CONDITIONS FOR THE EXISTENCE OF NEUTRAL SURFACE OF AN ELASTIC SHELL AND THE BOUNDARY VALUE PROBLEMS FOR GENERALIZED ANALYTIC FUNCTIONS

G. Akhalaia[1], T. Meunargia[2]

[1] I. Vekua Institute of Applied Mathematics of
Iv. Javakhishvili Tbilisi State University
2 University Str., Tbilisi 0186, Georgia
giaakha@gmail.com

[2] I. Vekua Institute of Applied Mathematics of
Iv. Javakhishvili Tbilisi State University
2 University Str., Tbilisi 0186, Georgia
tengizmeunargia37@gmail.com

(Received 12.02.2019; accepted 16.06.2019)

Abstract

In this paper the conditions for the existence of a neutral surface of elastic shells is consider, when the neutral surfaces are not the middle surface of the shell, it is the equidistant surface of the middle surface. Boundary value problems of the theory of generalized analytic functions are used for convex shells.

Keywords and phrases: The neutral surface, convex shells.

AMS subject classification (2010): 74B20.

1 Mixed Form of Stress-Strain Relation

Let us write the stress-strain relations in the form [1]

$$p_j^i = \lambda\theta g_j^i + 2\mu e_j^i \quad (i, j = 1, 2, 3, \;\; g_j^i = \delta_j^i) \tag{1.1}$$

where p_j^i and e_j^i are the mixed components, respectively, of stress and strain tensors, θ is the cubical dilatation which will be written as

$$\theta = e_i^i = \theta' + e_3^3, \quad \theta' = e_\alpha^\alpha, \quad (\alpha = 1, 2) \tag{1.2}$$

when $j = 3$ from (1.1) we have

$$p_3^\alpha = 2\mu e_3^\alpha, \quad p_3^3 = \lambda\theta + 2\mu e_3^3 = \lambda\theta' + (\lambda + 2\mu)e_3^3. \tag{1.3}$$

from (1.3)

$$e_3^\alpha = \frac{1}{2\mu} p_3^\alpha, \tag{1.3a}$$

$$e_3^3 = -\frac{\lambda}{\lambda + 2\mu}\theta' + \frac{1}{\lambda + 2\mu} p_3^3. \tag{1.3b}$$

By inserting expression (1.3b) into (1.2), we obtain

$$\theta = \frac{\lambda'}{\lambda}\theta' + \frac{\lambda}{\lambda + 2\mu} p_3^3, \tag{1.4}$$

where

$$\lambda' = \frac{2\lambda\mu}{\lambda + 2\mu}.$$

Substituting expression (1.4) into (1.1) we get

$$p_j^i = T_j^i + Q_j^i,$$

where

$$T_\beta^\alpha = \lambda'\theta' g_\beta^\alpha + 2\mu e_\beta^\alpha, \quad T_3^i = T_i^3 = 0, \quad \left(\theta' = e_\alpha^\alpha\right),$$

$$Q_\beta^\alpha = \sigma' p_3^3 g_\beta^\alpha, \quad Q_3^i = p_3^i, \quad Q_i^3 = p_i^3, \quad \left(\sigma' = \frac{\lambda}{\lambda + 2\mu}\right).$$

2 Derivation of the Basic Equation

Now assume that on some coordinate surface $\hat{S} : x^3 = const$, belonging to the shell Ω, the tangential stress field is identically zero, i.e.

$$T_{\alpha\beta} = 0, \quad (on\ \hat{S} : x^3 = const). \tag{2.1}$$

Then relations

$$\frac{1}{2}(\hat{\nabla}_\alpha U_\beta + \hat{\nabla}_\beta U_\alpha) - \hat{b}_{\alpha\beta} U_3 = \frac{1}{2\nu} T_{\alpha\beta} - \frac{\lambda'}{4\mu(\lambda' + \mu)} T_\gamma^\gamma g_{\alpha\beta}$$

imply that the displacement field satisfies the equations

$$\frac{1}{2}(\hat{\nabla}_\alpha U_\beta + \hat{\nabla}_\beta U_\alpha) - \hat{b}_{\alpha\beta} U_3 = 0. \quad (\hat{S} : x^3 = const). \tag{2.2}$$

Thus, the coordinate surface $\hat{S} : x^3 =$ const, for which this condition holds, is a neutral surface of the shell.

