ONE BOUNDARY VALUE PROBLEM FOR A MICROPOLAR POROUS ELASTIC BODY

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Abstract

In this paper the micropolar porous elastic body is considered. The two-dimensional system of equations corresponding to a plane deformation case is written in a complex form and its general solution is presented with using of two analytic functions of a complex variable and two solutions of the Helmholtz equations. One boundary value problems are solved for a circle.

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1 Introduction

The foundations of the theory of elastic materials with voids were first proposed by Cowin and Nunziato [1, 2]. They investigated the linear and nonlinear theories of elastic materials with voids. In these theories the independent variables are displacement vector field and the change of volume fraction of pores. Such materials include, in particular, rocks and soils, granulated and some other manufactured porous materials. In many cases, it is also important to calculate porous materials based on the theory of micropolar elasticity [3–8]

Various questions of elastic equilibrium of porous bodies with empty pores in the case of a classical elastic medium are further discussed in

[9–15]. The problems of porous elasticity for micropolar media with voids were considered in [16, 17].

The present paper deals with plane strain problem for micropolar porous elastic body with voids. The boundary value problem is solved for a circle.

2 Basic equations

Assume an elastic body with voids occupies the domain $\overline{\Omega} \in \mathbb{R}^3$. Denote by $x = (x_1,\ x_2,\ x_3)$ a point of the domain $\overline{\Omega}$ in the Cartesian coordinate system. Assume the domain $\overline{\Omega}$ is filled with an elastic Cosserat media having voids. Denote the volume of the macro point of x by $V(x)$, and the volume of pores at this point by $V_p(x)$. The value $v(x)$, which is defined by the equality $v(x) = V_p(x)/V(x)$, is called the relative volume of pores. In general, as a result of deformation of the body the relative volume of pores changes, too. The solid body characterized by the displacement vector $u = (u_1,\ u_2,\ u_3)$, the rotation vector $\omega = (\omega_1,\ \omega_2,\ \omega_3)$ and the change in volume fraction from the reference volume fraction [1, 2]

$$\varphi(x) = v(x) - v_R(x).$$

In this case, a system of static equilibrium equations is [2, 3, 16]

$$\begin{aligned} &-\partial_i\sigma_{ij}(x) = \rho F_j(x),\\ &-\partial_i\mu_{ij}(x) - \in_{jik}\sigma_{ik}(x) = \rho G_j(x),\\ &-\partial_i h_i(x) - g(x) = \rho l(x), \end{aligned} \tag{1}$$

where σ_{ij} are stress tensor components; ρ is material density; F_j are the components of the mass force vectors; μ_{ij} are moment stress tensor components; $\in_{ijk}$ is the Levi–Civita symbol; G_j are the components of the mass moment vectors; h_i is the equilibrated stress vector; g is the intrinsic equilibrated body force; l is the extrinsic equilibrated body force; $\partial_i \equiv \partial/\partial x_i$.

In the above formulas, the Latin indices take the values 1, 2, 3 and it is assumed that summation is carried out over the repeated indices. The same is also assumed below.

Formulas that interrelate functions σ_{ij}, μ_{ij}, h_i, g to the functions u_j, ω_j and φ have the form [2, 3, 16]

$$\begin{aligned} &\sigma_{ij} = (\lambda div\ u + \gamma\phi)\,\delta_{ij} + (\mu+\alpha)\,\partial_i u_j + (\mu-\alpha)\,\partial_j u_i - 2\alpha\in_{\mathrm{jik}}\omega_k,\\ &\mu_{ij} = \alpha\ div\ \omega\ \delta_{ij} + (\nu+\beta)\,\partial_i\omega_j + (\nu-\beta)\,\partial_j\omega_i,\\ &hi = \delta\partial_i\phi,\\ &g = -\xi\phi - \gamma div\ u, \end{aligned} \tag{2}$$

where λ and μ are the Lamé parameters; $\alpha,\ \beta,\ \nu,\ \sigma$ are the constants characterizing the microstructure of the discussed elastic media; $\delta,\ \xi,\ \gamma$

are the constants characterizing the body porosity; δ_{ij} is the Kronecker delta.

