SOME BVP IN THE PLANE THEORY OF THERMODYNAMICS WITH MICROTEMPERATURES

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Abstract

In this work we consider the two-dimensional version of statics of the linear theory of elastic materials with inner structure whose particles, in addition to the classical displacement and temperature fields, possess microtemperatures. The Dirichlet BVP is solved for a circle.

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1 Introduction

In the work we consider a two-dimensional system of differential equations describing the plane statical thermoelastic balance of homogenous isotropic elastic bodies. The linear theory of thermoelasticity with microtemperatures for materials with inner structure whose particles, in addition to the classical displacement and temperature fields, possess microtemperatures was presented by Iesan and Quintanilla [1]. The fundamental solutions of the equations of the theory of thermoelasticity with microtemperatures were constructed by Svanadze [2]. The exponential stability of solution of equations of the theory of thermoelasticity with microtemperatures was established by Casas and Quintanilla [3]. In [4, 5], the basic BVPs of steady vibrations were investigated using the potential method and the theory of singular integral equations.

Various issues of termoelastic equilibrium of isotropic homogeneous bodies taking into account the microtemperature are devoted to [6-19].

In the present paper we consider the two-dimensional system of the differential equations describing the plane statical thermoelastic balance of homogenous isotropic elastic bodies, the microelements of which have microtemperature in addition to the classical displacement and a temperature field. The general solution of this system of the equations is construed by means of analytic functions of complex variable and solutions of the equation of Helmholtz. We solve the Dirichlet boundary value problem for a circle.

2 Basic three-dimensional relations

Let (x_1, x_2, x_3) be the point of the Euclidean three-dimensional space.

The fundamental system of field equations in the linear equilibrium theory of thermoelasticity with microtemperatures consists of the equations of equilibrium [1]

$$\partial_i \sigma_{ij} + \rho F_j^{(1)} = 0, \tag{1}$$

the balance energy

$$\partial_i q_i + \rho S = 0, \tag{2}$$

the first moment of energy

$$\partial_i q_{ij} + q_j - Q_j + \rho F_j^{(2)} = 0, \tag{3}$$

the constitutive equations

$$\begin{aligned}
&\sigma_{ij} = \lambda e_{rr} \delta_{ij} + 2\mu e_{ij} - \beta T \delta_{ij}, \\
&q_j = k \partial_j T + k_1 w_j, \\
&q_{ij} = -k_4 \partial_r w_r \delta_{ij} - k_5 \partial_j w_i - k_6 \partial_i w_j, \\
&Q_j = (k_1 - k_2) w_j + (k - k_3) \partial_j T
\end{aligned} \tag{4}$$

and the geometrical equations

$$e_{ij} = \frac{1}{2}(\partial_j u_i + \partial_i u_j), \tag{5}$$

where $\mathbf{u} = (u_1, u_2, u_3)$ is the displacement vector, $\mathbf{w} = (w_1, w_2, w_3)$ is the microtemperature vector, T is the temperature measured from the constant absolute temperature T_0 ($T_0 > 0$), σ_{ij} is the stress tensor, ρ is the reference mass density ($\rho > 0$), $\mathbf{F}^{(1)} = (F_1^{(1)}, F_2^{(1)}, F_3^{(1)})$ is the body force, $\mathbf{q} = (q_1, q_2, q_3)$ is the heat flux vector, S is the heat supply, q_{ij} is first heat flux moment tensor, $\mathbf{Q} = (Q_1, Q_2, Q_3)$ is the mean heat flux vector, $\mathbf{F}^{(2)} = (F_1^{(2)}, F_2^{(2)}, F_3^{(2)})$ is first heat source moment vector, λ, μ, β, k, $k_1, ..., k_6$ are constitutive coefficients, δ_{ij} is the Kronecker delta, e_{ij} is the strain tensor, $i, j = 1, 2, 3$, and repeated indices are summed over the range (1,2,3).