We now pass to the study of the conditions ensuring the existence of a neutral surface among the coordinate surface $\hat{S} : x^3 = const$.

The vectorial equation of equilibrium

$$\frac{1}{\sqrt{g}}\partial_i(\sqrt{g}\boldsymbol{p}^i) + \boldsymbol{\Phi} = 0, \quad g = \det\{g_{ij}\}_{i,j=1}^{3} \tag{2.3}$$

may be written as

$$\frac{1}{\sqrt{g}}\partial_\alpha(\sqrt{g}\boldsymbol{T}^\alpha) + \frac{1}{\sqrt{g}}\partial_i(\sqrt{g}\boldsymbol{Q}^i) + \boldsymbol{\Phi} = 0. \tag{2.4}$$

Let

$$\overset{\circ}{\boldsymbol{T}}{}^\alpha = (\boldsymbol{T}^\alpha)_{x^3=0}, \quad \overset{\circ}{\boldsymbol{Q}}{}^\alpha = (\boldsymbol{Q}^\alpha)_{x^3=0}. \tag{2.5}$$

Then on the middle surface $\hat{S} : x^3 = 0$, equation (2.4) becomes

$$\frac{1}{\sqrt{a}}\partial_\alpha(\sqrt{a}\overset{\circ}{\boldsymbol{T}}{}^\alpha) + \frac{1}{\sqrt{a}}\partial_\alpha(\sqrt{a}\overset{\circ}{\boldsymbol{Q}}{}^\alpha) + \left(\frac{1}{\sqrt{g}}\partial_3(\sqrt{g}\boldsymbol{P}^3)\right)_{x^3=0} + \overset{\circ}{\boldsymbol{\Phi}} = 0, \tag{2.6}$$

where

$$\overset{\circ}{\boldsymbol{\Phi}} = (\boldsymbol{\Phi})_{x^3=0}. \tag{2.7}$$

Let the middle surface $S : x^3 = 0$ be the neutral surface of a shell. Then

$$\overset{\circ}{\boldsymbol{T}}{}^\alpha = 0, \quad \text{i.e} \quad \overset{\circ}{\boldsymbol{T}}{}^{\alpha\beta} = 0 \quad \text{on} \quad S \tag{2.8}$$

and equation (2.6) becomes

$$\frac{1}{\sqrt{a}}\partial_\alpha(\sqrt{a}\overset{\circ}{\boldsymbol{Q}}{}^\alpha) + \left(\frac{1}{\sqrt{g}}\partial_3(\sqrt{g}\boldsymbol{P}^3)\right)_{x^3=0} + \overset{\circ}{\boldsymbol{\Phi}} = 0. \tag{2.9}$$

Thus, the satisfaction on this equation is the condition necessary for the surface $S : x^3 = 0$ to be neutral.

In the sequel we consider thin shell of constant thickness $2h$. Denote the stress forces acting on the face surfaces S^+ and S^- by $\overset{(+)}{\boldsymbol{P}}$ and $\overset{(-)}{\boldsymbol{P}}$. If $\overset{(+)}{\boldsymbol{P}}$ and $\overset{(-)}{\boldsymbol{P}}$ are the pressure forces, then we egree to denote

$$\overset{(+)}{\boldsymbol{P}} = -p\boldsymbol{n}, \quad \overset{(-)}{\boldsymbol{P}} = q\boldsymbol{n}. \tag{2.9a}$$

where p and q are positive scalars. Accordingly, in the general case we assume that

$$\overset{(+)}{\boldsymbol{P}} = -(\boldsymbol{P}^3)_{x^3=h}, \quad \overset{(-)}{\boldsymbol{P}} = (\boldsymbol{P}^3)_{x^3=-h}. \tag{2.10}$$

