Civita symbol; G_j are the components of the mass moment vectors; h_i is the equilibrated stress vector; g is the intrinsic equilibrated body force; l is the extrinsic equilibrated body force; $\partial_i \equiv \partial/\partial x_i$.

Formulas that interrelate functions σ_{ij} , μ_{ij} , h_i , g to the functions u_j , ω_j and φ have the form [2, 8]

$$\begin{array}{l} \sigma_{ij} = (\lambda div\ u + \gamma\phi)\,\delta_{ij} + (\mu+\alpha)\,\partial_i u_j + (\mu-\alpha)\,\partial_j u_i - 2\alpha\in_{\text{jik}}\omega_k, \\ \mu_{ij} = \alpha\ div\ \omega\ \delta_{ij} + (\nu+\beta)\,\partial_i\omega_j + (\nu-\beta)\,\partial_j\omega_i, \\ hi = \delta\partial_i\phi, \\ g = -\xi\phi - \gamma div\ u, \end{array} \tag{2}$$

where λ and μ are the Lamé parameters; $\alpha,\ \beta,\ \nu,\ \sigma$ are the constants characterizing the microstructure of the discussed elastic media; $\delta,\ \xi,\ \gamma$ are the constants characterizing the body porosity; δ_{ij} is the Kronecker delta, $u = (u_1,\ u_2,\ u_3)$ is the displacement vector, $\omega = (\omega_1,\ \omega_2,\ \omega_3)$ is the rotation vector, ϕ is the change of volume fraction.

From the basic three-dimensional equations, we obtain the basic equations for the case of plane deformation. Let Ω be a sufficiently long cylindrical body with generatrix parallel to the Ox_3-axis. Denote by D the cross-section of this cylindrical body, thus $D \in \mathbb{R}^2$. In the case of plane deformation $u_3 = 0,\ \omega_1 = 0,\ \omega_2 = 0$, while the functions $u_1,\ u_2,\ \omega_3\ and\ \varphi$ do not depend on the coordinate x_3 [9]. We also assume $u_1,\ u_2,\ \omega_3,\ \varphi \in C^2\ (D) \cap\ C^1(\overline{D})$.

As follows from formula (2), in the case of plane deformation

$$\sigma_{\alpha 3} = 0,\ \sigma_{3\alpha} = 0,\ \mu_{\alpha\beta} = 0,\ \mu_{33} = 0,\ h_3 = 0, \ \ \alpha,\beta = 1,2.$$

If relations (2) are substituted into the system (1) then we obtain the following system of equilibrium equations with respect to the functions

$u_1,\ u_2,\ \omega_3$ and φ (the homogeneous system $(F_\alpha=0, G_\alpha=0, l=0)$)

$$\begin{array}{l}(\mu+\alpha)\,\Delta_2 u_1+(\lambda+\mu-\alpha)\,\partial_1\theta+2\alpha\partial_2\omega_3+\gamma\partial_1\varphi=0,\\(\mu+\alpha)\,\Delta_2 u_2+(\lambda+\mu-\alpha)\,\partial_2\theta-2\alpha\partial_1\omega_3+\gamma\partial_2\varphi=0,\\(\nu+\beta)\,\Delta_2\omega_3+2\alpha(\partial_1 u_2-\partial_2 u_1)-4\alpha\omega_3=0,\\(\delta\Delta_2-\xi)\varphi-\gamma\theta=0.\end{array}\tag{3}$$

On the plane Ox_1x_2, we introduce the complex variable $z=x_1+ix_2=re^{i\alpha}$ $\left(i^2=-1\right)$ and the operators $\partial_z=0.5\,(\partial_1-i\partial_2)$, $\partial_{\overline{z}}=0.5\,(\partial_1+i\partial_2)$, $\overline{z}=x_1-ix_2$, $\Delta_2=4\partial_z\partial_{\overline{z}}$.