From the basic three-dimensional equations, we obtain the basic equations for the case of plane deformation. Let Ω be a sufficiently long cylindrical body with generatrix parallel to the Ox_3-axis. Denote by D the cross-section of this cylindrical body, thus $D \in \mathbb{R}^2$. In the case of plane deformation $u_3 = 0,\ \omega_1 = 0,\ \omega_2 = 0$, while the functions $u_1,\ u_2,\ \omega_3$ *and* φ do not depend on the coordinate x_3 [18]. We also assume $u_1,\ u_2,\ \omega_3,\ \varphi \in C^2\ (D) \cap\ C^1(\overline{D})$.

As follows from formula (2), in the case of plane deformation

$$\sigma_{\alpha 3} = 0, \sigma_{3\alpha} = 0, \mu_{\alpha\beta} = 0, \mu_{33} = 0, h_3 = 0, \alpha = 1,2, \beta = 1,2.$$

Therefore the homogeneous system $(F_\alpha = 0, G_\alpha = 0, l = 0)$ of equilibrium Eqs. (2) takes the form

$$\begin{aligned}
&\partial_1\sigma_{11} + \partial_2\sigma_{21} = 0,\\
&\partial_1\sigma_{12} + \partial_2\sigma_{22} = 0,\\
&\partial_1\mu_{13} + \partial_2\mu_{23} + (\sigma_{12} - \sigma_{21}) = 0,\\
&\partial_1 h_1 + \partial_2 h_2 + g = 0,
\end{aligned} \tag{3}$$

where $\Delta_2 \equiv \partial_{11} + \partial_{22}$ is the two-dimensional Laplace operator.

Relations (2) are rewritten as (4)

$$\begin{aligned}
&\sigma_{11} = \gamma\varphi + \lambda\theta + 2\mu\partial_1 u_1,\\
&\sigma_{22} = \gamma\varphi + \lambda\theta + 2\mu\partial_2 u_2,\\
&\sigma_{12} = (\mu + \alpha)\,\partial_1 u_2 + (\mu + \alpha)\,\partial_2 u_1 - 2\alpha\omega_3,\\
&\sigma_{21} = (\mu + \alpha)\,\partial_2 u_1 + (\mu - \alpha)\,\partial_1 u_2 + 2\alpha\omega_3,\\
&\sigma_{33} = \gamma\varphi + \lambda\theta,\\
&\mu_{13} = (\nu + \beta)\,\partial_1\omega_3,\\
&\mu_{23} = (\nu + \beta)\,\partial_2\omega_3,\\
&\mu_{31} = (\nu - \beta)\,\partial_1\omega_3,\\
&\mu_{32} = (\nu - \beta)\,\partial_2\omega_3,\\
&h_1 = \delta\partial_1\varphi,\\
&h_2 = \delta\partial_2\varphi,\\
&g = -\xi\varphi - \gamma\theta,
\end{aligned} \tag{4}$$

where $\theta := \partial_1 u_1 + \partial_2 u_2$.

If relations (4) are substituted into the system (3) then we obtain the following system of equilibrium equations with respect to the functions $u_1,\ u_2,\ \omega_3$ and φ

$$\begin{aligned}
&(\mu + \alpha)\,\Delta_2 u_1 + (\lambda + \mu - \alpha)\,\partial_1\theta + 2\alpha\partial_2\omega_3 + \gamma\partial_1\varphi = 0,\\
&(\mu + \alpha)\,\Delta_2 u_2 + (\lambda + \mu - \alpha)\,\partial_2\theta - 2\alpha\partial_1\omega_3 + \gamma\partial_2\varphi = 0,\\
&(\nu + \beta)\,\Delta_2\omega_3 + 2\alpha(\partial_1 u_2 - \partial_2 u_1) - 4\alpha\omega_3 = 0,\\
&(\delta\Delta_2 - \xi)\varphi - \gamma\theta = 0.
\end{aligned}$$

On the plane Ox_1x_2, we introduce the complex variable $z = x_1 + ix_2 = re^{i\alpha}$ $\left(i^2 = -1\right)$ and the operators $\partial_z = 0.5\left(\partial_1 - i\partial_2\right)$, $\partial_{\overline{z}} = 0.5\left(\partial_1 + i\partial_2\right)$, $\overline{z} = x_1 - ix_2$, $\Delta_2 = 4\partial_z\partial_{\overline{z}}$.