If we approximately represent the transverse stress force by the formula

$$\boldsymbol{P}^3(x^1, x^2, x^3) \cong \overset{\circ}{\boldsymbol{P}}{}^3(x^1, x^2) + x^3 \overset{1}{\boldsymbol{P}}{}^3(x^1, x^2) \tag{2.10a}$$

where

$$\overset{\circ}{\boldsymbol{P}}{}^3 = (\boldsymbol{P}^3)_{x^3=0}, \quad \overset{1}{\boldsymbol{P}}{}^3 = (\partial_3 \boldsymbol{P}^3)_{x^3=0}, \tag{2.10b}$$

due to (2.10), we have

$$\overset{\circ}{\boldsymbol{P}}{}^3 = \frac{1}{2}\Big(\overset{(-)}{\boldsymbol{P}} - \overset{(+)}{\boldsymbol{P}}\Big) \quad \overset{1}{\boldsymbol{P}}{}^3 = -\frac{1}{2h}\Big(\overset{(+)}{\boldsymbol{P}} + \overset{(-)}{\boldsymbol{P}}\Big). \tag{2.11}$$

Consequently

$$\Big(\frac{1}{\sqrt{g}}\partial_3(\sqrt{g}\boldsymbol{P}^3)\Big)_{x^3=0} \cong H\Big(\overset{(+)}{\boldsymbol{P}} - \overset{(-)}{\boldsymbol{P}}\Big) - \frac{\overset{(+)}{\boldsymbol{P}} + \overset{(-)}{\boldsymbol{P}}}{2h}$$

$$= \frac{1}{2h}(2hH-1)\overset{(+)}{\boldsymbol{P}} - \frac{1}{2h}(2hH+1)\overset{(-)}{\boldsymbol{P}}. \tag{2.12}$$

Here we have made use of the formula

$$\Big(\frac{1}{\sqrt{g}}\partial_3(\sqrt{g})\Big)_{x^3=0} = (\partial_3 \ln \Theta)_{x^3=0} = -2H \tag{2.13}$$

and the boundary conditions (2.10), where H is middle curvatures of the shell. In view of equality (2.11) we have

$$\overset{\circ}{\boldsymbol{Q}}{}^\alpha = \boldsymbol{\sigma'}\overset{\circ}{P}{}^{33}\boldsymbol{r}^\alpha + \overset{\circ}{P}{}^\alpha_3\boldsymbol{n} = -\frac{1}{2}\sigma'\Big(\overset{(+)}{P}{}^3 - \overset{(-)}{P}{}^3\Big)\boldsymbol{r}^\alpha - \frac{1}{2}\Big(\overset{(+)}{P}{}^\alpha - \overset{(-)}{P}{}^\alpha\Big)\boldsymbol{n}, \tag{2.14}$$

where $\overset{(+)}{P}{}^i$ and $\overset{(-)}{P}{}^i$ are the contravariant components of the vectors $\overset{(+)}{\boldsymbol{P}}$ and $\overset{(-)}{\boldsymbol{P}}$ respectively. In view of equalities (2.14) and (2.11) we may write equation (2.9) as

$$\frac{1}{\sqrt{a}}\partial_\alpha\Big\{\sqrt{a}\Big[\sigma'\Big(\overset{(+)}{P}{}^3 - \overset{(-)}{P}{}^3\Big)\boldsymbol{r}^\alpha + \Big(\overset{(+)}{P}{}^\alpha - \overset{(-)}{P}{}^\alpha\Big)\boldsymbol{n}\Big]\Big\}$$

$$+\frac{1}{h}(1-2hH)\overset{(+)}{\boldsymbol{P}} + \frac{1}{h}(1+2hH)\overset{(-)}{\boldsymbol{P}} - 2\overset{(0)}{\boldsymbol{\Phi}} = 0. \tag{2.15}$$

Thus, if the middle surface of the thin elastic shell is neutral then the stresses $\overset{(+)}{\boldsymbol{P}}$ and $\overset{(+)}{\boldsymbol{P}}$, applied to the face surfaces of a shell, must satisfy the vector equation (2.15).