To write system (3) in the complex form, the second equation of this system is multiplied by i and summed up with the first equation

$$\begin{array}{l}2(\mu+\alpha)\partial_z\partial_{\overline{z}}u_+ +(\lambda+\mu-\alpha)\partial_{\overline{z}}\theta-2\alpha i\partial_{\overline{z}}\omega_3+\gamma\partial_{\overline{z}}\phi=0,\\2(\nu+\beta)\partial_z\partial_{\overline{z}}\omega_3+\alpha i(\theta-2\partial_z u_+)-2\alpha\omega_3=0,\\(4\partial_z\partial_{\overline{z}}-\xi)\phi-\gamma\theta=0.\end{array}\tag{4}$$

The general solution of the system of Eqs. (4) is represented using formulas [8, 9]

$$2\mu u_+=(\kappa+\kappa_0)\varphi(z)-(1-\kappa_0)z\overline{\varphi'(z)}-\overline{\psi(z)}+4\partial_{\bar z}(i\chi(z,\bar z)-\gamma\eta(z,\bar z)),\tag{5}$$

$$2\mu\omega_3=\frac{4\mu}{\nu+\beta}\chi(z,\overline{z})-\frac{\kappa+1}{2}i(\varphi'(z)-\varphi'(\overline{z}))\tag{6}$$

$$\phi=\frac{(\lambda+2\mu)\xi-\gamma^2}{\mu\delta}\eta(z,\overline{z})-\frac{\gamma}{(\lambda+\mu)\delta\zeta_2^2}(\varphi'(z)+\overline{\varphi'(z)})\tag{7}$$

where $\varphi(z)$ and $\psi(z)$ are the arbitrary analytic functions of z, $\chi(z,\bar z)$ and $\eta(z,\bar z)$ are the general solutions of the Helmholtz equations

$$\Delta_2\chi-\zeta_1^2\chi=0,\quad \Delta_2\eta-\zeta_2^2\eta=0$$

$$\zeta_1^2=\frac{4\mu\alpha}{(\nu+\beta)(\mu+\alpha)}>0,\quad \zeta_2^2=\frac{(\lambda+2\mu)\xi-\gamma^2}{(\lambda+2\mu)\delta}>0.$$

also

$$\kappa=\frac{\lambda+3\mu}{\lambda+\mu},\quad \kappa_0=\frac{\gamma^2\mu}{(\lambda+\mu)((\lambda+2\mu)\xi-\gamma^2)}.$$

3 The boundary value problem for a circle

Let us consider the elastic circle, consisting of Cosserat media with voids bounded by the circumference of radius R (Fig. 1). The origin of coordinates is at the center of the circle.

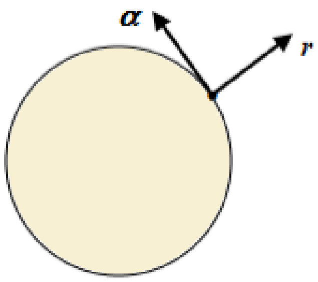

Figure 1: The elastic circle.

On the circumference, we consider the following boundary value problem

$$\sigma_{rr} - i\sigma_{r\alpha} = N - iT, \ \ \mu_{r3} = M, \ \ h_l = F, \ \ on\, r = R \tag{8}$$

where N, T, M and F are sufficiently smooth functions.

Substituting the formulas (5)–(7) into (8) we have [8, 9, 10]

$$\begin{gathered}\sigma_{rr} - i\sigma_{r\alpha} = (1-\kappa_0)\left(\varphi'(z) + \overline{\varphi'(z)}\right) + \varsigma_1^2 i\chi(z,\ \bar z) + \varsigma_2^2\gamma\eta(z,\ \bar z) \\ -e^{2i\alpha}\left[(1-\kappa_0)\,\bar z\varphi''(z) + \psi'(z) + 4\partial_z\partial_z\left(i\chi(z,\ \bar z) + \gamma\eta(z,\ \bar z)\right)\right]\end{gathered} \tag{9}$$

