EXPLICIT SOLUTION OF THE DIRICHLET TYPE QUASI-STATIC BOUNDARY VALUE PROBLEM OF ELASTICITY FOR POROUS CIRCULAR RING

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Abstract

This paper is concerned to study the Dirichlet type quasi-static boundary value problem of coupled theory of elasticity for porous circular ring. The obtained solution is represented as absolutely and uniformly convergent series.

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1 Introduction

Most of naturally or manufactured solids is not completely filled. In nearly every body there are empty interspaces, which are called pores through which the liquid or gas may flow. Many materials such as rocks, sand, soil etc., which occur on and below the surface of the earth, are known as porous materials. In some bodies there are immediately visible, in others the pores are recognized only with a magnifier. For example, the human skin has a larger number of pores, bone tissue could be assumed to be transversely isotropic and most closely describes mechanical anisotropy of bone and cancellous bone is considered as a porous material.

The foundations of the theory of elastic materials with voids were first proposed by Cowin and Nunziato [1, 2]. They investigated the linear and nonlinear theories of elastic materials with voids. In these theories the independent variables are displacement vector field and the change of volume fraction of pores. Such materials include, in particular, rocks and soils, granulated and some other manufactured porous materials.

Elastic materials which contain a multi-porous structure has a multitude of applications in real life. The history of development of porous body mechanics, the main results and the sphere of their application are set forth in detail in the monographs [3-6] (see references therein). The generalization of the theory of elasticity and thermoelasticity for materials with double void pores belongs to Ieşan and Quintanilla [7]. In [8] Svanadze consider the coupled linear model of porous elastic solids by combining the following three variables: the displacement vector field the volume fraction of pores; and the pressure of the fluid. The basic internal and external BVPs (boundary value problems) of steady vibrations are investigated, Green's formulas are obtained, the uniqueness and the existence theorems are proved by means of the potential method and the theory of singular integral equations (see references therein). In [9] the coupled linear quasi-static theory of elasticity for porous materials is considered. The fundamental solution is constructed, and its basic properties are established. Green's formulas are obtained, and the uniqueness theorems of the internal and external boundary value problems are proved, the existence theorems for classical solutions of the BVPs are proved by means of the potential method and the theory of singular integral equations.

For applications, it is especially important to construct the solutions of boundary value problems in explicit form. Questions related to this topic are considered, for example, in the works [10-27], where the explicit solutions are constructed for some boundary value problems of porous elasticity for the concrete domains.

This paper is concerned to study the Dirichlet type quasi-static boundary value problem of the coupled theory of elasticity for porous circular ring. The obtained solution is represented as absolutely and uniformly convergent series.

2 Basic Equations. Formulation of the Problems

Let $\mathbf{x} = (x_1, x_2)$ be a point in the Euclidean two-dimensional space E_2. Let us assume that D is a circular ring, $R_1 < |\mathbf{x}| < R_2$, centered at point $O(0,0)$ in the space E_2, S_1 is a circumference of radius R_1 , S_2 is a circumference of radius R_2 and $S = S_1 \cup S_2$. Let us assume that the domain D is filled with an isotropic porous materials.

Following the ideas proposed by Svanadze [8] and Mikelashvili [9], the basic system of equations of motion in the coupled linear quasi-static theory of elasticity for porous elastic materials expressed in terms of the displacement vector $\mathbf{u}$, the changes of volume fraction $\varphi(\mathbf{x})$ of pores and the change of fluid pressure in pore network $p(\mathbf{x})$ has the following form [8,9]

$$\begin{aligned} &\mu\Delta\mathbf{u} + (\lambda+\mu)\text{graddiv}\mathbf{u} + b\text{grad}\varphi - \beta\text{grad}p = 0,\\ &(\alpha\Delta - \alpha_1)\varphi - b\,\text{div}\mathbf{u} + mp = 0, \\ &(k\Delta + i\omega\, a)p + i\omega\beta\,\text{div}\mathbf{u} + i\omega\, m\varphi = 0, \end{aligned} \tag{1}$$

In the previous system $\mathbf{u} := (u_1, u_2)^\top$, is the displacement vector, λ and μ are the Lame constants, β is the effective stress parameter, $k = \frac{k'}{\mu'}$, μ' is the fluid viscosity, k' is the macroscopic intrinsic permeability associated with the pore network, α, b, m, α_1, are constitutive coefficients, the value a is measures the compressibility of pores, $\omega > 0$ is the oscillation frequency, Δ is the Laplacian. Throughout this paper assume that the superscript $^\top$ denotes transposition.