To write system (3) in the complex form, the second equation of this system is multiplied by i and summed up with the first equation

$$\begin{aligned}
&\partial_z(\sigma_{11} - \sigma_{22} + i(\sigma_{12} + \sigma_{21})) + \partial_{\overline{z}}(\sigma_{11} + \sigma_{22} + i(\sigma_{12} - \sigma_{21})) = 0\\
&\partial_z(\mu_{13} + i\mu_{23}) + \partial_{\overline{z}}(\mu_{13} - i\mu_{2,3}) + \sigma_{12} - \sigma_{21} = 0,\\
&\partial_z(h_1 + ih_2) + \partial_{\overline{z}}(h_1 - ih_2) + g = 0
\end{aligned} \tag{5}$$

where by formulas (4) we have

$$\begin{aligned}
&\sigma_{11} - \sigma_{22} + i(\sigma_{12} + \sigma_{21}) = 4\mu\partial_{\overline{z}}u_+\\
&\sigma_{11} + \sigma_{22} + i(\sigma_{12} - \sigma_{21})\\
&= 2(\lambda + \mu - \alpha)\theta - 4\alpha i\omega_3 + 2\gamma\phi + 4\alpha\partial_z u_+,\\
&\mu_{13} + i\mu_{23} = 2(\nu + \beta)\partial_{\overline{z}}\omega_3,\\
&\mu_{31} + i\mu_{23} = 2(\nu - \beta)\partial_{\overline{z}}\omega_3,\\
&h_1 + ih_2 = 2\delta\partial_{\overline{z}}\phi,\\
&u_+ := u_1 + iu_2,\ \overline{u}_+ = u_1 - iu_2,\ \theta = \partial_z + \partial_{\overline{z}}\overline{u}_+
\end{aligned} \tag{6}$$

If relations (6) are substituted into system (5), then system (3) is written in the complex form

$$\begin{aligned}
&2(\mu + \alpha)\partial_z\partial_{\overline{z}}u_+ + (\lambda + \mu - \alpha)\partial_{\overline{z}}\theta - 2\alpha i\partial_{\overline{z}}\omega_3 + \gamma\partial_{\overline{z}}\phi = 0,\\
&2(\nu + \beta)\partial_z\partial_{\overline{z}}\omega_3 + \alpha i(\theta - 2\partial_z u_+) - 2\alpha\omega_3 = 0,\\
&(4\partial_z\partial_{\overline{z}} - \xi)\phi - \gamma\theta = 0.
\end{aligned} \tag{7}$$

The general solution of the system of Eqs. (7) is represented using formulas [17, 19]

$$2\mu u_+ = (\kappa + \kappa_0)\varphi(z) - (1 - \kappa_0)z\overline{\varphi'(z)} - \overline{\psi(z)} + 4\partial_{\bar{z}}(i\chi(z,\bar{z}) - \gamma\eta(z,\bar{z})), \tag{8}$$

$$2\mu\omega_3 = \frac{4\mu}{\nu + \beta}\chi(z,\overline{z}) - \frac{\kappa + 1}{2}i(\varphi'(z) - \varphi'(\overline{z})) \tag{9}$$

$$\phi = \frac{(\lambda + 2\mu)\xi - \gamma^2}{\mu\delta}\eta(z,\overline{z}) - \frac{\gamma}{(\lambda + \mu)\delta\zeta_2^2}(\varphi'(z) + \overline{\varphi'(z)}) \tag{10}$$