$$\begin{gathered}\mu_{r3} = Re\left(\tfrac{(\kappa+1)(\nu+\beta)}{2\mu} i\overline{\varphi''(z)}\,e^{-i\alpha} + 4\partial_{\bar z}\chi(z,\ \bar z)\,e^{-i\alpha}\right), \\ h_l = Re\left(-\tfrac{2\gamma}{(\lambda+\mu)\varsigma_2^2}\overline{\varphi''(z)}e^{-i\alpha} + \tfrac{2(\lambda+2\mu)\xi-2\gamma^2}{\mu}\partial_{\bar z}\eta(z,\bar z)\,e^{-i\alpha}\right).\end{gathered} \tag{10}$$

The analytic functions $\varphi'(z), \psi'(z)$ and the metaharmonic functions $\chi(z,\overline{z}), \eta(z,\overline{z})$ are represented as the following series

$$\varphi'(z) = \sum_{n=0}^{\infty} a_n z^n, \ \ \psi'(z) = \sum_{n=0}^{\infty} b_n z^n \tag{11}$$

$$\chi(z,\overline{z}) = \sum_{-\infty}^{+\infty} \alpha_n I_n(\zeta_1 r)e^{in\alpha}, \ \ \eta(z,\overline{z}) = \sum_{-\infty}^{+\infty} \beta_n I_n(\zeta_2 r)e^{in\alpha}, \tag{12}$$

where $I_n(\zeta_1 r)$ and $I_n(\zeta_2 r)$ are the modified Bessel function of the first kind of n-th order.

Substituting (11), (12) in (9), (10) taking into account the boundary conditions (8) and assuming that the series converge on the circumference $r = R$, one finds

$$\begin{gathered}(1-\kappa_0)\sum_{n=0}^{+\infty} R^n\left((1-n)a_n e^{in\alpha} + \bar a_n e^{-in\alpha}\right) - \sum_{n=0}^{+\infty} R^n b_n e^{i(n+2)\alpha} \\ -\frac{2}{R}\sum_{-\infty}^{+\infty}(n-1)\left(\varsigma_1 I_{n-1}(\varsigma_1 R)\, i\alpha_n + \varsigma_2\gamma I_{n-1}(\varsigma_2 R)\,\beta_n\right)e^{in\alpha} = N - i\mathrm{T},\end{gathered} \tag{13}$$

$$2\zeta_1\sum_{-\infty}^{+\infty} I_n'(\zeta_1 R)\,\alpha_n e^{in\alpha}+\frac{(\kappa+1)(\nu+\beta)}{4\mu}\times\sum_{n=0}^{+\infty}R^{n-1}ni\left(\bar{a}_n e^{-in\alpha}-a_n e^{in\alpha}\right)=M, \tag{14}$$

$$\frac{2(\lambda+2\mu)\,\xi-2\gamma^2}{\mu}\frac{\zeta_2}{2}\sum_{-\infty}^{\infty}I_n'(\varsigma_2 r)\,\beta_n e^{in\alpha}-\\ -\frac{\gamma}{(\lambda+\mu)\,\varsigma_2^2}n\sum_{n=0}^{\infty}R^{n-1}\left(a_n e^{in\alpha}-\bar{a}_n R^{n-1}e^{-in\alpha}\right)=F. \tag{15}$$

As a conclusion of the previous relations, we used the following well-known formula

$$I_{n-1}(x)-I_{n+1}(x)=\frac{2n}{x}I_n(x),$$

$$I_{n-1}(x)+I_{n+1}(x)=2I_n'(x).$$

Expand the function $N-iT$, $2\mu M$ and F, given on $r=R$, in a complex Fourier series

$$N-iT=\sum_{-\infty}^{+\infty}N_n e^{in\alpha},\;\; 2\mu M=\sum_{-\infty}^{+\infty}M_n e^{in\alpha},\;\; F=\sum_{-\infty}^{+\infty}F_n e^{in\alpha}. \tag{16}$$