Then we have the equation

$$\frac{1}{\sqrt{a}}\partial_\alpha[\sqrt{a}(\sigma' p\boldsymbol{r}^\alpha + p^\alpha\boldsymbol{n})] + \frac{1}{h}(1-2hH)(p\boldsymbol{n} + p^\alpha\boldsymbol{r}_\alpha) + \tilde{\boldsymbol{\Phi}} = 0, \tag{2.16}$$

where

$$p = \overset{(+)}{P}{}^3 - \overset{(-)}{p}{}^3 \quad p^\alpha = \overset{(+)}{P}{}^\alpha - \overset{(-)}{p}{}^\alpha \tag{2.16a}$$

$$\tilde{\boldsymbol{\Phi}} = -2\overset{\circ}{\boldsymbol{\Phi}} + \frac{2}{h}\overset{(-)}{\boldsymbol{P}}. \tag{2.17}$$

Equation (2.16) is equivalent to the system of equations

$$\sigma' \partial_\alpha p + \frac{1}{h}[(1-2hH)a_{\alpha\beta} - hb_{\alpha\beta}]p^\beta + \tilde{\Phi}_\alpha = 0, \tag{2.18a}$$

$$\frac{1}{\sqrt{a}}\partial_\alpha(\sqrt{a}p^\alpha) + \frac{1}{h}[1-2h(1-\sigma')H]p + \tilde{\Phi}_3 = 0, \tag{2.18b}$$

where

$$\tilde{\Phi}_\beta = -2\overset{(\circ)}{\Phi}{}_\beta + \frac{2}{h}\overset{(-)}{P}{}_\beta \tag{2.18c}$$

$$\tilde{\Phi}_3 = -2\overset{(\circ)}{\Phi}{}_3 + \frac{2}{h}\overset{(-)}{P}{}_3 \tag{2.18d}$$

We assume that, on the face surface S, only normal forces act, of the form (i.e.pressure)

$$\overset{(-)}{\boldsymbol{P}} = q\boldsymbol{n} \tag{2.18e}$$

where q is some scalar function of the point at the surface S^-. Then formulae (2.18c,d) take the form

$$\Phi_\beta = -2\overset{(\circ)}{\Phi}{}_\beta, \quad \Phi_3 = -2\overset{(\circ)}{\Phi}{}_3 + \frac{2}{h}q. \tag{2.18f}$$

From the system of equations (2.18a) it is easy to derive the formula

$$p^\alpha = \overset{(+)}{p}{}^\alpha - \overset{(-)}{p}{}^\alpha = -d^{\alpha\beta}(\sigma'\partial_\beta p + \Phi_\beta) \tag{2.19}$$

where

$$d^{\alpha\beta} = \frac{h[a^{\alpha\beta}(1-4hH) + hb^{\alpha\beta}]}{1-2hH+kh^2+4hH(2Hh-1)}, \quad (k = b_1^1 b_2^2 - b_1^2 b_2^1). \tag{2.20}$$

Inserting expressions (2.19) into (2.18b) we obtain the equation

$$\sigma'\nabla_\alpha(d^{\alpha\beta}\nabla_\beta p) - \frac{1}{h}[1-2(1-\sigma')hH]p + \phi = 0, \tag{2.21}$$

where

$$\phi = -\phi_3 + \nabla_\alpha(d^{\alpha\beta}\phi_\beta)$$

$$= -\frac{2}{h}\overset{(-)}{p}{}^3 + \frac{2}{h}\nabla_\alpha(d^{\alpha\beta}\overset{(-)}{p}{}_\beta) + 2\overset{(\circ)}{\phi}{}_\beta - 2\nabla_\alpha(d^{\alpha\beta}\overset{(\circ)}{\phi}{}_\beta). \tag{2.22}$$