where $\varphi(z)$ and $\psi(z)$ are the arbitrary analytic functions of z, $\chi(z,\bar{z})$ and $\eta(z,\bar{z})$ are the general solutions of the Helmholtz equations

$$\Delta_2\chi - \zeta_1^2\chi = 0,\quad \Delta_2\eta - \zeta_2^2\eta = 0$$

$$\zeta_1^2 = \frac{4\mu\alpha}{(\nu + \beta)(\mu + \alpha)} > 0,\quad \zeta_2^2 = \frac{(\lambda + 2\mu)\xi - \gamma^2}{(\lambda + 2\mu)\delta} > 0.$$

also

$$\kappa = \frac{\lambda + 3\mu}{\lambda + \mu}, \quad \kappa_0 = \frac{\gamma^2 \mu}{(\lambda + \mu)((\lambda + 2\mu)\xi - \gamma^2)}.$$

Assume that mutually perpendicular unit vectors **l** and **s** be such that

$$\mathbf{l} \times \mathbf{s} = \mathbf{e}_3$$

where $\mathbf{e}_3$ is the unit vector directed along the x_3-axis. The vector **l** forms the angle α with the positive direction of the x_1-axis. Then the displacement components $u_l = \mathbf{u} \cdot \mathbf{l}$, $u_s = \mathbf{u} \cdot \mathbf{s}$, as well as the stress and moment stress components acting on an area of arbitrary orientation are expressed by the formulas [18]

$$\begin{aligned} & u_l + iu_s = e^{-i\alpha} u_+, \\ & \sigma_{ll} + i\sigma_{ls} = 0.5[\sigma_{11} + \sigma_{22} + i(\sigma_{12} - \sigma_{21}) \\ & +(\sigma_{11} - \sigma_{22} + i(\sigma_{12} + \sigma_{21}))e^{-2i\alpha}] \\ & \mu_{l3} = 0.5[(\mu_{13} + i\mu_{23})e^{-i\alpha} + (\mu_{13} - i\mu_{23})e^{i\alpha}] \\ & h_l = 0.5[(h_1 + ih_2)e^{-i\alpha} + (h_1 - ih_2)e^{i\alpha}]. \end{aligned} \tag{11}$$

3 The boundary value problem for a circle

Let us consider the elastic circle, consisting of Cosserat media with voids bounded by the circumference of radius R (Fig. 1). The origin of coordinates is at the center of the circle.

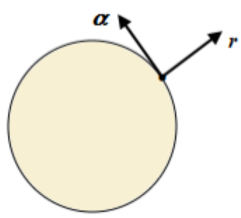

Figure 1: The elastic circle.

On the circumference, we consider the following boundary value problem

$$\sigma_{rr} - i\sigma_{r\alpha} = N - iT, \;\; \omega_3 = M, \;\; \phi = F, \;\; on\; r = R \tag{12}$$

where N, T, M and F are sufficiently smooth functions.

Substituting the formulas (8)–(10) into (6) and (11) we have

$$\begin{aligned} \sigma_{rr} - i\sigma_{r\alpha} = {} & (1 - \kappa_0)(\varphi'(z) + \overline{\varphi'(z)}) + \zeta_1^2 i\chi(z, \overline{z}) + \gamma\zeta_2^2\eta(z, \overline{z}) \\ & - e^{2i\alpha}[(1 - \kappa_0)\overline{z}\varphi''(z) + \psi'(z) + 4\partial(z)\partial(z)(i\chi(z, \overline{z}) + \gamma\eta(z\overline{z}))]. \end{aligned} \tag{13}$$

The analytic functions $\varphi'(z), \psi'(z)$ and the metaharmonic functions $\chi(z,\overline{z}), \eta(z,\overline{z})$ are represented as the following series

$$\varphi'(z) = \sum_{n=0}^{\infty} a_n z^n, \quad \psi'(z) = \sum_{n=0}^{\infty} b_n z^n \tag{14}$$

$$\chi(z,\overline{z}) = \sum_{-\infty}^{+\infty} \alpha_n I_n(\zeta_1 r) e^{in\alpha}, \quad \eta(z,\overline{z}) = \sum_{-\infty}^{+\infty} \beta_n I_n(\zeta_2 r) e^{in\alpha}, \tag{15}$$

where $I_n(\zeta_1 r)$ and $I_n(\zeta_2 r)$ are the modified Bessel function of the first kind of n-th order.