Comparing in (13)–(15) the coefficients of $e^{0i\alpha}$ we have (it is also assumed that a_0 is a real value [9])

$$2(1-\kappa_0)a_0+\frac{2\gamma}{R}\zeta_2 I_1(\zeta_2 R)\beta_0=N_0, \tag{17}$$

$$\beta_0=\frac{2\mu F_0}{(2(\lambda+2\mu)\,\xi-2\gamma^2)\zeta_2 I_1(\zeta_2 R)}, \tag{18}$$

$$\alpha_0=-\frac{R}{2\zeta_1 I_1(\zeta_1 R)}T_0=\frac{1}{2\zeta_1 I_0'(\zeta_1 R)}M_0. \tag{19}$$

In order for the problem to have a solution, the following condition must be met

$$M_0=-\frac{RI_0'(\varsigma_1 R)}{I_1(\varsigma_1 R)}T_0.$$

From Eqs. (17), (18) we determine the coefficients a_0

$$a_0=\frac{N_0}{2(1-\kappa_0)}-\frac{2\mu\gamma F_0}{R(1-\kappa_0)(2(\lambda+2\mu)\,\xi-2\gamma^2)}.$$

comparing the coefficients of $e^{in\alpha}$ $(n \neq 0)$, we have

$$\begin{gathered}(1-n)(1-\kappa_0)R^n a_n - R^{n-2}b_{n-2} - \\ \frac{2}{R}(n-1)(\zeta_1 I_{n-1}(\zeta_1 R)i\alpha_n + \zeta_2\gamma I_{n-1}(\zeta_2 R)\beta_n) = N_n,\ n \geq 2\end{gathered} \tag{20}$$

$$\begin{gathered}(1-\kappa_0)R^n a_n - \frac{2}{R}(n+1)(\zeta_1 I_{n+1}(\zeta_1 R)i\alpha_n - \zeta_2\gamma I_{n+1}(\zeta_2 R)\beta_n) \\ = \overline{N}_{-n},\ n > 0\end{gathered} \tag{21}$$

$$2\varsigma_1 I_n'(\varsigma_1 R)\,\alpha_n - \frac{(\kappa+1)(\nu+\beta)}{4\mu}R^{n-1}nia_n = M_n, \quad n \geq 1, \tag{22}$$

$$\frac{2(\lambda+2\mu)\,\xi-2\gamma^2}{2\mu}\varsigma_2 I'_n(\varsigma_2 R)\,\beta_n - \frac{\gamma}{(\lambda+\mu)\,\varsigma_2^2}R^{n-1}na_n = F_n; \quad n \geq 1. \tag{23}$$

From (20)–(23) one finds

$$a_n = \frac{\overline{N}_{-n} + k_{1n}M_n - k_{2n}F_n}{(1-\kappa_0)R^n + k_{3n} + k_{4n}}, \quad n > 0$$

$$\alpha_n = \frac{M_n + \frac{(\kappa+1)(\nu+\beta)}{4\mu}R^{n-1}nia_n}{2\zeta_1 I_n'(\zeta_1 R)}, \quad n > 0$$

$$\beta_n = \frac{2\mu\left(F_n + \frac{\gamma}{(\lambda+\mu)\zeta_2^2}R^{n-1}na_n\right)}{\zeta_2 I_n'(\varsigma_2 R)\,(2(\lambda+2\mu)\,\xi-2\gamma^2)}, \quad n > 0$$

$$b_{n-2} = (1-n)(1-\kappa_0)\,R^2 a_n$$

$$-\frac{2(n-1)}{R^{n-1}}(\varsigma_1 I_{n-1}(\varsigma_1 R)\,i\alpha_n + \varsigma_2\gamma I_{n-1}(\varsigma_2 R)\,\beta_n) - \frac{N_n}{R^{n-2}},\ n > 1.$$