If the stresses on S^- have the form (2.18e) then, in view of (2.18f) equality (2.21) takes the form

$$\phi = \frac{2}{h}q + 2\overset{(\circ)}{\phi}{}_3 - 2\nabla_\alpha(\tilde{d}^{\alpha\beta}\overset{(\circ)}{\phi}{}_\beta) \tag{2.22a}$$

In this way, in order to determine the normal component $\overset{(+)}{p}{}^3 = p + \overset{(-)}{p}{}^3$ of the stress force $\overset{(+)}{\boldsymbol{p}}$ applied to the face surface S^+, we have the second-order partial differential equation (2.21). It is easily seen that for thin and shallow shells equation (2.21) is of the elliptic type.

3 Formulation of Boundary Value Problems

We now proceed to equations of equilibrium of a continuous medium with respect to the components of the stress tensor and write them in the vector form (2.3)

$$\frac{1}{\sqrt{g}}\partial_i(\sqrt{g}\boldsymbol{p}^i) + \boldsymbol{\Phi} = 0. \tag{3.1}$$

Since $\boldsymbol{P}^i = P^{ij}\boldsymbol{R}_j$ we may rewrite this equation as

$$\frac{1}{\sqrt{g}}\partial_\alpha(\sqrt{g}p^{\alpha\beta}\boldsymbol{R}_\beta) + \frac{1}{\sqrt{g}}\partial_\alpha(\sqrt{g}p^{\alpha 3}\boldsymbol{n})$$

$$+\frac{1}{\sqrt{g}}\partial_3(\sqrt{g}\boldsymbol{p}^3) + \bar{\boldsymbol{\phi}} = 0,\ (\alpha,\beta = 1,2). \tag{3.2}$$

The vector $\boldsymbol{P}^3$ is previously fixed. Therefore the expression

$$\boldsymbol{X} = \frac{1}{\sqrt{g}}\partial_\alpha(\sqrt{g}p^{\alpha 3}\boldsymbol{n}) + \frac{1}{\sqrt{g}}\partial_3(\sqrt{g}\boldsymbol{P}^3) + \boldsymbol{\Phi}, \quad (p^{\alpha 3} = p^{3\alpha}) \tag{3.3}$$

represents the given vector field in the domain Ω. Consequently, we can write equation (3.2) on every coordinate surface $\hat{S} : x^3 = const$ in the following form:

$$\frac{1}{\sqrt{g}}\partial_\alpha(\sqrt{g}\boldsymbol{T}^\alpha) + \boldsymbol{X} = 0, \quad (on\ \hat{S} : x^3 = const) \tag{3.4}$$

where the formula

$$\boldsymbol{T}^\alpha = p^{\alpha\beta}\boldsymbol{R}_\beta, \quad p^{\alpha\beta} = p^{\beta\alpha}, \tag{3.4a}$$

expressed the desired tangential stress field, which is unknown.

Let us choose arbitrarily the coordinate surface $\hat{S} : x^3 = const$ and consider the following boundary value problems:

Problem A:

$$\frac{1}{\sqrt{g}}\partial_\alpha(\sqrt{g}\boldsymbol{T}^\alpha) + \boldsymbol{X} = 0 \quad (on\ \hat{S} : x^3 = const), \tag{3.5a}$$

$$P_{\ell\ell} \equiv p^{\alpha\beta}\ell_\alpha\ell_\beta = f_1 \ \ (on\ \ d\hat{S}\ \ (\ell_\alpha = \boldsymbol{l}\boldsymbol{r}_\alpha), \tag{3.5b}$$

Problem B:

$$\frac{1}{\sqrt{g}}\partial_\alpha(\sqrt{g}\boldsymbol{T}^\alpha) + \boldsymbol{X} = 0 \quad (on\ \hat{S} : x^3 = const), \tag{3.5a}$$

$$P_{\ell s} \equiv p^{\alpha\beta}\ell_\alpha s_\beta = f_2 \ \ (on\ \ d\hat{S}\ \ (s_\beta = \boldsymbol{s}\boldsymbol{r}_\beta,) \tag{3.5c}$$

where $P_{\ell\ell}$ - expresses the normal stresses and $P_{\ell s}$- the tangential stresses acting on the lateral surface of a shell.