Definition 1. A vector-function $\mathbf{U} = (\mathbf{u}, \varphi, p)$ defined in the domain D is called regular if

$$\mathbf{U} \in C^2(D) \cap C^1(\overline{D}),$$

For the system (1) we formulate the following BVP:

Problem 1. Find a regular solution $\mathbf{U} = (\mathbf{u}, \varphi, p)$ to system (1) in the domain D, satisfying the following boundary conditions on S :

$$\begin{aligned} &\mathbf{u}^+(\mathbf{z}) = \boldsymbol{F}^+(\mathbf{z}), \quad \varphi^+(\mathbf{z}) = f_3^+(\mathbf{z}), \quad p^+ = f_4^+(\mathbf{z}), \quad \mathbf{z} \in S_2.\\ &\mathbf{u}^-(\mathbf{z}) = \boldsymbol{F}^-(\mathbf{z}), \quad \varphi^-(\mathbf{z}) = f_3^-(\mathbf{z}), \quad p^- = f_4^-(\mathbf{z}), \quad \mathbf{z} \in S_1, \end{aligned}$$

where the vector-function $\mathbf{F}(\mathbf{z}) = (f_1, f_2)$, and the functions $f_3(\mathbf{z})$, $f_4(\mathbf{z})$ are prescribed functions on S, at $\mathbf{z}$, having the definite smoothness. The symbol $\mathbf{U}^+(\mathbf{U}^-)$ denotes the limits of $\mathbf{U}(\mathbf{x}) = (\mathbf{u}, \varphi, \psi)$ on $\mathbf{z} \in S$ from D

$$\mathbf{U}^+(\mathbf{z}) = \lim_{D \ni \mathbf{x} \to \mathbf{z} \in S_2} \mathbf{U}(\mathbf{x}), \quad \mathbf{U}^-(\mathbf{z}) = \lim_{D^- \ni \mathbf{x} \to \mathbf{z} \in S_1} \mathbf{U}(\mathbf{x}).$$

The following assertion holds [9]

Theorem 1. *The Problem 1 has one regular solution in D.*

The general solution of equations (1), which is useful in our subsequent, may be found in the paper [13] and we cite it without proof.

Theorem 2. *The regular solution $\boldsymbol{U} = (\boldsymbol{u}, \varphi, p)$ of the system (1) admits a representation*[13]

$$\begin{aligned} &\mathbf{u} = \mathbf{\Psi} - \text{grad}\left[(k_0 - 1)h_0 + \sum_{j=1}^{2} \frac{h_j}{\lambda_j^2}\right],\\ &\varphi = B_0 h + \sum_{j=1}^{2} B_j h_j, \quad p = C_0 h + \sum_{j=1}^{2} C_j h_j, \\ &\text{div}\mathbf{u} = h + \sum_{j=1}^{2} h_j, \quad \text{div}\mathbf{\Psi} = k_0 h, \end{aligned} \tag{2}$$

where

$$\Delta h_0 = h, \quad \Delta h = 0, \quad (\Delta + \lambda_j^2)h_j = 0, \ \ \Delta \boldsymbol{\Psi} = 0,$$

$$\begin{aligned}
&B_0 = \frac{ab + m\beta}{\delta_0}, \quad C_0 = \frac{\beta\alpha_1 - mb}{\delta_0}, \quad k_0 = \frac{A_2}{\mu\delta_0},\\
&B_j = \frac{i\omega\delta_0 B_0 - bk\lambda_j^2}{\delta_j}, \quad C_j = i\omega\frac{\delta_0 C_0 + \alpha\beta\lambda_j^2}{\delta_j}, \quad j = 1, 2,\\
&\delta_0 = -a\alpha_1 - m^2, \quad \delta_j = -(\alpha_1 + \alpha\lambda_j^2)(i\omega a - k\lambda_j^2) - i\omega m^2,\\
&\mu_0 + bB_0 - \beta C_0 = \frac{A_2}{\delta_0}, \quad bB_J - \beta C_j = -\mu_0,\\
&\alpha k\mu_0^2\delta_1\delta_2 = -i\omega\delta_0^2 K_0^2, \quad K_0 = kbC_0 + i\omega\alpha\beta B_0.
\end{aligned} \tag{3}$$