where

$$k_{1n} = \frac{n+1}{R}\frac{I_{n+1}(\varsigma_1 R)\,i}{I_n'(\varsigma_1 R)},$$

$$k_{2n} = \frac{n+1}{R}\frac{2\mu\gamma I_{n+1}(\zeta_2 R)}{((\lambda+2\mu)\,\xi-\gamma^2)\,I_n'(\zeta_2 R)},$$

$$k_{3n} = n(n+1)\frac{(\kappa+1)(\nu+\beta)\,R^{n-2}I_{n+1}(\varsigma_1 R)}{4\mu I_n'(\varsigma_1 R)},$$

$$k_{4n} = n(n+1)\frac{2\mu\gamma^2 R^{n-2}I_{n+1}(\zeta_2 R)}{(\lambda+\mu)\,\zeta_2^2\,((\lambda+2\mu)\,\xi-\gamma^2)\,I_n'(\zeta_2 R)}.$$

It is easy to prove the absolute and uniform convergence of the series obtained in the the circle (including the contours) when the functions set on the boundaries have sufficient smoothness.

4 The problem for the infinite plane with a circular hole

Now let we have an infinite plane with a circular hole (Fig. 2). Assume that the origin of coordinates is at the center of the hole of radius R.

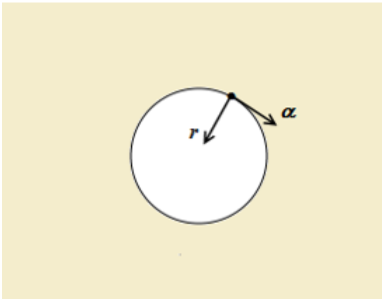

Figure 2: The infinite plane with a circular hole.

On the circle we consider the following boundary value problem

$$\begin{aligned} &2\mu u_{+} = N - iT, \\ &\omega_3 = M, \qquad\qquad r = R, \\ &\phi = F, \end{aligned} \tag{24}$$

where N, T, M and F are sufficiently smooth functions.

Substituting the formulas (5)–(7) into (24) we have

$$\begin{aligned} &(\kappa + \kappa_0)\,\varphi(z) - (1-\kappa_0)\, z\overline{\varphi'(z)} - \overline{\psi(z)} + 4i\partial_{\bar z}\chi\,(z,\hat z) \\ &-4\gamma\partial_{\hat z}\eta\,(z,\hat z) = \frac{N - iT}{2\mu} e^{i\alpha}, \\ &\frac{4\mu}{\nu+\beta}\chi\,(z,\overline{z}) - \frac{\kappa+1}{2} i \left(\varphi'\,(z) - \overline{\varphi'\,(z)}\right) = 2\mu M, \\ &\frac{(\lambda+2\mu)\,\xi - \gamma^2}{\mu\delta}\eta\,(z,\hat z) - \frac{\gamma}{(\lambda+\mu)\,\delta\varsigma_2^2}\left(\varphi'(z) + \overline{\varphi'(z)}\right) = F. \end{aligned} \tag{25}$$

Conditions at infinity

$$\begin{aligned} &\sigma_{11}^{(\infty)} = S_1, \qquad \sigma_{22}^{(\infty)} = S_2, \\ &\sigma_{12}^{(\infty)} = \sigma_{21}^{(\infty)} = S_3; \\ &\mu_{13}^{(\infty)} = \mu_{23}^{(\infty)} = 0; \qquad \phi = S_4, \end{aligned} \tag{26}$$

where S_1, S_2, S_3, S_4 are the constants.

In this case the analytic functions $\varphi(z)$, $\psi(z)$ and the metaharmonic functions $\chi(z,\ \hat{z})$, $\eta(z,\ \hat{z})$ are represented as a series

$$\begin{gathered}\varphi'(z)=\sum_{n=0}^{\infty}a_n z^{-n}\ ,\quad \psi'(z)=\sum_{n=0}^{\infty}b_n z^{-n},\\ \chi(z,\ \hat{z})=\sum_{-\infty}^{+\infty}\alpha_n K_n(\varsigma_1 r)\,e^{in\alpha}\ ,\quad \eta(z,\ \hat{z})=\sum_{-\infty}^{+\infty}\beta_n K_n(\varsigma_2 r)\,e^{in\alpha}\ ,\end{gathered}\tag{27}$$

where $K_n(\varsigma_1 r)$ and $K_n(\varsigma_2 r)$ are the modified Bessel function of the second kind of n-th order.