Problem A'_0:

$$\frac{1}{2}(\hat{\nabla}_\alpha V_\beta + \hat{\nabla}_\beta V_\alpha) - \hat{b}_{\alpha\beta}V_3 = 0 \ \ (\hat{S} : x^3 = const) \tag{3.6a}$$

$$V_{(S)} \equiv V^\alpha S_\alpha = 0 \quad (on\ \ \partial\hat{S}). \tag{3.6b}$$

Problem B'_0:

$$\frac{1}{2}(\hat{\nabla}_\alpha V_\beta + \hat{\nabla}_\beta V_\alpha) - \hat{b}_{\alpha\beta}V_3 = 0 \ \ (\hat{S} : x^3 = const) \tag{3.6a}$$

$$V_{(\ell)} \equiv V^\alpha \ell_\alpha = 0 \quad (on\ \ \partial\hat{S}). \tag{3.6c}$$

where boundary $D\hat{S}$ of the surface $\hat{S} : x^3 = const$, are prescribed for any coordinate surface. In other words, f_1 and f_2 are given functions of the point of the lateral surface of a shell. Moreover, by A_0 and B_0, we denote the homogeneous boundary value problems corresponding to Problems A and B:

Problem A_0:

$$\frac{1}{\sqrt{g}}\partial_\alpha(\sqrt{g}\boldsymbol{T}^\alpha) = 0 \quad (in\ \hat{S} : x^3 = const), \tag{3.6d}$$

$$P_{\ell\ell} = p^{\alpha\beta}\ell_\alpha\ell_\beta = 0 \ \ (on\ \partial\hat{S}). \tag{3.6e}$$

Problem B_0:

$$\frac{1}{\sqrt{g}}\partial_\alpha(\sqrt{g}\boldsymbol{T}^\alpha) = 0 \quad (in\ \hat{S} : x^3 = const), \tag{3.6f}$$

$$P_{\ell s} = p^{\alpha\beta}\ell_\alpha s_\beta = 0 \ \ (on\ d\hat{S}). \tag{3.6g}$$

It will be shown below that A_0 and A_0' as well as B_0 and B_0' are mutually adjoint homogeneous boundary value problems. Then from formula

$$\iint\limits_{\hat{S}} \boldsymbol{XV} d\hat{S} + \int\limits_{\partial\hat{S}} (P_{(\ell\ell)}V_{(\ell)} + P_{(\ell s)}V_{(s)})ds = 0 \tag{3.7a}$$

it immediately follows:

1) For the solvability of Problem $\boldsymbol{A}$, i.e. problem (3.5a,b), it is necessary that the following condition

$$\iint\limits_{\hat{S}} \boldsymbol{XV} d\hat{S} + \int\limits_{\partial\hat{S}} f_1 V_{(\ell)} ds = 0 \quad (on\ \hat{S} : x^3 = const) \tag{3.7b}$$

be satisfied, where the vector $\boldsymbol{V}$ is any solution of the homogeneous problem A_0', i.e problem (3.6a,b)

2) For the solvability of problem B, i.e. problem (3.6a,c) it is necessary that the condition

$$\iint\limits_{\hat{S}} \boldsymbol{xv} d\hat{S} + \int\limits_{\partial\hat{S}} f_2 V_{(S)} ds = 0 \quad (on\ \hat{S} : x^3 = const) \tag{3.7c}$$

be satisfied, where the vector $\boldsymbol{V}$ is any solution of the homogeneous problem B_0'.