By virtue of Eqs. (4) and (5), system (1)–(3) can be expressed in terms of the components of the displacement vector $\mathbf{u}$, of the components of the microtemperature vector $\mathbf{w}$ and the temperature T. We obtain the system of equations of the linear equilibrium theory of thermoelasticity with microtemperatures [1]

$$\begin{aligned} &\mu\Delta_3 u_i + (\lambda+\mu)\partial_i\Theta_1 - \beta\partial_i T = -\rho F_i^{(1)}, \\ &k_6\Delta_3 w_i + (k_4+k_5)\partial_i\Theta_2 - k_3\partial_i T - k_2 w_i = -\rho F_i^{(2)}, \\ &k\Delta_3 T + k_1\Theta_2 = -\rho S, \end{aligned} \tag{6}$$

where $\Delta_3 = \partial_{11} + \partial_{22} + \partial_{33}$ is the three dimensional Laplace operator, $\Theta_1 = \partial_1 u_1 + \partial_2 u_2 + \partial_3 u_3$, $\Theta_2 = \partial_1 w_1 + \partial_2 w_2 + \partial_3 w_3$.

3 The plane deformation case

Let the homogenous isotropic cylindrical body be classified by the Cartesian system of coordinates of x_1, x_2, x_3 in such a way that the generatrix of the lateral surface is parallel to an axis, x_3. In the case of a change in temperature T, and also the components of displacement vector u_1, u_2 and components of microtemperature vector w_1, w_2 along axes x_1, x_2, do not depend on coordinate x_3. Additionally, the components of displacement and microtemperature along axes x_3 (u_3 and w_3, respectively) are equal to zero, we have a case of the plane deformation state.

As follows from formulas (4)-(5), in the case of plane deformation

$$\sigma_{\alpha 3} = \sigma_{3\alpha} = 0, \quad q_{\alpha 3} = q_{3\alpha} = 0, \quad q_3 = Q_3 = 0.$$

Therefore the system of equilibrium equations (1-3) takes the form

$$\partial_\alpha \sigma_{\alpha\gamma} = 0, \tag{7}$$

$$\partial_\alpha q_\alpha = 0, \tag{8}$$

$$\partial_\alpha q_{\alpha\gamma} + q_\gamma - Q_\gamma = 0. \tag{9}$$

Relations (4) are rewritten as

$$\begin{aligned}
&\sigma_{\alpha\gamma} = \lambda\theta\delta_{\alpha\gamma} + \mu(\partial_\alpha u_\gamma + \partial_\gamma u_\alpha) - \beta T\delta_{\alpha\gamma},\\
&\sigma_{33} = \lambda\theta - \beta T,\\
&q_{\alpha\gamma} = -k_4\vartheta\delta_{\alpha\gamma} - k_5\partial_\gamma w_\alpha - k_6\partial_\alpha w_\gamma,\\
&q_{33} = -k_4\vartheta,\\
&q_\alpha = k\partial_\alpha T + k_1 w_\alpha,\\
&Q_\alpha = (k_1 - k_2)w_\alpha + (k - k_3)\partial_\alpha T \quad \alpha,\gamma = 1,2,
\end{aligned} \tag{10}$$

where $\theta = \partial_1 u_1 + \partial_2 u_2$, $\vartheta = \partial_1 w_1 + \partial_2 w_2$.

If relations (10) are substituted into system (7-9), then we obtain the following system of equilibrium equations with respect to the functions u_α, w_α and T

$$\begin{aligned}
&\mu\Delta u_\alpha + (\lambda+\mu)\partial_\alpha\theta - \beta\partial_\alpha T = 0,\\
&k_6\Delta w_\alpha + (k_4 + k_5)\partial_\alpha\vartheta - k_3\partial_\alpha T - k_2 w_\alpha = 0,\\
&k\Delta T + k_1\vartheta = 0,
\end{aligned} \tag{11}$$

where $\Delta = \partial_{11} + \partial_{22}$ is the tow dimensional Laplace operator.

On the plane Ox_1x_2 a complex variable $z = x_1 + ix_2$, where i the imaginary unit, and the following operators $\partial_z = 0.5(\partial_1 - i\partial_2)$, $\partial_{\bar z} = 0.5(\partial_1 + i\partial_2)$ are introduced. Then the system consisting of the equations (1) can be written in complex form as follows

$$\begin{aligned}
&\mu\Delta u_+ + 2(\lambda+\mu)\partial_{\bar z}\theta - 2\beta\partial_{\bar z}T = 0,\\
&k_6\Delta w_+ + 2(k_4 + k_5)\partial_{\bar z}\vartheta - 2k_3\partial_{\bar z}T - k_2 w_+ = 0,\\
&k\Delta T + k_1\vartheta = 0,
\end{aligned} \tag{12}$$

where $\Delta = 4\partial_z\partial_{\bar z}$; $u_+ := u_1 + iu_2$; $w_+ := w_1 + iw_2$.