Substituting (27) in (25) taking into account the boundary conditions (24) and assuming that the series to converge on the circumference $r=R$, one finds

$$\begin{gathered}(\kappa+\kappa_0)\left(Ra_0 e^{i\alpha}+lnR\ a_1+i\alpha a_1+\sum_{n=2}^{\infty}\frac{R^{-n+1}}{-n+1}a_n e^{i(-n+1)\alpha}\right)\\ -(1-\kappa_0)\sum_{n=0}^{\infty}\frac{\bar{a}_n}{R^{n-1}}e^{i(n+1)\alpha}-R\bar{b}_0 e^{i\alpha}-lnR\ \bar{b}_1+i\alpha\bar{b}_1\\ -\sum_{n=2}^{\infty}\frac{R^{-n+1}}{-n+1}\hat{b}_n e^{i(n-1)\alpha}-2\varsigma_1 i\sum_{-\infty}^{\infty}K_{n+1}(\varsigma_1 R)\,\alpha_n e^{i(n+1)\alpha}\\ +2\eta\varsigma_2\sum_{-\infty}^{\infty}K_{n+1}(\varsigma_2 R)\,\beta_n e^{i(n+1)\alpha}=\frac{N-iT}{2\mu}e^{i\alpha},\end{gathered}\tag{28}$$

$$\begin{gathered}\frac{2}{\nu+\beta}\sum_{-\infty}^{\infty}\alpha_n K_n(\zeta_1 R)\,e^{in\alpha}-\frac{\kappa+1}{4\mu}i\sum_{n=0}^{\infty}(a_n e^{-in\alpha}-\bar{a}_n e^{in\alpha})R^{-n}\\ =M,\end{gathered}\tag{29}$$

$$\begin{gathered}\frac{(\lambda+2\mu)\,\xi-\gamma^2}{\mu\delta}\sum_{-\infty}^{+\infty}K_n(\varsigma_2 R)\,\beta_n e^{in\alpha}\\ -\frac{\gamma}{(\lambda+\mu)\,\delta\ \varsigma_2^2}\ \sum_{n=0}^{+\infty}\frac{1}{R^n}\left(a_n e^{-in\alpha}+\bar{a}_n\ e^{in\alpha}\right)=F.\end{gathered}\tag{30}$$

Expand the function $N-iT$, M and F, given on $r=R$, in a complex Fourier series

$$\begin{aligned}&\frac{N-iT}{2\mu}e^{i\alpha}=\sum_{-\infty}^{+\infty}N_n\ e^{in\alpha},\\ &2\mu M=\sum_{-\infty}^{+\infty}M_n e^{in\alpha},\\ &F=\sum_{-\infty}^{+\infty}F_n e^{in\alpha}.\end{aligned}\tag{31}$$

Due to the fact that $\chi,\ \eta,\ M$ and F are real functions, we have

$$\alpha_n=\bar{\alpha}_{-n},\qquad \beta_n=\bar{\beta}_{-n},$$
$$B_n=\bar{B}_{-n},\qquad C_n=\bar{C}_{-n}.$$

It is known that [9]

$$a_0=\mathit{\Gamma},\qquad b_0=\mathit{\Gamma}^{/},\tag{32}$$

where $\mathit{\Gamma},\ \mathit{\Gamma}^{/}$ are known quantities, specifying the stress distribution at infinity (It is also assumed that a_0 is a real value [9]). As follows from formulas (5), (6), (7) and conditions (26) [8]

$$Re\mathit{\Gamma}=\frac{S_1+S_2}{4\left(1-\kappa_0\right)}=\frac{\left(\lambda+\mu\right)\delta\varsigma_2^2S_4}{2},$$
$$Re\mathit{\Gamma}^{/}=\frac{S_2-S_1}{2},\qquad Im\mathit{\Gamma}^{/}=S_3.$$