$\lambda_j^2, j = 1, 2$, are roots of the following equation

$$\begin{aligned}
&\alpha\mu_0 k\xi^2 - A_1\xi + i\omega A_2 = 0, \quad \mu_0 = \lambda + 2\mu,\\
&A_1 = \mu_0(a\alpha i\omega - \alpha_1 k) + kb^2 + i\omega\alpha\beta^2,\\
&A_2 = \mu_0(-\alpha_1 a - m^2) + ab^2 - \alpha_1\beta^2 + 2bm\beta.
\end{aligned} \tag{4}$$

We assume that λ_j^2, $(j = 1, 2)$ are distinct and different from zero. We may assume without loss of generality that $Im\lambda_j^2 > 0$. [9]

3 Explicit Solution of Problem 1

Let us introduce the polar coordinates

$$x_1 = \rho\cos\vartheta, \quad x_2 = \rho\sin\vartheta, \quad \rho = \sqrt{x_1^2 + x_2^2}, \quad 0 \le \vartheta \le 2\pi.$$

Taking into account the identity $\mathbf{x}\cdot\text{grad} = \rho\frac{\partial}{\partial\rho}$, from (2) we obtain

$$(\mathbf{x}\cdot\mathbf{u}) = (\mathbf{x}\cdot\boldsymbol{\Psi}) - \rho\frac{\partial}{\partial\rho}\left[(k_0 - 1)h_0 + \sum_{j=1}^{2}\frac{h_j}{\lambda_j^2}\right]. \tag{5}$$

It is easily verified, that the function $(\mathbf{x}\cdot\boldsymbol{\Psi})$ is a solution of the equation

$$\Delta(\mathbf{x}\cdot\boldsymbol{\Psi}) = 2\text{div}\ \boldsymbol{\Psi} = 2k_0 h.$$

Finding the function $(\mathbf{x}\cdot\boldsymbol{\Psi})$ yields:

$$(\mathbf{x}\cdot\boldsymbol{\Psi}) = \Omega + 2k_0 h_0, \tag{6}$$

where Ω is an arbitrary harmonic function $\Delta\Omega = 0$ and $\Delta h_0 = h$.

Substituting (6), into (5), gives

$$\begin{aligned} &(\mathbf{x}\cdot\mathbf{u}) = \Omega + 2k_0h_0 - \rho\frac{\partial}{\partial\rho}\left[(k_0-1)h_0 + \sum_{j=1}^{2}\frac{h_j}{\lambda_j^2}\right],\\ &\varphi = B_0h + \sum_{j=1}^{2}B_jh_j, \quad p = C_0h + \sum_{j=1}^{2}C_jh_j, \quad \mathrm{div}\mathbf{u} = h + \sum_{j=1}^{2}h_j. \end{aligned} \tag{7}$$

Let us assume that functions $h,\ h_j,\ \ j=1,2,$ and Ω are sought in the following form

$$\begin{aligned} &h(\mathbf{x}) = E_0' + M_0'\ln\rho + \sum_{k=1}^{\infty}\left(\frac{R_1}{\rho}\right)^k (E_k'\cos k\vartheta + M_k'\sin k\vartheta)\\ &+\sum_{k=1}^{\infty}\left(\frac{\rho}{R_2}\right)^k (E_k\cos k\vartheta + M_k\sin k\vartheta),\\ &h_j(\mathbf{x}) = C_{0j}J_0(\lambda_j\rho) + B_{0j}H_0^{(1)}(\lambda_j\rho)\\ &+\sum_{k=1}^{\infty}J_k(\lambda_j\rho)(\lambda_j\rho)(C_{jk}\cos k\vartheta + C_{jk}'\sin k\vartheta)\\ &+\sum_{k=1}^{\infty}H_k^{(1)}(\lambda_j\rho)(D_{jk}\cos k\vartheta + D_{jk}'\sin k\vartheta),\\ &\Omega(\mathbf{x}) = a_0' + b_0'\ln\rho + \sum_{k=1}^{\infty}\left(\frac{R_1}{\rho}\right)^k (a_k'\cos k\vartheta + b_k'\sin k\vartheta)\\ &+\sum_{k=1}^{\infty}\left(\frac{\rho}{R_2}\right)^k (a_k\cos k\vartheta + b_k\sin k\vartheta), \end{aligned} \tag{8}$$

respectively, where E_n, E_n', $M_n, M_n', ...$ are the unknown quantities, $J_k(\lambda_j\rho)$ is the Bessel's function, $H_k^{(1)}(\lambda_j\rho) = J_k(\lambda_j\rho) + iN_k(\lambda_j\rho)$ is the Hankel's function with the index k; On the basis of equation $\Delta h_0 = h$, the function h_0 can be represented in the following form