For the positive definiteness of the corresponding quadratic form will satisfy the conditions

$$k_4 + k_5 + k_6 > 0, \ \ k_2 > 0, \ \ k_1k_3 - kk_2 < 0, \ \ k > 0.$$

In [19] it is shown that the general solution of system (12) is represented as follows:

$$
\begin{aligned}
&2\mu u_+ = \varkappa\phi(z) - z\overline{\phi'(z)} - \overline{\psi(z)} \\
&+\frac{\mu\beta}{\lambda+2\mu}\left\{\frac{k_2}{2k_3}\left[\varphi(z) + z\overline{\varphi'(z)}\right] - \frac{2k_1}{kk^*}\partial_{\bar z}\chi_1(z,\bar z)\right\}, \\
&w_+ = -\overline{\varphi''(z)} + \partial_{\bar z}[\chi_1(z\bar z) + i\chi_2(z\bar z)], \\
&T = \frac{k_2}{2k_3}[\varphi'(z) + \overline{\varphi'(z)}] - \frac{k_1}{2k}\chi_1(z\bar z),
\end{aligned}
\tag{13}
$$

where $\varkappa = \dfrac{\lambda+3\mu}{\lambda+\mu}$, $\varphi(z)$, $\phi(z)$ and $\psi(z)$ are the arbitrary analytic function of a complex variable z, $\chi_1(z\bar z)$ is a general solution of the following Helmholtz equation $\Delta\chi_1 - k^*\chi_1 = 0$, $k^* = \dfrac{k_2k - k_1k_3}{k(k_4+k+5+k_6)} > 0$; $\chi_2(z\bar z)$ is a general solution of the following Helmholtz equation $\Delta\chi_2 - \tilde k\chi_2 = 0$, $\tilde k = \dfrac{k_2}{k_6}$.

4 A problem for a circle.

In this section, we solve a concrete boundary value problem for a circle of radius R (Fig. 1).

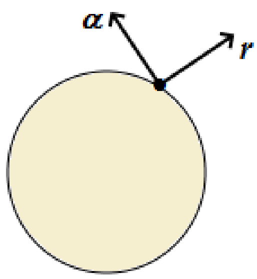

Figure 1:

We consider the following problem

$$
\begin{aligned}
2\mu u_+|_{r=R} &= 2\mu(G_1 + iG_2), \\
w_+|_{r=R} &= H_1 + iH_2, \quad T|_{r=R} = Q.
\end{aligned}
\tag{14}
$$

The expressions $2\mu(G_1+iG_2)$, H_1+iH_2, Q may be represented by the series

$$
\begin{aligned}
2\mu(G_1 + iG_2) &= \sum_{-\infty}^{+\infty} A_n e^{in\alpha},\\
H_1 + iH_2 &= \sum_{-\infty}^{+\infty} B_n e^{in\alpha}, \\
Q = \sum_{-\infty}^{+\infty} C_n e^{in\alpha}, &\quad C_n = \overline{C}_{-n}.
\end{aligned}
\tag{15}
$$

The analytic function $\phi(z)$, $\psi(z)$, $\varphi(z)$ and the metaharmonic functions $\chi_1(z,\bar z)$, $\chi_2(z,\bar z)$ are represented as a series

$$
\begin{aligned}
\phi(z) = \sum_{n=1}^{\infty} a_n z^n, \quad \psi(z) = \sum_{n=0}^{\infty} b_n z^n, \quad \varphi(z) = \sum_{n=1}^{+\infty} c_n z^n,\\
\chi_1(z,\bar z) = \sum_{-\infty}^{+\infty} \alpha_n I_n(\sqrt{k^*}r)e^{in\alpha}, \quad \chi_2(z,\bar z) = \sum_{-\infty}^{+\infty} \beta_n I_n(\sqrt{\tilde k}r)e^{in\alpha},
\end{aligned}
\tag{16}
$$

where $I_n(\cdot)$ is modified Bessel function of n-th order.