We use the condition of single-valuedness of the displacements which in the present case is expressed as

$$\left(\kappa+\kappa_0\right)a_1+\bar{b}_1=0.\tag{33}$$

After introducing (31) into (28)–(30), and comparing the coefficients of $e^{in\alpha}$, we have

$$\left(\kappa+\kappa_0\right)lnR\ a_1-lnR\ \bar{b}_1-2\varsigma_1iK_0\left(\varsigma_1R\right)\alpha_{-1}+2\eta\varsigma_2K_0\left(\varsigma_2R\right)\beta_{-1}=N_0,\tag{34}$$

$$\frac{2}{\nu+\beta}K_0\left(\varsigma_1R\right)\alpha_0=M_0,\tag{35}$$

$$\frac{\left(\lambda+2\mu\right)\xi-\gamma^2}{\mu\delta}K_0\left(\varsigma_2R\right)\beta_0-\frac{\gamma}{\left(\lambda+\mu\right)\delta\varsigma_2^2}\bar{a}_0=F_0.\tag{36}$$

$$\begin{aligned}&\left(\kappa+\kappa_0\right)Ra_0-\left(1-\kappa_0\right)R\bar{a}_0-R\bar{b}_0+\frac{1}{R}\bar{b}_2-2\varsigma_1iK_1\left(\varsigma_1R\right)\alpha_0\\&+2\eta\varsigma_2K_1\left(\varsigma_2R\right)\beta_0=N_1,\end{aligned}\tag{37}$$

$$\begin{aligned}&-\left(1-\kappa_0\right)\frac{\bar{a}_{n-1}}{R^{n-2}}+\frac{1}{nR^n}\bar{b}_{n+1}-2\varsigma_1iK_n\left(\varsigma_1R\right)\alpha_{n-1}\\&+2\eta\varsigma_2K_n\left(\varsigma_2R\right)\beta_{n-1}=N_n,\ \ n\geq2,\end{aligned}\tag{38}$$

$$\begin{aligned}&\left(\kappa+\kappa_0\right)\frac{R^{-n}}{-n}a_{n+1}-2\varsigma_1iK_{-n}\left(\varsigma_1R\right)\alpha_{-n-1}+2\eta\varsigma_2K_{-n}\left(\varsigma_2R\right)\beta_{-n-1}\\&=N_{-n},\ \ n\geq1,\end{aligned}\tag{39}$$

$$\frac{2}{\nu+\beta}K_n\left(\varsigma_1R\right)\alpha_n+\frac{\left(\kappa+1\right)i}{4\mu R^n}\bar{a}_n=M_n,n\geq1,\tag{40}$$

$$\frac{(\lambda+2\mu)\,\xi-\gamma^2}{\mu\delta}K_n\left(\varsigma_2 R\right)\beta_n-\frac{\gamma}{(\lambda+\mu)\,\delta\varsigma_2^2 R^n}\bar{a}_n=F_n, n\geq 0. \qquad (41)$$

From (32) and (35-37) one finds

$$\alpha_0=\frac{(\nu+\beta)\,M_0}{2K_0\left(\varsigma_1 R\right)},$$

$$\beta_0=\frac{\mu\delta N_0}{(\lambda+2\mu)\,\xi-\gamma^2}\frac{1}{K_0\left(\varsigma_2 R\right)}-\frac{2\gamma\mu\Gamma}{(\lambda+\mu)\left((\lambda+2\mu)\,\xi-\gamma^2\right)\varsigma_2^2}.$$

$$b_2=RN_1+\left(1-\kappa-2\kappa_0\right)R^2\Gamma+R^2\Gamma'+2\varsigma_1 iK_1\left(\varsigma_1 R\right)\frac{(\nu+\beta)\,M_0}{2K_0\left(\varsigma_1 R\right)}$$

$$-2\eta\varsigma_2 RK_1\left(\varsigma_2 R\right)\frac{\mu\delta N_0}{(\lambda+2\mu)\,\xi-\gamma^2}\frac{1}{K_0\left(\varsigma_2 R\right)}-\frac{2\gamma\mu\Gamma}{(\lambda+\mu)\left((\lambda+2\mu)\,\xi-\gamma^2\right)\varsigma_2^2}.$$