Conditions $(3.7a.b)$ are to be fulfilled for any coordinate surfaces $x^3 = const$

Thus, problem A_0' (3.6a,b) (respectively, B_0' (3.6ac) is the adjoint homogeneous boundary value problem with respect to the boundary value problem A : (3.5a,b) (respectively, B_0' (3.6a,c) is the adjoint homogeneous boundary value problem with respect to the boundary value problem A : (3.5a,b) (respectively, B : (3.5a,c)). For convex shells the boundary value problems formulated are reduced to boundary value problems for generalized analytic functions.

4 Isometric-Conjugate Coordinates

In the following we shall deal only with convex shells. Then the coordinate surfaces $\hat{S} : X^3 = const$ are convex and we may take on them, for Gaussian parameters, the isometric-conjugate coordinates x, y, with respect to which the second fundamental quadratic form is

$$II = k_s d\hat{S}^2 = \hat{\Lambda}(dx^2 + dy^2) = \hat{\Lambda} dz d\bar{z}, \quad (z = x + iy, \ \ \bar{z} = x - iy) \tag{4.1}$$

where

$$\hat{\Lambda} = \hat{b}_{11}\hat{b}_{22} = \sqrt{g\hat{k}}, \ \ \hat{b}_{12}\hat{b}_{21} = 0. \tag{4.2}$$

Since $g = a\theta^2$ and $\hat{K} = \theta^{-1}K$, where K is the principal curvature of the surface s, a is the discriminant of its metric quadratic form and $\vartheta = (1 - k_1x_3)(1 - k_2x_3)$ the first equality in (4.2) may be written as

$$\hat{\Lambda} = \vartheta^{\frac{1}{2}}\Lambda = \sqrt{\vartheta a k}, \tag{4.3}$$

where $\Lambda = \sqrt{aK}$ is the coefficient of the second fundamental quadratic form of the base surface $S : x^3 = 0$ with respect to the isometric-conjugate coordinates.

For convex shells of class TS (thin and shallow), in view of approximate equalities

$$1 - k_1x_3 \cong 1, \quad 1 - k_2x_3 \cong 1.$$

Then on every coordinate surface $\hat{S} : x^3 = const$ the second fundamental quadratic form $\hat{II}$ may be identified with the corresponding quadratic form II of the surface S, i.e. we may put

$$\hat{II} \cong II = \Lambda(dx^2 + dy^2) = \Lambda dz d\bar{z},$$

$$\Lambda = \sqrt{a\boldsymbol{K}} \ \ (z = x + iy). \tag{4.1}$$

Thus, In the case of shells the isometric-conjugate coordinates of any coordinate surface $\hat{S} : x^3 = const$ may be identified with the corresponding coordinates of the surface S, representing the parametrization base of Ω.

5 Reduction of Tatically Definable Problems of the Tangential Stress Field to the Boundary Value Problems for Generalized Analytic Functions

Let $\hat{G}$ be a domain of the complex plane $z = x + iy$ onto which the coordinate surface $\hat{S} : x^3 = const$ is topologically mappped by means of the corresponding isometric-conjugate coordinates x and y. The value problems A and B: (3.5 a, b, c) and also A'_0 B'_0: (3.6 a, b, c) may be formulated, respectively, as follows:

Problem $\hat{A}$:

$$\partial_{\bar{z}}\omega' - \bar{\hat{B}}\bar{\omega}' = \hat{F} \qquad (in\ \hat{G}), \tag{5.1a}$$

$$Re\Big[\omega'\Big(\frac{dz}{ds}\Big)^2\Big] = -\hat{K}^{1/4}f_1 - \frac{1}{2}\hat{k}_s\hat{K}^{3/4}X_3 \quad (on\ \partial\hat{G}) \tag{5.1b}$$