From (13), substituted (15), (16) in the boundary conditions (14) we have $(z|_R = Re^{i\alpha})$

$$
\begin{aligned}
&\varkappa\sum_{n=1}^{\infty} a_n R^n e^{in\alpha} - \bar a_1 Re^{i\alpha} - \sum_{n=0}^{\infty}(n+2)\bar a_{n+2}R^{n+2}e^{-in\alpha} - \sum_{n=0}^{\infty}\bar b_n R^n e^{-in\alpha}\\
&+\frac{\mu\beta}{\lambda+2\mu}\left\{\frac{k_2}{2k_3}\left[\sum_{n=1}^{\infty} c_n R^n e^{in\alpha} + \bar c_1 Re^{i\alpha} + \sum_{n=0}^{\infty}(n+2)\bar c_{n+2}R^{n+2}e^{-in\alpha}\right]\right.\\
&\left.-\frac{k_1}{k\sqrt{k^*}}\sum_{-\infty}^{+\infty}\alpha_n I_{n+1}(\sqrt{k^*}R)e^{i(n+1)\alpha}\right\} = \sum_{-\infty}^{+\infty} A_n e^{in\alpha},
\end{aligned}
$$

$$
\begin{aligned}
&\frac{k_2}{2k_3}\sum_{n=1}^{\infty}\left[nc_nR^{n-1}e^{i(n-1)\alpha} + n\bar c_n R^{n-1}e^{-i(n-1)\alpha}\right]\\
&-\frac{k_1}{2k}\sum_{-\infty}^{+\infty}\alpha_n I_n(\sqrt{k^*}R)e^{in\alpha} = \sum_{-\infty}^{+\infty} C_n e^{in\alpha},
\end{aligned}
$$

$$
\begin{aligned}
&-\sum_{n=2}^{\infty} n(n-1)\bar c_n R^{n-2}e^{-i(n-2)\alpha} + \frac{\sqrt{k^*}}{2}\sum_{-\infty}^{+\infty}\alpha_n I_{n+1}(\sqrt{k^*}R)e^{i(n+1)\alpha}\\
&+i\frac{\sqrt{\tilde k}}{2}\sum_{-\infty}^{+\infty}\beta_n I_{n+1}(\sqrt{\tilde k}R)e^{i(n+1)\alpha} = \sum_{-\infty}^{+\infty} B_n e^{in\alpha}.
\end{aligned}
$$

Compare the coefficients at identical degrees. We obtain the following systems of equations

$$
\varkappa Ra_1 - R\bar a_1 + \frac{\mu\beta}{\lambda+2\mu}\left[\frac{k_2R}{2k_3}(c_1+\bar c_1) - \frac{k_1}{k\sqrt{k^*}}I_1(\sqrt{k^*}R)\alpha_0\right] = A_1, \tag{17}
$$

$$\varkappa R^n a_n + \frac{\mu\beta}{\lambda+2\mu}\left[\frac{k_2 R^n}{2k_3}c_n - \frac{k_1}{k\sqrt{k^*}}I_n(\sqrt{k^*}R)\alpha_{n-1}\right] = A_n, \quad n>1, \tag{18}$$

$$\begin{gathered}-(n+2)R^{n+2}a_{n+2} - R^n b_n + \frac{\mu\beta}{\lambda+2\mu} \\ \times\left[\frac{k_2 R^{n+2}}{2k_3}(n+2)c_{n+2} - \frac{k_1}{k\sqrt{k^*}}I_n(\sqrt{k^*}R)\alpha_{n+1}\right] = \bar{A}_{-n}, \quad n\geq 0,\end{gathered} \tag{19}$$

$$\frac{k_2}{2k_3}(c_1+\bar{c}_1) - \frac{k_1}{2k}I_0(\sqrt{k^*}R)\alpha_0 = C_0, \tag{20}$$

$$\frac{k_2}{2k_3}(n+1)R^n c_{n+1} - \frac{k_1}{2k}I_n(\sqrt{k^*}R)\alpha_n = C_n, \quad n>0, \tag{21}$$

$$\frac{\sqrt{k^*}}{2}I_1(\sqrt{k^*}R)\alpha_0 + i\frac{\sqrt{\tilde{k}}}{2}I_1(\tilde{k}R)\beta_0 = B_1, \tag{22}$$

$$\frac{\sqrt{k^*}}{2}I_{n+1}(\sqrt{k^*}R)\alpha_n + i\frac{\sqrt{\tilde{k}}}{2}I_{n+1}(\tilde{k}R)\beta_n = B_{n+1}, \quad n>0, \tag{23}$$

$$\begin{gathered}-n(n+1)R^{n-1}c_{n+1} + \frac{\sqrt{k^*}}{2}I_{n-1}(\sqrt{k^*}R)\alpha_n - i\frac{\sqrt{\tilde{k}}}{2}I_{n-1}(\tilde{k}R)\beta_n \\ = \bar{B}_{-n+1}, \quad n\geq 1.\end{gathered} \tag{24}$$