$$\begin{aligned} h_0 = &\frac{E_0'\rho^2}{4} + \frac{M_0'\rho^2}{4}(\ln\rho - 1)\\ &-\frac{\rho^2}{4}\sum_{k=2}^{\infty}\frac{1}{k-1}\left(\frac{R_1}{\rho}\right)^k (E_k'\cos k\vartheta + M_k'\sin k\vartheta)\\ &+\frac{\rho^2}{4}\sum_{k=1}^{\infty}\frac{1}{k+1}\left(\frac{\rho}{R_2}\right)^k (E_k\cos k\vartheta + M_k\sin k\vartheta). \end{aligned} \tag{9}$$

For convenience we introduce the following functions

$$(\mathbf{x}\cdot\mathbf{F})^{\pm} = g_1^{\pm}, \quad (\mathrm{div}\mathbf{F})^{\pm} = g_2^{\pm}, \quad \varphi^{\pm} = g_3^{\pm}, \quad p^{\pm} = g_4^{\pm}.$$

In what follows we assume that the functions g_k, $k = 1,..,4$, can be expanded into the Fourier series.

From (7), passing to the limit as $\rho \longrightarrow R_2$, and $\rho \longrightarrow R_1$, we obtain

$$\begin{aligned}
&\Omega^+ + 2k_0h_0^+ - \left\{\rho\frac{\partial}{\partial\rho}\left[(k_0-1)h_0 + \sum_{j=1}^{2}\frac{h_j}{\lambda_j^2}\right]\right\}_{\rho=R_2} = g_1^+,\\
&h^+ + \sum_{j=1}^{2} h_j^+ = g_2^+, \quad B_0h^+ + \sum_{j=1}^{2} B_jh_j^+ = g_3^+,\\
&C_0h^+ + \sum_{j=1}^{2} C_jh_j^+ = g_4^+,\\
&\Omega^- + 2k_0h_0^- - \left\{\rho\frac{\partial}{\partial\rho}\left[(k_0-1)h_0 + \sum_{j=1}^{2}\frac{h_j}{\lambda_j^2}\right]\right\}_{\rho=R_1} = g_1^-,\\
&h^- + \sum_{j=1}^{2} h_j^- = g_2^-, \quad B_0h^- + \sum_{j=1}^{2} B_jh_j^- = g_3^-,\\
&C_0h^- + \sum_{j=1}^{2} C_jh_j^- = g_4^-,
\end{aligned} \tag{10}$$

Following Theorem 1 we conclude that the determinant of system (10) is different from zero and the system (10) is uniquely solvable and we can find the functions $h^{\pm}$, $h_j^{\pm}$ and $\Omega^{\pm}$

$$\begin{aligned}
&h^+ = \frac{\delta_0}{A_2}[\mu_0g_2^+ + bg_3^+ - \beta g_4^+] = G^+,\\
&h_j^+ = \frac{(-1)^j}{\sigma}\left[g_2^+ - G^+ - \frac{B_1B_2}{B_j}(g_3^+ - G^+)\right] = G_j^+, \quad j = 1,2,\\
&\Omega^+ = g_1^+ - 2k_0h_0^+ + R_2\left\{\frac{\partial}{\partial\rho}\left((k_0-1)h_0 + \sum_{j=1}^{2}\frac{h_j}{\lambda_j^2}\right)\right\}_{\rho=R_2}\\
&= G_3^+,
\end{aligned} \tag{11}$$

$$\begin{aligned}
&h^- = \frac{\delta_0}{A_2}[\mu_0g_2^- + bg_3^- - \beta g_4^-] = G^-,\\
&h_j^- = \frac{(-1)^j}{\sigma}\left[g_2^- - G^- - \frac{B_1B_2}{B_j}(g_3^- - G^-)\right] = G_j^-, \quad j = 1,2,\\
&\Omega^- = g_1^- - 2k_0h_0^- + R_1\left\{\frac{\partial}{\partial\rho}\left((k_0-1)h_0 + \sum_{j=1}^{2}\frac{h_j}{\lambda_j^2}\right)\right\}_{\rho=R_1}\\
&= G_3^-,
\end{aligned} \tag{12}$$

where

$$\begin{aligned}
&\sigma = -i\omega\frac{(\lambda_1^2-\lambda_2^2)K_0A_2}{\mu_0\delta_1\delta_2} = \frac{\alpha k\mu_0(\lambda_1^2-\lambda_2^2)A_2}{\delta_0^2K_0},\\
&K_0 = kbC_0 + \alpha\beta i\omega B_0, \quad \alpha k\mu_0^2\delta_1\delta_2 = -i\omega\delta_0^2K_0^2.
\end{aligned} \tag{13}$$