Substituting (14), (15) in (9), (10), (13), taking into account the boundary conditions (12) and assuming that the series converge on the circumference $r = R$, one finds

$$(1-\kappa_0)\sum_{n=0}^{\infty} R^n((1-n)a_n e^{in\alpha} + \overline{a}_n e^{-in\alpha}) - \sum_{n=0}^{\infty} R^n b_n e^{i(n+2)\alpha}$$
$$-\frac{2}{R}\sum_{-\infty}^{+\infty}(n-1)(\zeta_1 I_{n-1}(\zeta_1 R) i\alpha_n + \zeta_2 \gamma I_{n-1}(\zeta_2 R)\beta_n) e^{in\alpha} = N - iT \tag{16}$$

$$\frac{4\mu}{\nu+\beta}\sum_{-\infty}^{+\infty} I_n(\zeta_1 R)\alpha_n e^{in\alpha} +$$
$$\frac{\kappa+1}{2} i \sum_{n=0}^{\infty} R^n (a_n e^{in\alpha} - \overline{a}_n e^{-in\alpha}) = 2\mu M, \tag{17}$$

$$\frac{(\lambda+2\mu)\xi - \gamma^2}{\mu\delta}\sum_{-\infty}^{+\infty} I_n(\zeta_2 R)\beta_n e^{in\alpha}$$
$$-\frac{\gamma}{(\lambda+\mu)\delta\zeta_2^2}\sum_{n=0}^{+\infty} R^n (a_n e^{in\alpha} + \overline{a}_n e^{-in\alpha}) = F. \tag{18}$$

As a conclusion of the previous relations, we used the following well-known formula

$$I_{n-1}(x) - I_{n+1}(x) = \frac{2n}{x} I_n(x).$$

Expand the function $N - iT$, $2\mu M$ and F, given on $r = R$, in a complex Fourier series

$$N - iT = \sum_{-\infty}^{+\infty} A_n e^{in\alpha}, \quad 2\mu M = \sum_{-\infty}^{+\infty} B_n e^{in\alpha}, \quad F = \sum_{-\infty}^{+\infty} C_n e^{in\alpha}.$$

Comparing in (16)–(18) the coefficients of $e^{0i\alpha}$ we have (it is also assumed that a_0 is a real value [18])

$$2(1-\kappa_0)a_0+\frac{2\gamma}{R}\zeta_2 I_1(\zeta_2 R)\beta_0=N_0, \tag{19}$$

$$-\frac{2\gamma}{(\lambda+\mu)\delta\zeta_2^2}a_0+\frac{(\lambda+2\mu)\xi-\gamma^2}{\mu\delta}I_0(\zeta_2 R)\beta_0=F_0, \tag{20}$$

$$\alpha_0=-\frac{R}{2\zeta_1 I_1(\zeta_1 R)}T_0=\frac{\nu+\beta}{4\mu I_0(\zeta_1 R)}M_0.$$

In order for the problem to have a solution, the following condition must be met

$$M_0=-\frac{4\mu R I_0(\zeta_1 R)}{2(\nu+\beta)\zeta_1 I_1(\zeta_1 R)}T_0.$$

From Eqs. (19), (20) we determine the coefficients a_0 and β_0

$$a_0=\frac{\frac{(\lambda+2\mu)\xi-\gamma^2}{\mu\delta}I_0(\zeta_2 R)N_0-\frac{2\gamma}{R}\zeta_2 I_1(\zeta_2 R)F_0}{\frac{2(\lambda+2\mu)((\lambda+\mu)\xi-\gamma^2)I_0(\zeta_2 R)}{(\lambda+\mu)\mu\delta}+\frac{4\gamma^2 I_1(\zeta_2 R)}{(\lambda+\mu)\delta\zeta_2 R)}},$$