From (22) and (23)

$$\begin{gathered}\alpha_0 = \frac{2\,\mathrm{Re}B_1}{\sqrt{k^*}I_1(\sqrt{k^*}R)}, \quad \beta_0 = \frac{2\,\mathrm{Im}B_1}{\sqrt{\tilde{k}}I_1(\tilde{k}R)}, \\ c_1+\bar{c}_1 = \frac{2k_3}{k_2}C_0 + \frac{2k_1k_3}{kk_2}\frac{\mathrm{Re}B_1}{\sqrt{k^*}I_1(\sqrt{k^*}R)}.\end{gathered} \tag{25}$$

From (17) and (25) we have

$$a_1 = \frac{\varkappa A_1' + \bar{A}_1'}{(\varkappa^2-1)R},$$

where

$$A_1' = A_1 - \frac{\mu\beta}{\lambda+2\mu}\left[\frac{k_2R}{2k_3}(c_1+\bar{c}_1) - \frac{k_1}{k\sqrt{k^*}}I_1(\sqrt{k^*}R)\alpha_0\right].$$

(remembering that always $\varkappa>1$).

From (21), (23) and (24)

$$c_n = \frac{\bar{B}_{-n+2} + \dfrac{I_{n-2}(\sqrt{\tilde{k}}R)}{I_n(\sqrt{\tilde{k}}R)}B_n + \Gamma_n\dfrac{k\sqrt{k^*}C_{n-1}}{k_1I_{n-1}(\sqrt{k^*}R)}}{-(n-1)nR^{n-2} + \Gamma_n\dfrac{kk_2\sqrt{k^*}nR^{n-1}}{2k_1k_3I_{n-1}(\sqrt{k^*}R)}}, \ n>1,$$

$$\alpha_n = \frac{kk_2(n+1)R^n}{k_1k_3I_n(\sqrt{k^*}R)}c_{n+1} - \frac{2k}{k_1I_n(\sqrt{k^*}R)}C_n,\ n>0,$$

$$\beta_n = i\frac{\sqrt{k^*}I_{n+1}(\sqrt{k^*}R)}{\sqrt{\tilde{k}}I_{n+1}(\sqrt{\tilde{k}}R)}\alpha_n - i\frac{2}{\sqrt{\tilde{k}}I_{n+1}(\sqrt{\tilde{k}}R)}B_{n+1},\ n>0,$$

where

$$\Gamma_n = I_{n-2}(\sqrt{k^*}R) + \frac{I_{n-2}(\sqrt{\tilde{k}}R)I_n(\sqrt{k^*}R)}{I_n(\sqrt{\tilde{k}}R)}.$$

The coefficients a_n are determined by these formulae (18)

$$a_n = \frac{A_n}{\varkappa R^n} - \frac{\mu\beta}{(\lambda+2\mu)\varkappa R^n}\left[\frac{k_2R^n}{2k_3}c_n - \frac{k_1}{k\sqrt{k^*}}I_n(\sqrt{k^*}R)\alpha_{n-1}\right],\ n>1.$$

finally, (19) determines all coefficients b_n for $n \geq 0$:

$$b_n = -\bar{A}_{-n} - (n+2)R^2a_{n+2} + \frac{\mu\beta}{\lambda+2\mu}$$
$$\times\left[\frac{k_2R^2}{2k_3}(n+2)c_{n+2} - \frac{k_1}{k\sqrt{k^*}R^n}I_n(\sqrt{k^*}R)\alpha_{n+1}\right].$$

It is easy to prove the absolute and uniform convergence of the series obtained in the circle (including the contours) when the functions set on the boundaries have sufficient smoothness.

Similarly the problem can be solved when on the boundary of the considered domain the values of stresses are given.

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