On the other hand, it is evident that,when $\rho = R_1$ we have

$$\begin{aligned}
&E_0' + M_0' \ln R_1 + \sum_{k=1}^{\infty} (E_k' \cos k\vartheta + M_k' \sin k\vartheta) \\
&+ \sum_{k=1}^{\infty} \left(\frac{R_1}{R_2}\right)^k (E_k \cos k\vartheta + M_k \sin k\vartheta) = G^-, \\
&C_{0j} J_0(\lambda_j R_1) + B_{0j} H_0^{(1)}(\lambda_j R_1) + \sum_{k=1}^{\infty} [J_k(\lambda_j R_1)[C_{jk} \cos k\vartheta + C_{jk}' \sin k\vartheta] \\
&+ \sum_{k=1}^{\infty} H_k^{(1)}(\lambda_j R_1)[D_{jk} \cos k\vartheta + D_{jk}' \sin k\vartheta] = G_j^-, \\
&a_0' + b_0' \ln R_1 + \sum_{k=1}^{\infty} (a_k' \cos k\vartheta + b_k' \sin k\vartheta) \\
&+ \sum_{k=1}^{\infty} \left(\frac{R_1}{R_2}\right)^k (a_k \cos k\vartheta + b_k \sin k\vartheta) = G_3^-,
\end{aligned} \tag{14}$$

when $\rho = R_2$

$$\begin{aligned}
&E_0' + M_0' \ln R_2 + \sum_{k=1}^{\infty} \left(\frac{R_1}{R_2}\right)^k (E_k' \cos k\vartheta + M_k' \sin k\vartheta) \\
&+ \sum_{k=1}^{\infty} (E_k \cos k\vartheta + M_k \sin k\vartheta) = G^+, \\
&C_{0j} J_0(\lambda_j R_2) + B_{0j} H_0^{(1)}(\lambda_j R_2) \\
&+ \sum_{k=1}^{\infty} [J_k(\lambda_j R_2)[C_{jk} \cos k\vartheta + C_{jk}' \sin k\vartheta] \\
&+ \sum_{k=1}^{\infty} H_k^{(1)}(\lambda_j R_2)[D_{jk} \cos k\vartheta + D_{jk}' \sin k\vartheta] = G_j^+, \\
&a_0' + b_0' \ln R_2 + \sum_{k=1}^{\infty} \left(\frac{R_1}{R_2}\right)^k (a_k' \cos k\vartheta + b_k' \sin k\vartheta) \\
&+ \sum_{k=1}^{\infty} (a_k \cos k\vartheta + b_k \sin k\vartheta) = G_3^-,
\end{aligned} \tag{15}$$

From here we get

$$\begin{aligned}
E_0' + M_0' \ln R_1 = q_0^- &= \frac{1}{2\pi} \int_0^{2\pi} G^-(\eta) d\eta, \\
E_0' + M_0' \ln R_2 = q_0^+ &= \frac{1}{2\pi} \int_0^{2\pi} G^+(\eta) d\eta, \\
a_0' + b_0' \ln R_1 = q_3^- &= \frac{1}{2\pi} \int_0^{2\pi} G_3^-(\eta) d\eta,
\end{aligned} \tag{16}$$

$$a_0' + b_0' \ln R_2 = q_3^+ = \frac{1}{2\pi}\int_0^{2\pi} G_3^+(\eta)d\eta,$$

$$\begin{aligned}
&E_k' + \left(\frac{R_1}{R_2}\right)^k E_k = q_k^- = \frac{1}{\pi}\int_0^{2\pi} G^-(\eta)\cos k\eta d\eta,\\
&\left(\frac{R_1}{R_2}\right)^k E_k' + E_k = q_k^+ = \frac{1}{\pi}\int_0^{2\pi} G^+(\eta)\cos k\eta d\eta,\\
&M_k' + \left(\frac{R_1}{R_2}\right)^k M_k = g_k^- = \frac{1}{\pi}\int_0^{2\pi} G^-(\eta)\sin k\eta d\eta,\\
&\left(\frac{R_1}{R_2}\right)^k M_k' + M_k = g_k^+ = \frac{1}{\pi}\int_0^{2\pi} G^+(\eta)\sin k\eta d\eta,
\end{aligned} \tag{17}$$