From equations (33), (34), (40), (41) we get the following system of equations with respect to a_1, b_1, α_1 and β_1:

$$\left(\kappa+\kappa_0\right)lnR\ \bar{a}_1-lnR\ b_1+2\varsigma_1 iK_0\left(\varsigma_1 R\right)\alpha_1+2\eta\varsigma_2 K_0\left(\varsigma_2 R\right)\beta_1=\bar{N}_0,$$

$$\left(\kappa+\kappa_0\right)\bar{a}_1+b_1=0,$$

$$\frac{2}{\nu+\beta}K_1\left(\varsigma_1 R\right)\alpha_1+\frac{(\kappa+1)\,i}{4\mu R}\bar{a}_1=M_1,$$

$$\frac{(\lambda+2\mu)\,\xi-\gamma^2}{\mu\delta}K_1\left(\varsigma_2 R\right)\beta_1-\frac{\gamma}{(\lambda+\mu)\,\delta\varsigma_2^2 R}\bar{a}_1=F_1.$$

From (38-41) one finds

$$\bar{a}_n=\frac{\bar{N}_{-n+1}-J_{1n}M_n-J_{2n}F_n}{J_{3n}-J_{4n}-J_{5n}}, n\geq 2$$

$$\alpha_n=\frac{M_n-\frac{(\kappa+1)i}{4\mu R^n}\bar{a}_n}{\frac{2}{\nu+\beta}K_n\left(\varsigma_1 R\right)}\ ,\quad n\geq 1,$$

$$\beta_n=\frac{F_n+\frac{\gamma}{(\lambda+\mu)\delta\varsigma_2^2 R^n}\bar{a}_n}{\frac{(\lambda+2\mu)\xi-\gamma^2}{\mu\delta}K_n\left(\varsigma_2 R\right)},\quad n\geq 0$$

$$\bar{b}_{n+1}=nR^n\left(N_n+\left(1-\kappa_0\right)\frac{\bar{a}_{n-1}}{R^{n-2}}+2\varsigma_1 iK_n\left(\varsigma_1 R\right)\alpha_{n-1}-2\eta\varsigma_2 K_n\left(\varsigma_2 R\right)\beta_{n-1}\right),$$

where

$$J_{1n}=\frac{i\varsigma_1 K_{n-1}\left(\varsigma_1 R\right)(\nu+\beta)}{K_n\left(\varsigma_1 R\right)},$$

$$J_{2n}=\frac{2\eta\varsigma_2\mu\delta K_{n-1}\left(\varsigma_2 R\right)}{\left((\lambda+2\mu)\,\xi-\gamma^2\right)K_n\left(\varsigma_2 R\right)},$$

$$J_{3n}=\frac{\kappa+\kappa_0}{R^{n-1}\left(1-n\right)},\quad J_{4n}=\frac{\varsigma_1\left(\kappa+1\right)\left(\nu+\beta\right)K_{n-1}\left(\varsigma_1 R\right)}{4\mu R^n K_n\left(\varsigma_1 R\right)},$$

$$J_{5n}=-\frac{2\eta\mu\gamma K_{n-1}\left(\varsigma_2 R\right)}{\left(\lambda+\mu\right)\left(\left(\lambda+2\mu\right)\xi-\gamma^2\right)\varsigma_2 K_n\left(\varsigma_2 R\right)}.$$

It is easy to prove the absolute and uniform convergence of the series obtained in the infinite plane with a circular hole (including the contours) when the functions set on the boundaries have sufficient smoothness.

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