Problem $\hat{B}$:

$$\partial_{\bar{z}} w' - \bar{\hat{B}}\bar{w}' = \hat{F} \qquad (in\ \hat{G}) \tag{5.1c}$$

$$Re\Big[w'\frac{dz}{d\ell}\frac{dz}{ds}\Big] = \hat{K}^{1/4}f_2 + \frac{1}{2}\hat{\tau}_s\hat{K}^{-3/4}X_3 \qquad (on\ \partial\hat{G}) \tag{5.1d}$$

Problem $\tilde{A}_0'$:

$$\partial_{\bar{z}} w + \hat{B}\bar{w} = 0 \qquad (in\ \hat{G}) \tag{5.2a}$$

$$Re\Big[w\frac{d\bar{z}}{ds}\Big] = 0 \qquad (on\ \partial\hat{G}) \tag{5.2b}$$

Problem $\tilde{B}_0'$:

$$\partial_{\bar{z}} w + \hat{B}\bar{w} = 0 \qquad (in\ \hat{G}) \tag{5.2c}$$

$$Re\Big[w\frac{d\bar{z}}{d\ell}\Big] = 0 \qquad (on\ \partial\hat{G}) \tag{5.2d}$$

where $\partial\hat{G}$ denotes the boundary of the $\hat{G}$. It will be assumed that $\partial\hat{G}$ as the topological image of $d\hat{S}$.

Let us also consider the homogeneous problems, corresponding to problems $\hat{A}$ and $\hat{B}$

Problem $\hat{A}_0$:

$$\partial_{\bar{z}} w' - \bar{\hat{B}}\bar{w}' = 0 \qquad (in\ \hat{G}) \tag{5.2e}$$

$$Re\Big[w'\Big(\frac{dz}{ds}\Big)^2\Big] = 0 \qquad (on\ \partial\hat{G}) \tag{5.2f}$$

Problem $\hat{B}_0$:

$$\partial_{\bar{z}} w' - \bar{\hat{B}}\bar{w}' = 0 \qquad (in\ \hat{G}) \tag{5.2g}$$

$$Re\Big[w'\frac{dz}{d\ell}\frac{dz}{ds}\Big] = 0 \qquad (on\ \partial\hat{G}) \tag{5.2h}$$

Using the general theorems of the theory of generalized analytic functions we can derive the following necessary and sufficient conditions for the solvability of boundary value problems $\hat{A}$ and $\hat{B}$ (see [2])

$$\frac{1}{2i}\int\limits_{\partial\hat{g}} \frac{\sqrt{g\hat{K}}}{\hat{k}_s}[K^{\hat{1}/4}f_1 + \frac{1}{2}\hat{k}_s\hat{K}^{-3/4}\hat{X}^3]\tilde{w}d\bar{z} + Re\iint\limits_{\hat{G}}\tilde{\omega}\hat{F}d\hat{G} = 0, \tag{5.3a}$$

$$\frac{1}{2i}\int\limits_{\partial\hat{g}} \frac{\sqrt{g\hat{k}}}{\hat{k}_\ell}[K^{\hat{1}/4}f_2 + \frac{1}{2}\hat{\tau}_s\hat{K}^{-3/4}\hat{X}^3]\tilde{w}\frac{d\bar{z}}{d\ell}ds - Re\iint\limits_{\hat{G}}\tilde{w}\hat{F}d\hat{G} = 0. \tag{5.3b}$$

In equality (5.3a) (respectively (5.3b)) $\hat{w}$ is an arbitrary solution of the boundary value problems $\hat{A}_0'$: $(5.2a, b)$ (respectively problems $\hat{B}_0'$: $(5.2c, d)$).

Acknowledgement

The designated project has been fulfilled by a financial support of Shota Rustaveli National Science Foundation (Grant SRNSF/FR/358/5-109/14). Any idea in this publication is possessed by the author and may not represent the opinion of Shota Rustaveli National Science Foundation itself.

References

1. Vekua I.N. *Shell Theory: General Methods of onstruction*, Pitman Advanced Publishing Program, Boston-London-Melbourne (1985).

2. Vekua I.N. *Generalized Analitic Functions.* Mockow, 1959, Fizmatgiz.