$$\begin{aligned}
&a_k' + \left(\frac{R_1}{R_2}\right)^k a_k = q_1^- = \frac{1}{\pi}\int_0^{2\pi} G_3^-(\eta)\cos k\eta d\eta,\\
&\left(\frac{R_1}{R_2}\right)^k a_k' + a_k = q_1^+ = \frac{1}{\pi}\int_0^{2\pi} G_3^+(\eta)\cos k\eta d\eta,\\
&b_k' + \left(\frac{R_1}{R_2}\right)^k b_k = g_1^- = \frac{1}{\pi}\int_0^{2\pi} G_3^-(\eta)\sin k\eta d\eta,\\
&\left(\frac{R_1}{R_2}\right)^k b_k' + b_k = g_1^+ = \frac{1}{\pi}\int_0^{2\pi} G_3^+(\eta)\sin k\eta d\eta,
\end{aligned} \tag{18}$$

$$C_{jk}J_k(\lambda_j R_1) + D_{jk}H_k^{(1)}(\lambda_j R_1) = \frac{1}{\pi}\int_0^{2\pi} G_j^-(\eta)\cos k\eta d\eta = q_{jk}^-,$$

$$C_{jk}J_k(\lambda_j R_2) + D_{jk}H_k^{(1)}(\lambda_j R_2) = \frac{1}{\pi}\int_0^{2\pi} G_j^+(\eta)\cos k\eta d\eta = q_{jk}^+,$$

$$\begin{aligned}
&C_{jk}'J_k(\lambda_j R_1) + D_{jk}'H_k^{(1)}(\lambda_j R_1) = \frac{1}{\pi}\int_0^{2\pi} G_j^-(\eta)\sin k\eta d\eta = g_{jk}^-,\\
&C_{jk}'J_k(\lambda_j R_2) + D_{jk}'H_k^{(1)}(\lambda_j R_2) = \frac{1}{\pi}\int_0^{2\pi} G_j^+(\eta)\sin k\eta d\eta = g_{jk}^+,
\end{aligned} \tag{19}$$

$$
\begin{aligned}
&C_{0j}J_0(\lambda_j R_1) + B_{0j}H_0^{(1)}(\lambda_j R_1)) = \frac{1}{2\pi}\int_0^{2\pi} G_j^-(\eta)d\eta = q_{j0}^-,\\
&C_{0j}J_k(\lambda_j R_2) + B_{0j}H_0^{(1)}(\lambda_j R_2)) = \frac{1}{2\pi}\int_0^{2\pi} G_j^+(\eta)d\eta = q_{j0}^+,
\end{aligned} \tag{20}
$$

The determinants of systems(16),(17),(18),(19) and (20) are the following

$$
\alpha_0 = \ln\frac{R_2}{R_1} \neq 0, \quad \alpha_k = 1 - \left(\frac{R_1}{R_2}\right)^{2k} \neq 0, \quad k = 1, 2, ..
$$

$$
d_k = iJ_k(\lambda_j R_1)N_k(\lambda_j R_2) - iJ_k(\lambda_j R_2)N_k(\lambda_j R_1) \neq 0, \quad k = 0, 1, 2, ...
$$

because λ_j is a complex number.

Thus, from the last systems, after making some elementary transformations, we uniquely define the unknown coefficients. Through inserting the obtained values into (8),(2) we get the final form for solution of the considered Problem 1.

We assume that the functions $\mathbf{F}^{\pm}$, $f_j^{\pm}$, $j = 3, 4$, satisfy the following conditions on S

$$
\mathbf{F}^{\pm}, f_j \in C^{1,\alpha}(S), \ \alpha < 1, \ j = 3, 4.
$$

Under these conditions the resulting series are absolutely and uniformly convergent.

Remark. The potential users of the obtained results will be the scientists and engineers working on the problems of solid mechanics, micro and nanomechanics, mechanics of materials, engineering mechanics, engineering medicine, biomechanics, engineering geology, geomechanics, hydroengineering, applied and computing mechanics.

4 Conclusions

In this paper the coupled linear quasi-static equations of theory of elasticity are considered for porous elastic materials, in which the basic equations are expressed in terms of the displacement vector $\mathbf{u}$, the changes of the volume fraction φ of pores and the fluid pressure p in pore network. The following results are obtained:

1. Efficient solutions of the Dirichlet type BVP are obtained for a porous circular ring

2. The obtained solutions are represented as absolutely and uniformly convergent series.

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