$$\beta_0=\frac{\frac{2\gamma}{(\lambda+\mu)\delta\zeta_2^2}N_0+2(1-\kappa_0)F_0}{\frac{2(\lambda+2\mu)((\lambda+\mu)\xi-\gamma^2)I_0(\zeta_2 R)}{(\lambda+\mu)\mu\delta}+\frac{4\gamma^2 I_1(\zeta_2 R)}{(\lambda+\mu)\delta\zeta_2 R)}}.$$

comparing the coefficients of $e^{in\alpha}$ $(n\neq 0)$, we have

$$\begin{aligned}&(1-n)(1-\kappa_0)R^n a_n-R^{n-2}b_{n-2}-\\&\frac{2}{R}(n-1)(\zeta_1 I_{n-1}(\zeta_1 R)i\alpha_n+\zeta_2\gamma I_{n-1}(\zeta_2 R)\beta_n)=N_n,\ n\geq 2\end{aligned} \tag{21}$$

$$\begin{aligned}&(1-\kappa_0)R^n a_n-\frac{2}{R}(n+1)(\zeta_1 I_{n+1}(\zeta_1 R)i\alpha_n-\zeta_2\gamma I_{n+1}(\zeta_2 R)\beta_n)\\&=\overline{N}_{-n},\ n>0\end{aligned} \tag{22}$$

$$\frac{4\mu}{\nu+\beta}I_n(\zeta_1 R)\alpha_n+\frac{\kappa+1}{2}iR^n a_n=M_n,\ n\geq 1 \tag{23}$$

$$\frac{(\lambda+2\mu)\xi-\gamma^2}{\mu\delta}I_n(\zeta_2 R)\beta_n-\frac{\gamma}{(\lambda+\mu)\delta\zeta_2^2}R^n a_n=F_n,\ n\geq 1. \tag{24}$$

From (21)–(24) one finds

$$a_n=\frac{\overline{N}_{-n}+k_{1n}M_n-k_{2n}F_n}{(1-\kappa_0)R^n-k_{3n}+k_{4n}},$$

$$\alpha_n=\frac{\nu+\beta}{4\mu I_n(\zeta_1 R)}\left(M_n-\frac{\kappa+1}{2}iR^n a_n\right),\ n\geq 1$$

$$\beta_n = \frac{\mu\delta}{((\lambda+2\mu)\xi-\gamma^2)I_n(\zeta_2 R)}\left(F_n + \frac{\gamma}{(\lambda+\mu)\delta\zeta_2^2}R^n a_n\right),\ n \geq 1.$$

$$b_{n-2} = R^{2-n}((1-n)(1-\kappa_0)R^n a_n$$

$$-\frac{2}{R}(n-1)(\zeta_1 I_{n-1}(\zeta_1 R)i\alpha_n + \zeta_2\gamma I_{n-1}(\zeta_2 R)\beta_n) - N_n), n \geq 2$$

where

$$k_{1n} = \frac{(n+1)\zeta_1 I_{n+1}(\zeta_1 R)(\nu+\beta)i}{2\mu R I_n(\zeta_1 R)},$$

$$k_{2n} = \frac{2(n+1)\zeta_2\gamma\mu\delta I_{n+1}(\zeta_2 R)}{(2(\lambda+2\mu)\xi-\gamma^2)R I_n(\zeta_2 R)},$$

$$k_{3n} = \frac{(n+1)\zeta_1(\kappa+1)(\nu+\beta)R^n I_{n+1}(\zeta_1 R)}{4\mu R I_n(\zeta_1 R)},$$

$$k_{4n} = \frac{2(n+1)\gamma^2\mu I_{n+1}(\zeta_2 R)R^n}{((\lambda+2\mu)\xi-\gamma^2)(\lambda+\mu)\zeta_2 R I_n(\zeta_2 R)}.$$

It is easy to prove the absolute and uniform convergence of the series obtained in the the circle (including the contours) when the functions set on the boundaries have sufficient smoothness.

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